Dr. Z.'s Math 251 Basic Skills and Math Etiquette Worksheet

Due: Dec. 11, 2006. (As one of the conditions for the "deal").

1. Trivial arithmetical simplifications

example: 1+1 should be replaced by 2

3-3 should be replaced by 0.

5/5 should be replaced by 1.

162/20 should be replaced by 81/10.

Do right now!:

Simplify as much as you can:

$300/62 =$,

$300/33 =$

$11/11 =$

$\sqrt{9} =$

$\sqrt{25} =$

$\sqrt{16} - (10/2)(\sqrt{4}) + (60/10) =$

Warning: if I will see something like 202/62 unsimplified, I'll take-off points.

2. Absolute value:

$|a| = a$ if a is positive, $|-a|$=a if a is negative.

Do right now!: Simplify the following numerical expressions featuring the absolute-value sign:

$|11| =$

$|-6| =$

$|9| + |-11| =$

$|11 - 7| - |6 - 10| + |22/11| - |-55/11|$.

Warning: if I see as final answer something like $\ln|7|$ (instead of $\ln 7$), I'll take-off points.

3. Simple values of $\sin x, \cos x, e^x, \ln x$.

Recall that: $\sin 0 = 0$, $\sin \pi/2 = 1$, $\cos 0 = 1$, $\cos \pi/2 = 0$, $\ln 1 = 0$, $\ln e = 1$, $e^0 = 1$, $e^1 = e$.

Do right now!: Simplify the following numerical expressions:

$\ln(\cos 0) =$

$e^{\sin 0} =$

$\ln e^{\cos 0} =$

$\ln(\sin(\pi/2)) + \cos^3(-\pi/2) + e^{\cos 0} - e^{\sin 0} =$

4. Simple numerical expressions with square-roots

Recall that $x^{1/2} = \sqrt{x}, x^{-1/2} = 1/\sqrt{x}$,

Example: how to simplify $4^{-3/2}$?

$$4^{-3/2} = \frac{1}{4^{3/2}} = \frac{1}{(4^{1/2})^3} = \frac{1}{(\sqrt{4})^3} = \frac{1}{(2)^3} = \frac{1}{8} \quad .$$

Do right now!:

$9^{1/2} =$

$27^{2/3} =$

$16^{-1/2} =$

$4^{3/2} =$

5. **Simple algebra**: Recall the famous formula: $a^2 - b^2 = (a - b)(a + b)$.

Do right now!: Factorize

$x^2 - y^2 =$

$x^2 - 9y^2 =$

$36x^2 - 25y^2 =$

$x^4 - y^4 =$

Quick review of differentiation:

Basic differentiations that you should memorize:

$$(x^n)' = nx^{n-1} \quad , \quad (\sin x)' = \cos x \quad , \quad (\cos x)' = -\sin x$$

$$, \quad (e^x)' = e^x \quad , \quad (\ln x)' = \frac{1}{x} \quad .$$

6.1 The product rule: $(fg)' = f'g + fg'$.

Example:

$$(x^2 \sin x)' = (x^2)'(\sin x) + (x^2)(\sin x)' = (2x)\sin x + (x^2)(\cos x) = 2x \sin x + x^2(\cos x) \quad .$$

Do right now:

$(x^5 \sin x)' =$

$(e^x \sin x)' =$

$(x^5 e^x)' =$

$((\ln x)(\sin x)(e^x))' =$

6.2 The Chain Rule:

$$[f(g(x))]' = f'(g(x)) \cdot g'(x) \quad ,$$

Or, better-put:

$$[OUTSIDE(INSIDE)]' = OUTSIDE'(INSIDE) \cdot INSIDE' \quad ,$$

Examples:

$$(e^{x^3})' = (e^{x^3})(x^3)' = (e^{x^3})(3x^2) = 3x^2 e^{x^3} \quad ,$$

$$(\sin(e^x))' = \cos(e^x) \cdot (e^x)' = \cos(e^x) \cdot (e^x) = e^x \cos(e^x) \quad .$$

Do right now:

$(e^{\sin 5x})' =$

$(e^{x^5})' =$

$(\ln(x^6 + 9))' =$

$(\cos(2x^2 + 5cx))' =$

6.3 Combination of The Chain Rule and Product Rule

Example:

$$(x\sin(x^4))' = x'\sin(x^4) + x(\sin(x^4))' = \sin(x^4) + x((\cos(x^4))(x^4)') =$$

$$\sin(x^4) + x((\cos(x^4))(4x^3) = \sin(x^4) + 4x^4\cos(x^4) \quad .$$

Do Right Now:

$(e^x \sin^3 x)' =$

$(e^{x^2} \sin(x^3))' =$

$(e^{x^5+6} \cos(x^3+5))' =$

$(x^5 e^{\sin(2x^3)})' =$

7. Quick review of integration:

7.1: Substitution

Recall that

$$\int e^u du = e^u + C \quad ,$$

$$\int \sin u du = \cos u + C \quad ,$$

$$\int \cos u du = \sin u + C \quad ,$$

$$\int \frac{du}{u} = \ln|u| + C \quad .$$

Often we can do a **short-cut substitution** by expressing an integral as a "composition".

If u is a function of x, then $du = u'(x)\,dx$.

Examples:

$$\int e^{x^3}\, 3x^2\, dx = \int e^{x^3}\, d(x^3)\, dx = e^{x^3} + C$$

$$\int \sin^2 \cos x\, dx = \int \sin^2 x\, d(\sin x) = (1/3)\sin^3 x + C \quad .$$

$$\int \cos^2 \sin x\,dx = \int \cos^2 x\,d(-\cos x) = -\int \cos^2 x\,d(\cos x) = -(1/3)\cos^3 x + C \quad .$$

Sometimes we have to make minor adjustments

$$\int e^{x^3} x^2\,dx = (1/3)\int e^{x^3} 3x^2\,dx = (1/3)\int e^{x^3} d(x^3)\,dx = (1/3)e^{x^3} + C$$

(here x^2 is 'not quite' $(x^3)'$ so we stick a $(1/3)$ in front and replace the x^2 by $3x^2$.

Do right now!:

$\int \sin(e^x)\, e^x\,dx =$

$\int \sin(x^5)\, x^4\,dx =$

$\int e^{\sin x} \cos x\,dx =$

$\int \frac{x^2}{x^3+11}\,dx =$

$\int \tan x\,dx =$

(Hint: $\tan x = \sin x / \cos x$)

$\int \frac{x^5}{x^6+100}\,dx =$

$\int 3x^4\, e^{x^5}\,dx =$

$\int \frac{3(\ln x)^2}{x} e^{(\ln x)^3}\,dx =$

7.2 Integration by Parts:

$$\int u dv = uv - \int v du \quad .$$

Example: $\int x e^x\,dx = \int x\,d(e^x) = xe^x - \int e^x\,dx = xe^x - x + C$

Do right now:

$\int x \sin x\,dx =$

$\int x \cos x\,dx =$

7.3 powers of trig-functions

$\int \sin^{odd} x\,dx$ and $\int \cos^{odd} x\,dx$

are handled via **substitution**, e.g.

$$\int \sin^3 x\,dx = \int (\sin^2 x)\,(\sin x)\,dx = -\int (\sin^2 x)\,d(\cos x)$$

$$= -\int (1-\cos^2 x)\, d(\cos x) = -\int d(\cos x) + \int (\cos^2 x) d(\cos x) = -\cos x + (\cos^3 x)/3 + C \quad .$$

$\int \sin^{even} x\, dx$ and $\int \cos^{even} x\, dx$ are handled via **trig identities**

$$\cos^2 x = \frac{1+\cos 2x}{2} \quad , \quad \sin^2 x = \frac{1-\cos 2x}{2} \quad ,$$

and then taking advantage of $\int \sin(ax)\, dx = -(\cos ax)/a + C$, $\int \cos(ax)\, dx = \sin(ax)/a + C$.

Example:

$$\int \cos^2 x\, dx = \int \frac{1+\cos 2x}{2}\, dx = \int \frac{1}{2}\, dx + \int \frac{\cos 2x}{2}\, dx = \frac{x}{2} + \frac{\sin 2x}{4} + C \quad .$$

Do right now:

$\int \sin^2 x\, dx =$

$\int \sin^5 x\, dx =$

$\int \cos^4 x\, dx =$

(Hint: write $\cos^4 x = (\cos^2 x)^2$ and use the trig-identity $\cos^2 w = (1+\cos 2w)/2$ twice, first with $w = x$, and after using algebra and expanding, with $w = 2x$)

$\int \cos^5 x\, dx =$