Dr. Z's Math251 Handout #12.2 [Vectors]

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Problem Type 12.2a: A person walks due west (or east) on the deck of a ship at a mi/h. This ship is moving north (or south, or whatever) at b mi/h. Find the speed and direction of the person relative to the surface of the water.

Example Problem 12.2a: A girl walks due east on the deck of a ship at 5 mi/h. This ship is moving south at 12 mi/h. Find the speed and direction of the girl relative to the surface of the water.

Steps

1. Find the velocity vectors of the person and the ship. Add them up to get the composite velocity.

(East is in the direction of the **positive** x axis so it is a postive multiple of the vector $\mathbf{i}$ (or $\langle 1, 0\rangle$) West is in the direction of the **negative** x axis so it is a postive multiple of the vector -$\mathbf{i}$ (or $\langle -1, 0\rangle$) North is in the direction of the **positive** y axis so it is a postive multiple of the vector $\mathbf{j}$ (or $\langle 0, 1\rangle$) South is in the direction of the **negative** y axis so it is a postive multiple of the vector -$\mathbf{j}$ (or $\langle 0, -1\rangle$)).

2. If the composite velocity vector is $a\mathbf{i} + b\mathbf{j}$ (or $\langle a, b\rangle$), then the **speed** is the **magnitude**, which is $\sqrt{a^2 + b^2}$, and the direction is at angle $\theta = \tan^{-1} b/a$ with the positive x-axis (alias East).

Example

1. The velocity vector of the girl is $5\mathbf{i}$, and the velocity vector of the ship is $-12\mathbf{j}$. So the velocity vector relative to the water is the sum $\mathbf{5i - 12j}$ (or $\langle 5, -12\rangle$).

2. speed= $|5\mathbf{i} - \mathbf{12j}| = \sqrt{\mathbf{5^2 + 12^2}} = \mathbf{13}$ and the direction is $\theta = \tan^{-1}(-12/5) = -\tan^{-1} 12/5$.