My
wonderful
Experience
A 2 ect

SLIDE 1

A:

A question about pairs of lines in 3D projective space

Asked 7 years, 2 months ago Active 7 years, 2 months ago Viewed 555 times

6

3

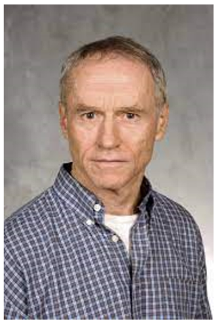
Consider a 3-dimensional projective space X.

Let m be the smallest number so that there are m pairs of lines $\ell_1, \ell_1', \ell_2, \ell_2', \dots, \ell_m, \ell_m'$ in X:

a) For every $i = 1, 2, \dots, m$, $\ell_i \cap \ell_i' = \emptyset$.

b) For every $i, j \le m$, $i \ne j$, $\ell_i \cap \ell_j' \ne \emptyset$.

(If there is no upper bound on m we let $m = \infty$.)



Algebra, Amitai, Amitsur, Avinoam

Questions:

> **a)** Is it always the case that $m = 6$?
>
> **b)** Is it (at least) true that either $m = 6$ or $m = \infty$?
>
> **c)** What is the answer for the projective space over the Quaternions?

- over commutative field. (Pappusian PS) $m \le 6$. Exterior algebra Lovasz

$V_1 \dots V_m \in \mathbb{R}^4$

$U_1 \dots U_m$

$$U_i \cap V_j = \begin{cases} \{0\} & i = j \\ \ne \{0\} & i \ne j \end{cases}$$

$U \longrightarrow f_U \in \Lambda^2 \mathbb{R}^4$ $f_{V_1} \dots f_{V_m}$ $f_{U_1} \dots f_{U_m}$

$f_{V_i} \wedge f_{U_i} \ne 0$

$f_{V_i} \wedge f_{U_j} = 0$ $i \ne j$

$\Longrightarrow f_{V_1} \dots f_{V_m}$ linearly independent

Several conversations with Amitzur

- Answer by David Speyer:

Now bound over general division rings

7,8 ~~over~~ for quaternions

things I learned from A:

Rings with PI

Role of Cayley Hamilton thm

Move from: $(\mathbb{Z}_2^n)$

to S_n

$(n-k)! \to$ stirling # ← my first paper $n-k \to n-k+p$ SLIDE 2

Z

Bijective proofs for Abel Sum identities

$$\sum_{k=0}^{n} \binom{n}{k} n^k (n-k)! = \sum_{k=0}^{n} \binom{n}{k} k^k (n-k)^{n-k}$$

1978
1993
Z60
ZGL
ZPL

Count (f, A) $f : [n] \to [n]$
$f(A) = A$

82 81
(Sister Celine's method)

Count (f, B) $f : [n] \to [n]$
$f(B) \subseteq B$
$f([n] \setminus B) \subseteq ([n] \setminus B)$

For every f # A's = # B's

· post card.

Weighted enumeration of HD trees

$$\Sigma\ |H_{d-1}(K, Z)|^2 = n^{\binom{n-2}{d}}$$

The sum is taken over all d-dimensional Q-acyclic simplicial complexes with n vertices with complete (k-1)-dimensional skeletons.

SLIDE 3

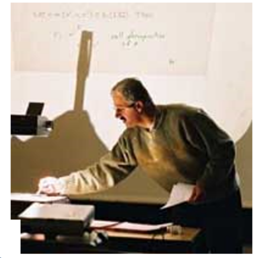
SLIDE 4

In EC Can you gain mileage by adding weights?

SLIDE 5

Recent interviews by Taofik Mansour

Toufik



Enumerative Combinatorics and Applications ecajournal.haifa.ac.il ECA 1:2 (2021) Interview #S3I6

Interview with Amitai Regev

Toufik Mansour

Amitai Regev is the Herman P. Taubman Professor of mathematics at the Weizmann Institute of Science. He received his doctorate from the Hebrew University of Jerusalem in 1972, under the direction of Shimshon Amitsur. Regev has made significant contributions to the theory of polynomial identity rings. He developed the so-called "Regev theory" that connects polynomial identity rings to representations of the symmetric group, and hence to Young tableaux. He has made seminal contributions to the asymptotic enumeration of Young tableaux and tableaux of thick hook shape, and together with William Beckner proved the Macdonald-Selberg conjecture for the infinite Lie algebras of type B, C, and D.

Mansour: Professor Regev, first of all, we would like to thank you for accepting this interview. What do you think about the development of the relations between combinatorics and the rest of mathematics?

Regev: Often there is an algebraic structure with a corresponding combinatorial structure, which is induced by the corresponding algebra

Regev: For my Ph.D.[1] research (under Amitsur) I chose the $A\otimes B$ problem, which was then open. At an early stage of that research, I realized I should determine the case $G\otimes G$ where G is the Grassmann algebra[2]. After that, plus lots of work, the general case – followed. I arrive in combinatorics later in my mathematical career, so I feel lack the overview.

Enumerative Combinatorics and Applications ecajournal.haifa.ac.il ECA 1:1 (2021) Interview #S3I3

Interview with Doron Zeilberger

Toufik Mansour

Doron Zeilberger received a B.Sc. in mathematics from the University of London in 1972, and a Ph.D. in mathematics from the Weizmann Institute of Science in 1976, under the supervision of Harry Dym. Professor Zeilberger has important contributions to the fields of hypergeometric summation and q-Series and was the first to prove the alternating sign matrix conjecture. He is considered a champion of using computers and algorithms to do mathematics quickly and efficiently, and his results have been used extensively in modern computer algebra software. Professor Zeilberger's distinctions include the Lester R. Ford Award in 1990, Leroy P. Steele Prize for Seminal Contributions to Research in 1998 for the development of WZ theory with Herbert Wilf, and the Euler Medal in 2004. In 2016 he received, together with Manuel Kauers and Christoph Koutschan, the David P. Robbins Prize of the American Mathematical Society. Professor Zeilberger was a member of the inaugural 2013 class of fellows of the American Mathematical Society.

Mansour: Professor Zeilberger, first of all we would like to thank you for accepting this interview. Would you tell us broadly what combinatorics is?

Zeilberger: Combinatorics is **everything**. All our worlds, the physical, mathematical, and even spiritual, are inherently **finite** and **discrete**, and so-called *infinities*, be their *actual* or *po-*

goals of your research?

Zeilberger: Since, very soon, pure human-generated mathematics will be done better and faster by computers, I dedicate my research to teaching computers to do mathematics, that in my case is mostly combinatorics and number theory. In fifty years, computers will not need us, but until then, it is fun to act as "coaches".

SLIDE 6

ecajournal.haifa.ac.il Enumerative Combinatorics and Applications *ECA* **2:2** (2022) Interview #S3I7

Interview with Gil Kalai

Toufik Mansour

Photo by Muli Safra

Gil Kalai received his Ph.D. at the Einstein Institute of the Hebrew University in 1983, under the supervision of Micha A. Perles. After a postdoctoral position at the Massachusetts Institute of Technology (MIT), he joined the Hebrew University in 1985, (Professor Emeritus since 2018) and he holds the Henry and Manya Noskwith Chair. Since 2018, Kalai is a Professor of Computer Science at the Efi Arazi School of Computer Science in IDC, Herzliya. Since 2004, he has also been an Adjunct Professor at the Departments of Mathematics and Computer Science, Yale University. He has held visiting positions at MIT, Cornell, the Institute of Advanced Studies in Princeton, the Royal Institute of Technology in Stockholm, and in the research centers of IBM and

Mansour: What advice would you give to young people thinking about pursuing a research career in mathematics?

Kalai:

a) Learn to use, to master, and to enjoy computer programming;

b) Learn to use, to master, and to enjoy the English language. Here one can even add

c) Learn touch typing.

Actually, these are pieces of advice that I would also give myself at present, and I hope they might be suitable for the elderly as well.

SLIDE 7

3B1

Flag number inequalities

(Fantasy) Bounds on diameter

(Fantasy) EC for lattices of p-subgroups A.

Theorem 4.3 *Every 2-simplicial d-polytope ($d \geq 7$) has a 3-face with less than 7 vertices.*

Proof: Again it suffices to prove the theorem for 7-polytopes. Assume that every 3-face of a 2-simplicial 7-polytope has 7 or more vertices (inequality $f_0^3 - 7 \geq 0$ in the bottom interval $[-1, 3]$) and that every 2-face is triangular (inequality $3 - f_0^2 \geq 0$ in the interval $[-1, 2]$). Note that $g_1^2 = f_0^2 - 3 \geq 0$ and therefore $f_0^2 = 3$. Consider the following 15 inequalities for 7-polytopes obtained by convolutions of the g-numbers, their duals and the added inequalities. The theorem follows again from the infeasibility of this system of linear inequalities.

$$
\begin{array}{rccl}
[1] & (3 - f_0) * g_0^0 * g_1^2 * g_0^0 & = & 18 f_{03} - 36 f_3 + 18 f_{24} + 3 f_{035} - 6 f_{35} - 6 f_{13} \\
 & & & - 6 f_{024} - f_{135} \geq 0 \\
[2] & (f_0 - 7) * g_1^2 * g_0^0 & = & - 6 f_{03} - f_{035} + 3 f_{024} + 42 f_3 + 7 f_{35} - 21 f_{24} \\
 & & & + 15 f_{14} - 15 f_{04} \geq 0 \\
[3] & g_0^1 * g_1^4 * g_0^0 & = & - f_{024} + f_{025} - 10 f_1 - 3 f_{13} + 5 f_{14} - 5 f_{15} \\
 & & & + 5 f_{02} \geq 0 \\
[4] & g_0^1 * g_2^4 * g_0^0 & = & 3 f_{024} - f_{025} + 20 f_1 + 4 f_{13} - 10 f_{14} + 4 f_{15} \\
 & & & - 10 f_{02} \geq 0 \\
[5] & g_1^6 * g_0^0 & = & - 14 + 7 f_0 + 7 f_2 - 7 f_3 + 7 f_4 - 7 f_5 - f_{02} \\
 & & & + f_{03} - f_{04} + f_{05} - 5 f_1 \geq 0 \\
[6] & (3 - f_0) * g_1^2 * g_0^1 & = & 6 f_{35} - 3 f_{035} + f_{135} - 9 f_{25} + 3 f_{025} \geq 0 \\
[7] & (f_0 - 7) * g_0^0 * g_1^2 & = & - 3 f_{024} + 2 f_{035} + 15 f_{04} - 15 f_{14} + 21 f_{24} \\
 & & & - 14 f_{35} \geq 0 \\
[8] & (3 - f_0) * g_0^1 * g_1^2 & = & f_{135} - 3 f_{035} + 6 f_{35} - 9 f_{24} + 3 f_{024} \geq 0 \\
[9] & g_0^1 * g_1^2 * g_1^2 & = & - 3 f_{024} - 6 f_{15} - f_{135} + 3 f_{025} + 9 f_{14} \geq 0 \\
[10] & g_0^4 * g_1^2 & = & 2 f_5 - f_{05} + f_{15} - f_{25} + f_{35} - 3 f_4 \geq 0 \\
[11] & (f_0 - 7) * g_0^3 & = & f_{03} - 7 f_3 \geq 0 \\
[12] & g_0^2 * g_2^4 & = & - 8 f_3 + 4 f_{03} - 4 f_{13} + 2 f_{35} - f_{035} + f_{135} \\
 & & & + f_{24} - 3 f_{25} + 10 f_2 \geq 0 \\
[13] & g_1^2 * g_2^4 & = & 24 f_3 - 12 f_{03} - 6 f_{35} + 3 f_{035} + 4 f_{13} - f_{135} \\
 & & & + f_{024} - 3 f_{025} + 10 f_{02} - 3 f_{24} + 9 f_{25} \\
 & & & - 30 f_2 \geq 0 \\
[14] & (3 - f_0) * g_0^4 & = & - f_{02} + 3 f_2 \geq 0 \\
[15] & g_0^0 * \overline{g_1}^6 & = & - f_{02} + f_{03} - f_{04} + f_{05} + 2 f_1 - 7 f_0 \geq 0
\end{array}
$$

Every 9-polytope has a 3-face with at most 77 2-faces.

$$
\begin{array}{lcl}
[1] \quad g_0^1 * g_1^2 * g_0^0 * g_1^2 * g_0^0 & = & 6 f_{0246} - 18 f_{025} + 6 f_{135} + 36 f_{15} - 18 f_{146} - 3 f_{0257} + f_{1357} \\
 & & +6 f_{157} \\
[2] \quad g_1^2 * g_1^2 * g_1^2 * g_0^0 & = & -54 f_{25} - 9 f_{257} - 54 f_{36} + 27 f_{036} + 54 f_{26} - 18 f_{035} + 36 f_{35} \\
 & & +9 f_{246} - 3 f_{0357} + 6 f_{357} + 18 f_{025} + 3 f_{0257} - 9 f_{136} \\
 & & -18 f_{026} + 6 f_{135} - 3 f_{0246} + f_{1357} \\
[3] \quad g_0^0 * g_2^4 * g_1^2 * g_0^0 & = & -60 f_{05} - 10 f_{057} - 60 f_{16} - 30 f_{036} + 60 f_{06} + 48 f_{15} + 8 f_{157} \\
 & & -12 f_{135} - 2 f_{1357} + 18 f_{035} + 3 f_{0357} + 12 f_{046} - 6 f_{146} \\
 & & -6 f_{025} - f_{0257} + 18 f_{136} + 12 f_{026} \\
[4] \quad g_0^0 * g_1^2 * g_1^4 * g_0^0 & = & -6 f_{146} + 6 f_{046} + 6 f_{147} - 6 f_{047} + 30 f_{03} + 9 f_{035} - 15 f_{036} \\
 & & +15 f_{037} + 30 f_{14} - 30 f_{04} + f_{0246} - f_{0247} - 20 f_{13} - 5 f_{024} \\
 & & -6 f_{135} + 10 f_{136} - 10 f_{137} \\
[5] \quad g_0^0 * g_1^2 * \overline{g_1}^4 * g_0^0 & = & 30 f_{03} + 15 f_{035} - 15 f_{036} + 9 f_{037} + 30 f_{14} - 30 f_{04} - 20 f_{13} \\
 & & -5 f_{024} - 10 f_{135} + 10 f_{136} - 6 f_{137} \\
[6] \quad g_0^3 * g_2^4 * g_0^0 & = & 3 f_{246} - 3 f_{146} + 3 f_{046} - f_{247} + f_{147} - f_{047} + 20 f_3 + 4 f_{35} \\
 & & -10 f_{36} + 4 f_{37} - 10 f_{24} + 10 f_{14} - 10 f_{04} \\
[7] \quad (f_0 - 78) * g_2^4 * g_0^0 & = & 3 f_{0246} - f_{0247} + 20 f_{03} + 4 f_{035} - 10 f_{036} + 4 f_{037} - 10 f_{024} \\
 & & -234 f_{246} + 228 f_{146} - 228 f_{046} + 78 f_{247} - 76 f_{147} + 76 f_{047} \\
 & & -1560 f_3 - 312 f_{35} + 780 f_{36} - 312 f_{37} + 780 f_{24} - 760 f_{14} \\
 & & +760 f_{04} \\
[8] \quad (f_2 - 78) * g_1^4 * g_0^0 & = & -10 f_{13} + 10 f_{03} - 3 f_{135} + 3 f_{035} + 5 f_{136} - 5 f_{036} - 5 f_{137} \\
 & & +5 f_{037} + 76 f_{246} - 78 f_{146} + 78 f_{046} - 76 f_{247} + 78 f_{147} \\
 & & -78 f_{047} + 760 f_3 + 228 f_{35} - 380 f_{36} + 380 f_{37} - 380 f_{24} \\
 & & +390 f_{14} - 390 f_{04} \\
[9] \quad (f_2 - 78) * \overline{g_1}^4 * g_0^0 & = & -10 f_{13} + 10 f_{03} - 5 f_{135} + 5 f_{035} + 5 f_{136} - 5 f_{036} - 3 f_{137} \\
 & & +3 f_{037} + 760 f_3 + 380 f_{35} - 380 f_{36} + 228 f_{37} - 380 f_{24} \\
 & & +390 f_{14} - 390 f_{04}
\end{array}
$$

Bernan Venet

Happy Birthday

Amitai and Dr. Z