

# **A HANDBOOK OF INTEGER SEQUENCES**

**N. J. A. Sloane**

*Mathematics Research Center  
Bell Telephone Laboratories, Inc.  
Murray Hill, New Jersey*



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*To John Riordan*

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**PREFACE**

In spite of the large number of existing mathematical tables, until now there has been no table of sequences of integers. Thus someone coming across the sequence 1, 2, 5, 15, 52, 203, 877, 4140, . . . would have had difficulty in finding out that these are the Bell numbers, and that they have been extensively studied. This handbook remedies this situation. The main table contains a list of some 2300 sequences of integers, collected from all branches of mathematics and science. The sequences are arranged in numerical order, and for each one a brief description and a reference is given.

The first part of the book describes how to use the table, gives methods for analyzing unknown sequences, and contains an illustrated description of the most important sequences.

Who will use this handbook? Anyone who has ever been confronted with a strange sequence, whether in an intelligence test in high school, e.g.,

1, 8, 11, 69, 88, 96, 101, 111, 181, 609, . . .

(guess!<sup>1)</sup>, or in solving a mathematical problem, e.g.,

1, 2, 5, 14, 42, 132, 429, 1430, . . .

(the Catalan numbers), or from a counting problem, e.g.,

1, 1, 2, 4, 9, 20, 48, 115, 286, 719, . . .

(the number of rooted trees with  $n$  points), or in physics, e.g.,

1, 0, 3, 22, 192, 2046, 24853, . . .

<sup>1</sup>For many more terms and the explanation, see the main table.

coefficients of the partition function for a cubic lattice), or in chemistry, e.g.,

1, 1, 1, 2, 3, 5, 9, 18, 35, 75, 159, . . .

the number of distinct hydrocarbons of the methane series), or in electrical engineering, e.g.,

3, 7, 46, 4336, 134281216, . . .

the number of Boolean functions of  $n$  variables), will find this handbook useful.

Besides identifying sequences, the handbook will serve as an index to the literature for locating references on a particular problem, and for quickly finding numbers like  $7^{12}$ , the number of partitions of 30, the 18th Catalan number, or the expansion of  $\pi$  to 60 decimal places. It might also be useful to have around when the first signals arrive from Betelgeuse sequence **2311** for example would be a friendly beginning).

## ACKNOWLEDGMENTS

This book was begun at Cornell University in the years 1965-1969, and finished at Bell Telephone Laboratories from 1969 to 1972. During that time I have been sustained by the support and encouragement of Richard Guy of the University of Alberta, Ron Graham and Henry Pollak of Bell Telephone Laboratories, John Riordan of Rockefeller University, and Ann Snitow of Rutgers University. Most of the sequences were found by searching through the stacks of the libraries of Cornell University, Brown University, and Bell Telephone Laboratories, and I thank the staffs of these excellent libraries for their patience and cooperation. Other sequences were suggested by friends and correspondents, to all of whom I am most grateful. E. R. Berkamp, J. J. Cannon, D. G. Cantor, B. Ganter, F. Harary, D. E. Knuth, Shen Lin, W. F. Lunnon, R. C. Read, P. R. Stein, and J. W. Wrench, Jr. have been especially helpful. Finally I thank Eleanor Potter and Herman P. Robinson for a thorough reading of the manuscript.

The table was produced by first recording the sequences on punched cards, and (except when the sequence was generated by the author) comparing a listing of the cards with the original tables. These cards were then stored on magnetic tape, and the table has been typeset automatically from this tape. My thanks are due to the staff of the Bell Laboratories computation center at Murray Hill, especially the keypunch operators, for their untiring assistance.

## ABBREVIATIONS

Abbreviations of the references are listed in the bibliography

[ $a$ ]	the largest integer $\leq a$
$a ** b$	$a^b$
$C(i, j)$	the binomial coefficient $\binom{i}{j}$
$\exp(a)$	$e^a$
gf	generating function
LCM	least common multiple
REF	reference(s)
seq.	sequence

CHAPTER  
I

DESCRIPTION OF THE BOOK

It is the fate of those who toil at the lower employments of life, to be driven rather by the fear of evil, than attracted by the prospect of good; to be exposed to censure, without hope of praise; to be disgraced by miscarriage, or punished for neglect, where success would have been without applause, and diligence without reward.

Among these unhappy mortals is the writer of dictionaries; whom mankind have considered, not as the pupil, but the slave of science, the pioneer of literature, doomed only to remove rubbish and clear obstructions from the paths of Learning and Genius, who press forward to conquest and glory, without bestowing a smile on the humble drudge that facilitates their progress. Every other author may aspire to praise; the lexicographer can only hope to escape reproach, and even this negative recompense has yet been granted to very few.

Samuel Johnson, Preface to the "Dictionary," 1755

1.1 DESCRIPTION OF A TYPICAL ENTRY

The main table is a list of about 2300 sequences of integers. A typical entry is:

256 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, 610, 987, 1597, 2584, 4181, 6765, 10946,  
17711, 28657, 46368, 75025, 121393, 196418, 317811, 514229, 832040, 1346269  
FIBONACCI NUMBERS  $A(N) = A(N-1) + A(N-2)$ , REF HW1 148, REC 11 20 62, HO1.

and consists of the following items:

256	the sequence identification number
1, 2, 3, 5, 8, 13, 21, ...	the sequence itself
FIBONACCI NUMBERS	a name or descriptive phrase (in this case a recurrence)
$A(N) = A(N-1) + A(N-2)$	for the sequence
REF	references

## 1 DESCRIPTION OF THE BOOK

G. H. Hardy and E. M. Wright, "An Introduction to the Theory of Numbers," Oxford Univ. Press, 3rd ed., page 148, 1954

*Recreational Mathematics Magazine*, Volume 11, page 20, 1962.

V. E. Hoggatt, Jr., "Fibonacci and Lucas Numbers," Houghton Mifflin, Boston, 1969

and in four journals:  
*American Mathematical Monthly* [AMM]  
*Fibonacci Quarterly* [FQ]  
*Journal of Combinatorial Theory* [JCT]  
*Journal of Combinatorial Theory* [MTAC]

Unusual sequences may send the reader to more exotic sources, but in any case he should first check Chapter III where additional information about some of the commoner sequences is given, and the index to see if other sequences (and hence references) of a similar type are listed.

**Journal references usually give volume, page, and year, in that order.** (See the example at beginning of this chapter.) Years after 1899 are abbreviated, by dropping the 19. Earlier years are not abbreviated. Sometimes to avoid ambiguity we use the more expanded form of: journal name (series number), volume number (issue number), page number, year.

**References to books give volume (if any) and page.** (See the example at the beginning of this chapter.)

The references do not attempt to give the discoverer of a sequence, but rather the most extensive table of the sequence that has been published.

## 1.5 WHAT SEQUENCES ARE INCLUDED?

**Rule 1** The sequence must consist of nonnegative integers. (Sequences alternating in sign have been replaced by their absolute values. Interesting sequences of fractions have been entered by numerators and denominators separately. Some sequences of real numbers have been replaced by their integer parts, others by the nearest integers.)

**Rule 2** The sequence must be infinite.

A few, like the Mersenne primes, have been given the benefit of the doubt.

**Rule 3** The first two terms must be 1,  $n$ , where  $n$  is between 2 and 999.

An initial 1 has been silently inserted before the first term if this is greater than 1, and extra 1's and 0's at the beginning have been silently deleted. (See the beginning of Chapter II for examples.)

**Rule 3** Enough terms must be known to separate the sequence from its neighbors in the table.

**Rule 4** The sequence should have appeared in the scientific literature, and must be well-defined and interesting.

The selection has inevitably been subjective, but the goal has been to

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The entries are arranged in lexicographic order, so that sequences beginning 1, 2, 1 come before those beginning 1, 2, 2, and so on.

## 1.2 ARRANGEMENT

## 1.3 NUMBER OF TERMS GIVEN

Whenever possible enough terms are given to fill two lines. If fewer terms are given, it is because they have never been calculated so far as the author knows. (He would be very pleased to be corrected.) Finding the next term in the following sequences is known to be difficult (others of similar type can be located via the index):

1, 48, 66-68, 124, 125, 129, 142, 143, 149, 150, 181, 189, 195, 226, 246, 48, 271, 304, 309, 317, 321-325, 329, 330, 358, 373, 380, 393, 435, 450, 65, 477, 516, 559-561, 580, 581, 595, 596, 614, 615, 621, 648, 650, 730, 31, 745, 757, 782, 788, 809, 812, 911, 954, 972, 994, 998, 1052, 1099, 115, 1133, 1167, 1210, 1244, 1245, 1339, 1340, 1403, 1404, 1467, 1518, 537, 1803, 2248, 2342.

These sequences all represent unsolved problems.

## 1.4 REFERENCES

To conserve space, journal references are extremely abbreviated. They usually give the exact page on which the sequence may be found, but either the author nor the title of the article. To find out more the reader must go to a library; this book is meant to be used in conjunction with a library. Quite a small one will do. A considerable fraction of the sequences will be found in the following nine great works:

Dickson [D12]  
 Lehmer [LE1]  
 Fletcher, Miller, Rosenhead, and Comrie [FMR]

Davis [DA2]  
 Abramowitz and Stegun [AS1]  
 David, Kendall, and Barton [DKB]

Riordan [R1]  
 David and Barton [DB1]  
 Comrie [CO1]



ude a broad variety of sequences and as many as possible.

### 1.6 HOW ARE ARRAYS OF NUMBERS TREATED?

Arrays of numbers (binomial coefficients, Stirling numbers of the first, etc.) have been entered by rows, columns, or diagonals, whichever is most appropriate.

### 1.7 SUPPLEMENTS

It is planned to issue supplements to the Handbook from time to time, containing new sequences and corrections and extensions to the original ones. Readers wishing to receive these supplements should notify the author.

## CHAPTER

## II

### HOW TO HANDLE A STRANGE SEQUENCE

#### 2.1 HOW TO SEE IF A SEQUENCE IS IN THE TABLE

Obtain as many terms of the sequence as possible. The initial terms are handled as follows: Recall that the sequence must begin 1,  $n$ , where  $n$  is between 2 and 999. Find the first term in the sequence that is greater than 1, and replace all the terms that come before it by a single 1. Then look it up in the table. The initial 1 is just a marker, and need not be in the original sequence. For example, if the sequence begins

1, 2, 3, 5, 8, 13, ...	see under 1, 2, 3, 5, 8, 13, ...
2, 3, 5, 8, 13, ...	see under 1, 2, 3, 5, 8, 13, ...
-1, 1, 0, 1, 1, 2, 3, 5, 8, ...	see under 1, 2, 3, 5, 8, ...
1, 0, 0, 2, 24, 552, 21280, ...	see under 1, 2, 24, 522, 21280, ...

#### 2.2 IF THE SEQUENCE IS NOT IN THE TABLE

(i) Try changing or redefining the sequence. Some typical changes are inserting or deleting an initial term (e.g., seq. 46 occurs as both 1, 2, 1, 2, 3, 6, 9, 18, ... and 1, 2, 3, 6, 9, 18, ...); adding or subtracting 1 or 2 from all the terms (e.g., seq. 309 occurs as both 1, 2, 3, 6, 20, 168, ... and 1, 4, 18, 166, ...); and multiplying all the terms by 2 or dividing by any common factor.

(ii) If all these methods fail, and it seems certain that the sequence is not in this handbook, please send the sequence and anything that is known about it, including appropriate references, to the author for possible inclusion in later editions.<sup>1</sup>

<sup>1</sup>Address: Mathematics Research Center, Bell Telephone Laboratories, Inc., Murray Hill, New Jersey 07974.

2.3 FINDING THE NEXT TERM

Suppose the beginning of a sequence is given as

0	1	2	3	4	5	6	7
$a_0$	$a_1$	$a_2$	$a_3$	$a_4$	$a_5$	$a_6$	$a_7$

A rule or explanation for it is desired. If nothing is known about the story of the sequence or if it is an arbitrary sequence, nothing can be said and any continuation is possible. (Any  $n + 1$  points can be fitted by an  $n$ th degree polynomial.)

But the sequences normally encountered, and those in this handbook, distinguished in that they have been produced in some intelligent and systematic way. Occasionally such sequences have a simple explanation, but if so, the methods given below may help to find it. These methods can be divided roughly into two classes: those which look for a systematic way of generating the  $n$ th term  $a_n$  from the terms  $a_0, \dots, a_{n-1}$  before it, i.e.,  $a_n = a_{n-1} + a_{n-2}$ , i.e., methods which seek an internal explanation; those which look for a systematic way of going from  $n$  to  $a_n$ , e.g.,  $a_n$  is number of divisors of  $n$ , or the number of trees with  $n$  nodes, or the prime number, i.e., methods which seek an external explanation. The former methods are described in the rest of this chapter, the latter in chapter III.

In practice it is usually clear for one reason or another when a correct explanation for a sequence has been found.

For the related problems of defining the complexity of a sequence, extrapolating a sequence of real numbers, see the interesting work Martin-Lof [IC 9 602 66] and Fine [IC 16 331 70 and F11.]

2.4 LOOK FOR A RECURRENCE

Let the sequence be  $a_0, a_1, a_2, a_3, \dots$ . Is there a systematic way of finding the  $n$ th term  $a_n$  from the preceding terms  $a_{n-1}, a_{n-2}, \dots$ ? A rule doing this, such as  $a_n = a_{n-1}^2 - a_{n-2}$ , is called a *recurrence*, and of course provides a method for getting as many terms of the sequence as needed.

In studying sequences and recurrences it is useful to define a *generating function* (gf) associated with the sequence, usually an ordinary gf:

$$A(x) = a_0 + a_1x + a_2x^2 + a_3x^3 + \dots,$$

sometimes an exponential gf:

$$E(x) = a_0 + a_1 \frac{x}{1!} + a_2 \frac{x^2}{2!} + a_3 \frac{x^3}{3!} + \dots$$

(These are formal power series having the sequence as coefficients; questions of convergence do not arise.)

Once a recurrence has been found for the sequence, techniques for solving it will be found in the works by Riordan [R1 19], Batchelder [BAT], and Levy and Lessman [LE2].

For example, consider seq. 256, the Fibonacci numbers: 1, 1, 2, 3, 5, 8, 13, 21, 34, . . . . These are generated by the recurrence  $a_n = a_{n-1} + a_{n-2}$ , and from this it is not difficult to obtain the generating function

$$1 + x + 2x^2 + 3x^3 + 5x^4 + \dots = \frac{1}{1 - x - x^2},$$

and the explicit formula for the  $n$ th term

$$a_n = \frac{1}{\sqrt{5}} \left[ \left( \frac{1 + \sqrt{5}}{2} \right)^{n+1} - \left( \frac{1 - \sqrt{5}}{2} \right)^{n+1} \right].$$

2.4.1 METHOD OF DIFFERENCES

This is the standard method for finding recurrences. In simple cases, it will even find an explicit formula for the  $n$ th term of a sequence, e.g., if this is a polynomial (such as  $a_n = n^2 + 1$ ) or a simple exponential (such as  $a_n = 2^n + n + 1$ ).

If the sequence is

$$a_0, a_1, a_2, a_3, a_4, \dots,$$

its first differences are the numbers

$$\Delta a_0 = a_1 - a_0, \quad \Delta a_1 = a_2 - a_1, \quad \Delta a_2 = a_3 - a_2, \quad \dots,$$

its second differences are

$$\Delta^2 a_0 = \Delta a_1 - \Delta a_0, \quad \Delta^2 a_1 = \Delta a_2 - \Delta a_1, \quad \Delta^2 a_2 = \Delta a_3 - \Delta a_2, \quad \dots,$$

and so on. The 0th differences are the original sequence:  $\Delta^0 a_0 = a_0$ ,  $\Delta^0 a_1 = a_1$ ,  $\Delta^0 a_2 = a_2, \dots$ ; and the  $m$ th differences are

$$\Delta^m a_n = \Delta^{m-1} a_{n+1} - \Delta^{m-1} a_n$$

or, in terms of the original sequence,

$$\Delta^m a_n = \sum_{i=0}^m (-1)^i \binom{m}{i} a_{n+m-i}. \tag{1}$$

Therefore if the differences of some order can be identified, Eq. (1) gives a recurrence for the sequence.

furthermore, if the differences  $a_k, \Delta a_k, \Delta^2 a_k, \Delta^3 a_k, \dots$  are known for a fixed value of  $k$ , then a formula for the  $n$ th term is given by

$$a_{n+k} = \sum_{m=0}^n \binom{n}{m} \Delta^m a_k. \quad (2)$$

**Example (i)** Seq. 1562

$n$	1	2	3	4	5	6	7	8
$a_n$	1	5	12	22	35	51	70	92
$a_n$	4	7	10	13	16	19	22	
$a_n$	3	3	3	3	3	3	3	
$a_n$	0	0	0	0	0	0	0	

Since  $\Delta^2 a_n = 3$ ,  $\Delta a_{n+1} - \Delta a_n = 3$ , or  $a_{n+2} - 2a_{n+1} + a_n = 3$ , a recurrence for the sequence. An explicit formula is obtained from Eq. (2) with 1:

$$a_{n+1} = 1 + 4 \binom{n}{1} + 3 \binom{n}{2} = 1 + 4n + \frac{3}{2}n(n-1) = \frac{1}{2}(n+1)(3n+2).$$

In general, if the  $m$ th differences are zero,  $a_n$  is a polynomial in  $n$  of degree  $m-1$ .

**Example (ii)** Seq. 1382

$n$	1	2	3	4	5	6	7
$a_n$	1	4	11	26	57	120	247
$\Delta a_n$	3	7	15	31	63	127	
$\Delta^2 a_n$	4	8	16	32	64		

Here  $\Delta^2 a_n = 2^{n+1}$ ,  $\Delta a_n = 2^{n+1} - 1$ , and  $a_n = 2^{n+1} - n - 2$ . Equation gives the same answer.

**Example (iii)** Seq. 552 (the Pell numbers)

$n$	1	2	3	4	5	6	7
$a_n$	1	2	5	12	29	70	169
$\Delta a_n$	1	3	7	17	41	99	
$\Delta^2 a_n$	2	4	10	24	58		
$\frac{1}{2} \Delta^2 a_n$	1	2	5	12	29		

Since  $\frac{1}{2} \Delta^2 a_n = a_n$ , Eq. (1) gives the recurrence  $a_{n+2} - 2a_{n+1} - a_n = 0$ . Calculating further differences shows that  $\Delta^m a_n = 2^{\lfloor m/2 \rfloor}$  and so Eq. (2) gives the formula

$$a_{n+1} = \sum_{m=0}^n \binom{n}{m} 2^{\lfloor m/2 \rfloor}.$$

**Example (iv)** Seq. 469

$n$	1	2	3	4	5	6	7	8
$a_n$	1	2	4	10	26	76	232	764
$\Delta a_n$	1	2	6	16	50	156	532	
$n^{-1} \Delta a_n$	1	1	2	4	10	26	76	

Notice that  $\Delta a_n$  is divisible by  $n$ , and in fact  $n^{-1} \Delta a_n = a_{n-1}$ , so that  $a_{n+1} = a_n + n a_{n-1}$ . Again Eq. (2) gives a formula for  $a_n$ .

### 2.4.2 OTHER METHODS OF ATTACK

Is the sequence close to a known sequence, such as the powers of 2? If so, try subtracting off the known sequence. For example, seq. 1382 (again): 1, 4, 11, 26, 57, 120, 247, 502, 1013, 2036, 4083, ... The last four numbers are close to powers of 2: 512, 1024, 2048, 4096; and then it is easy to find  $a_n = 2^n - n - 1$ .

Is a simple recurrence such as  $a_n = \alpha a_{n-1} + \beta a_{n-2}$  likely? For this to happen, the ratio  $\rho_n = a_{n+1}/a_n$  of successive terms must approach a constant as  $n$  increases. Use the values  $a_2$  to  $a_8$  to determine  $\alpha$  and  $\beta$  and then see if  $a_6, a_7, \dots$  are generated correctly.

If the ratio  $\rho_n$  has first differences which are approximately constant, this suggests a recurrence of the type  $a_n = \alpha n a_{n-1} + \dots$ . For example, seq. 704: 1, 2, 7, 30, 157, 972, 6961, 56660, 516901, ... has successive ratios 2, 3.5, 4.29, 5.23, 6.19, 7.16, 8.14, 9.12, ... with differences approaching 1, suggesting  $a_n = n a_{n-1} + ?$ . Subtracting  $n a_{n-1}$  from  $a_n$ , we obtain the original sequence 0, 1, 2, 7, 30, 157, 972, ... again, so  $a_n = n a_{n-1} + a_{n-2}$ .

This example illustrates the principle that whenever  $\rho_n = a_{n+1}/a_n$  seems to be close to a recognizable sequence  $r_n$ , one should try to analyze the sequence  $b_n = a_{n+1} - r_n a_n$ .

A recurrence of the form  $a_n = n a_{n-1} +$  (small term) can be identified by the fact that the 10th term is approximately 10 times the 9th. For example, seq. 766: 0, 1, 2, 9, 44, 265, 1854, 14833, 133496, 1334961, ... ,  $a_n = n a_{n-1} + (-1)^n$ .

The recurrence  $a_n = a_{n-1}^2 + \dots$  is characterized by the fact that each term is about twice as long as the one before. For example, seq. 331: 1, 2, 3, 7, 43, 1807, 3263443, 10650056950807, ... , and  $a_n = a_{n-1}^2 - a_{n-1} + 1$ .

### 2.4.3 FACTORIZING

Does the sequence, or one obtained from it by some simple operation, have many factors?

**Example (i)** Seq. 1614: 1, 5, 23, 119, 719, 5039, 40319, ... . As it stands, the sequence cannot be factored, since 719 is prime, but the addi-

$n$  of 1 to all the terms gives the highly composite sequence  $2, 6 = 2 \cdot 3, = 2 \cdot 3 \cdot 4, 120 = 2 \cdot 3 \cdot 4 \cdot 5, \dots$ , which are the factorial numbers (see section 3.13).

The presence of only small primes may also suggest binomial coefficients:

*Example (ii)* Seq. 577 (the Catalan numbers):  $1, 2, 5, 14 = 2 \cdot 7, = 2 \cdot 3 \cdot 7, 132 = 4 \cdot 3 \cdot 11, 429 = 3 \cdot 11 \cdot 13, 1430 = 2 \cdot 5 \cdot 11 \cdot 13, 4862 = 2 \cdot 11 \cdot 13 \cdot 17, \dots$  and

$$a_n = \frac{1}{n+1} \binom{2n}{n}$$

(see Section 3.5).

Sequences arising in number theory are sometimes *multiplicative*, i.e., the property that  $a_{mn} = a_m a_n$ , whenever  $m$  and  $n$  have no common factor. For example, seq. 86:  $1, 2, 2, 3, 2, 4, 2, 4, \dots$  the number of divisors of  $n$ .

#### 1.4 SELF-GENERATING SEQUENCES

This section describes some recurrences of a simple yet unusual type. They have been called (rather arbitrarily) *self-generating*.

In the first two examples let  $\mathcal{A} = \{a_0 = 1, a_1, a_2, \dots\}$  be a sequence 1's and 2's.

- (i) If every 1 in  $\mathcal{A}$  is replaced by 1, 2 and every 2 by 2, 1 a new sequence  $\mathcal{A}'$  is obtained. Imposing the condition that  $\mathcal{A} = \mathcal{A}'$  forces  $\mathcal{A}$  to be: 1: 71: 1, 2, 2, 1, 2, 1, 1, ... Sequences 21 and 36 are of the same type.
- (ii) Let  $\mathcal{A}'' = \{b_0, b_1, b_2, \dots\}$ , where  $b_n$  is the length of the  $n$ th run in  $\mathcal{A}$ . (A run is a maximal string of identical symbols.) The condition  $\mathcal{A} = \mathcal{A}''$  forces  $\mathcal{A}$  to be seq. 70: 1, 2, 2, 1, 1, 2, 1, 2, 2, 1, ...

In the remaining examples,  $\mathcal{A} = \{a_0 = 1, a_1, a_2, \dots\}$  is a nondecreasing sequence of integers.

- (iii) Let  $c_n$  be the number of times  $n$  occurs in  $\mathcal{A}$ , for  $n = 1, 2, \dots$ . If  $\mathcal{A} = \mathcal{A}''$ ,  $\mathcal{A}$  is seq. 89: 1, 2, 2, 3, 3, 3, 4, 4, 4, 4, ... If  $c_n = a_{n-1}$ ,  $\mathcal{A}$  is seq. 91: 2, 2, 3, 3, 4, 4, 4, 5, 5, 5, 6, 6, 6, 6, ... (Seq. 965 is related to the latter sequence.)

- (iv) The condition that  $a_{n+1} - a_n$  be the smallest positive integer not equal to  $a_i - a_j$  for any  $i, j \leq n$  forces  $\mathcal{A}$  to be seq. 416: 1, 2, 4, 8, 13, 21, ... The conditions  $a_0 = 1, a_2 = 2$ , and that  $a_n$  be the smallest integer which can be written uniquely as the sum of two distinct preceding terms force  $\mathcal{A}$  to be seq. 201: 1, 2, 3, 4, 6, 8, 11, 13, ... Sequences 231, 254, 425, and 9 have similar explanations.

## CHAPTER III

### ILLUSTRATED DESCRIPTION OF SOME IMPORTANT SEQUENCES

While Chapter II studied ways of getting the  $n$ th term of a sequence from the preceding terms, this chapter considers externally generated sequences, such as the sequences in which the  $n$ th term is the number of graphs with  $n$  nodes or the  $n$ th triangular number. An informal and illustrated description is given of some of the most important such sequences.

#### 3.1 GRAPHS AND TREES

Stated informally, a *graph* consists of a finite set of points (or nodes) some of which are joined by lines (or edges). Figure 1 illustrates seq. 479, the number of graphs with  $n$  nodes.

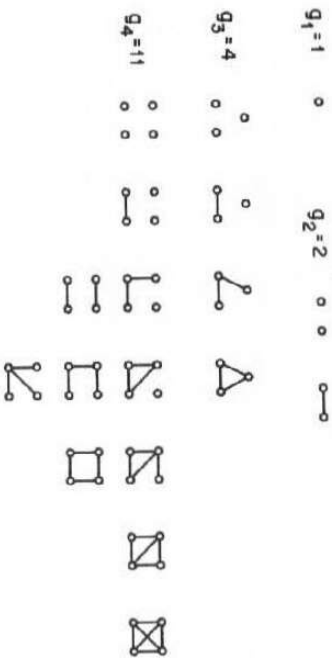


Fig. 1. Seq. 479, graphs or reflexive symmetric relations.

A *digraph*, or directed graph, is a graph with arrows on the edges (Fig. 2, seq. 1229). Figure 3 shows seq. 1069, digraphs of functions, i.e., digraphs with exactly one arrow directed out of each node.

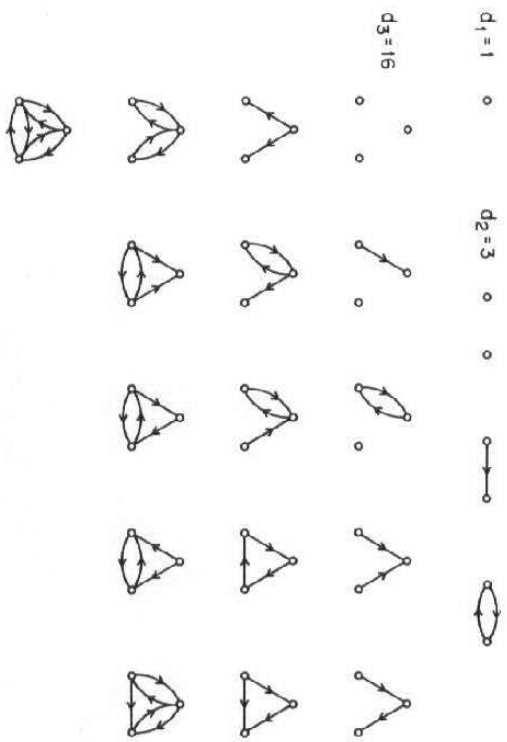


Fig. 2. Seq. 1229, digraphs or reflexive relations.

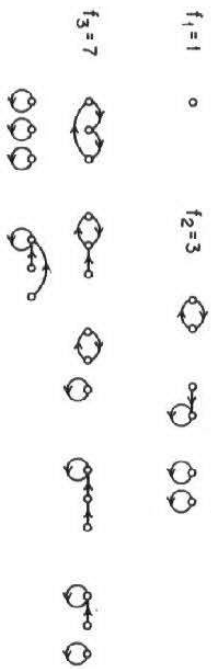


Fig. 3. Seq. 1069, functional digraphs.

A tree is a connected graph containing not closed paths (Fig. 4, seq. 99). A rooted tree is a tree with a distinguished node called Eve, or the root. Figure 5 illustrates seq. 454, the number of rooted trees with  $n$  nodes. The generating function (gf) of this sequence is

$$r(x) = x + x^2 + 2x^3 + 4x^4 + 9x^5 + \dots$$

and satisfies

$$r(x) = x \exp[r(x) + \frac{1}{2}r(x^2) + \frac{1}{3}r(x^3) + \dots].$$

The generating function for trees,

$$t(x) = x + x^2 + x^3 + 2x^4 + 3x^5 + 6x^6 + \dots$$

is then given by

$$t(x) = r(x) - \frac{1}{2}r^2(x) + \frac{1}{3}r^3(x^2).$$

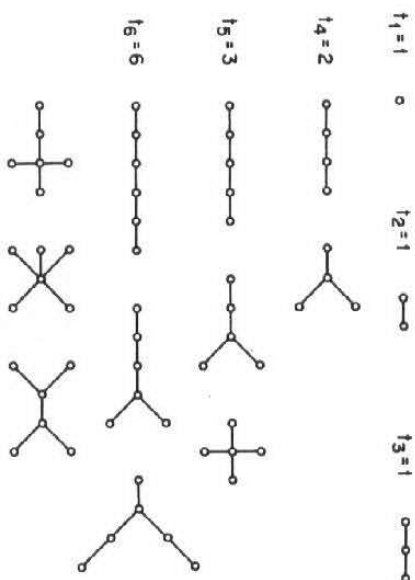


Fig. 4. Seq. 299, trees.

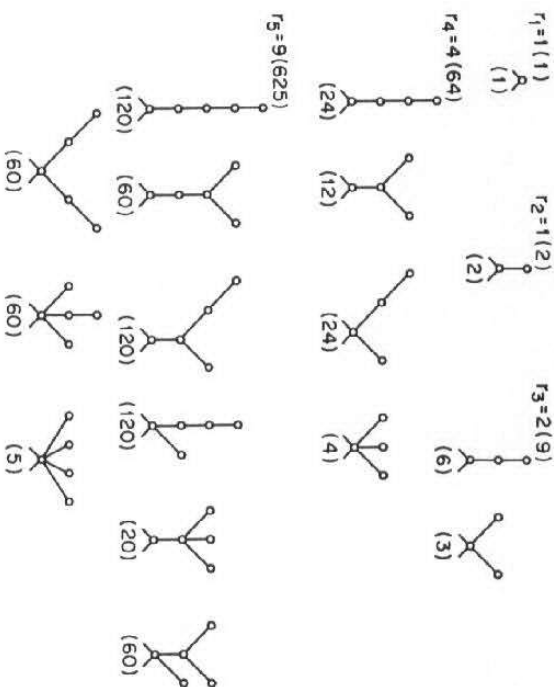


Fig. 5. Seq. 454, rooted trees. (The numbers in parentheses give seq. 771, labeled rooted trees.)

Any of these graphs may be labeled by (if there are  $n$  nodes) attaching the numbers from 1 to  $n$  to the nodes. For example in Fig. 5, the numbers in parentheses give the number of ways of labeling each tree, and then the total number of labeled rooted trees with  $n$  nodes is  $n^{n-1}$ , seq. 771. Usually when graphs are mentioned in the main table they are unlabeled unless stated otherwise.

The degree of a node is the number of edges meeting it. Figure 6 shows seq. 118, series-reduced trees, or trees without nodes of degree 2. For further information about the preceding sequences and for the enumeration of other kinds of graphs, see Riordan [R1] and Harary [HA5].

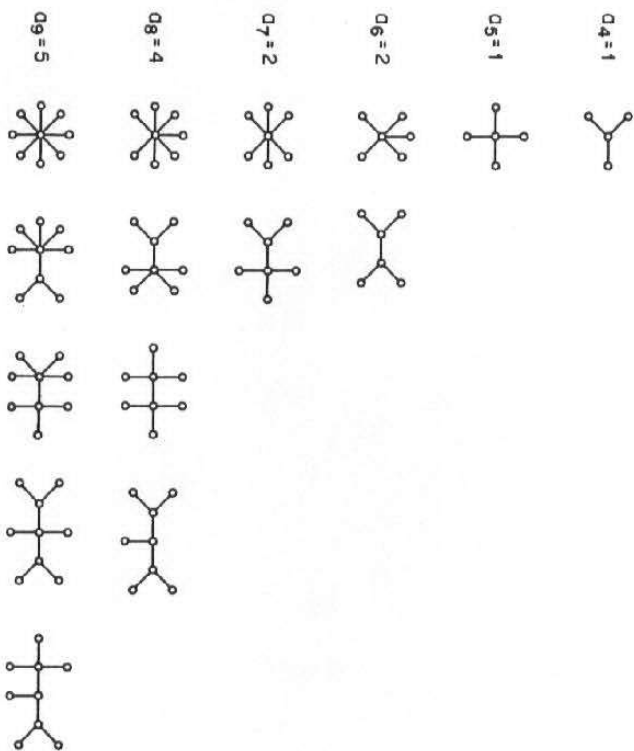


Fig. 6. Seq. 118, series-reduced trees.

### 3.2 RELATIONS

A relation  $R$  on a set  $S$  is any subset of  $S \times S$ , and  $xRy$  means  $(x, y) \in R$  or "x is related to y". A relation is reflexive if  $xRx$  for all  $x$  in  $S$ , symmetric if  $xRy \Rightarrow yRx$ , antisymmetric if  $xRy$  and  $yRx \Rightarrow x = y$ , and transitive if  $xRy$  and  $yRz \Rightarrow xRz$ .

The most important types of relations are:

- (1) unrestricted, or digraphs with loops of length 1 allowed (seq. 784: 10, 104, 3044, 291968, ...);
- (2) symmetric, or graphs with loops of length 1 allowed (seq. 646: 6, 20, 90, 544, 5096, 79264, ...);
- (3) reflexive, or digraphs (Fig. 2, seq. 1229 again);
- (4) reflexive symmetric, or graphs (Fig. 1, seq. 479 again);

- (5) reflexive transitive, or topologies (Fig. 7, seq. 1133: 1, 3, 9, 33, 139, 718, 4535, ?). For the connection between digraphs and topologies, see Birkhoff [B11 117]);
- (6) reflexive symmetric transitive, or partitions (Fig. 20, p. 24, seq. 244);
- (7) reflexive antisymmetric transitive, or partially ordered sets (Fig. 8, seq. 588: 1, 2, 5, 16, 63, 318, 2045, ?).

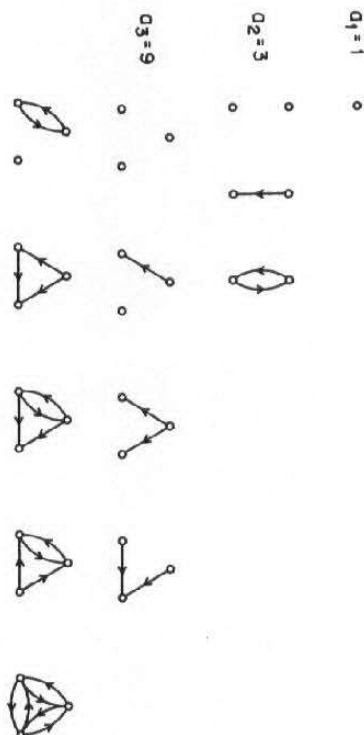


Fig. 7. Seq. 1133, topologies.

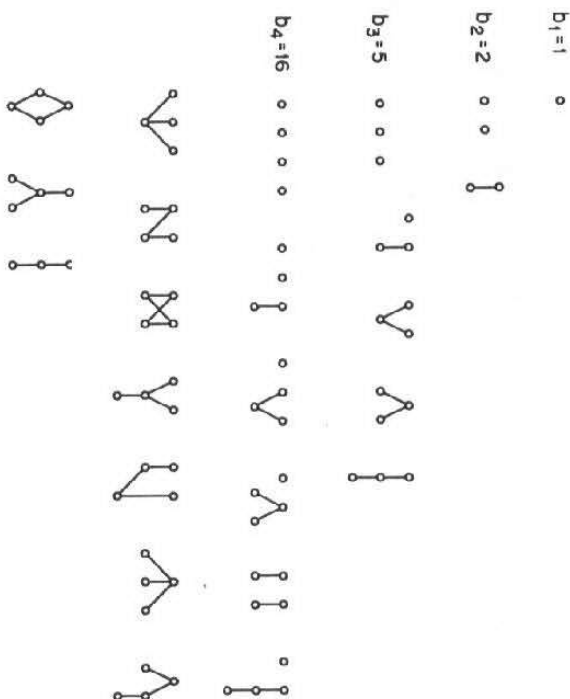


Fig. 8. Seq. 588, partially ordered sets.

This assumes that the graphs are unlabeled, i.e., that the elements of the set  $S$  are indistinguishable. If the elements of  $S$  are labeled 1 through  $n$ , the corresponding numbers are:

- (1)  $2^{n^2}$ ;
- (2)  $2^{n(n+1)/2}$ ;
- (3)  $2^{n(n-1)}$ ;
- (4)  $2^{\binom{n}{2}}$

These four [(1)-(4)] are not in the table, but the sequences of their exponents are):

- (5) seq. 1476: 1, 4, 29, 355, 6942, 209527, 9535241, ?;
- (6) seq. 585: the Bell numbers or the number of equivalence relations on a set of  $n$  objects (see Fig. 22, p. 25);
- (7) seq. 1244: 1, 3, 19, 219, 4231, 130023, 6129859, ?.

### 3.3 GEOMETRIES

The numbers of topologies were shown in Fig. 7; the following are also basic geometrical sequences:

A *linear space* is a system of (abstract) points and lines such that every two points lie on a unique line, and every line contains at least two points. A *geometry* is a system of points, lines, planes, ... with an analogous definition. Figure 9 shows seq. 462: 1, 1, 2, 4, 9, 26, 101, 950, ?, the number of geometries with  $n$  points. (See Crapo and Rota [JM2 49 127 0]). The \* denotes 5 points in general position in 4-dimensional space.) The planar figures in Fig. 9 form seq. 271: 1, 1, 2, 3, 5, 10, 24, 69, 384, ?, the number of linear spaces (Doyen [BSM 19 424 67]).

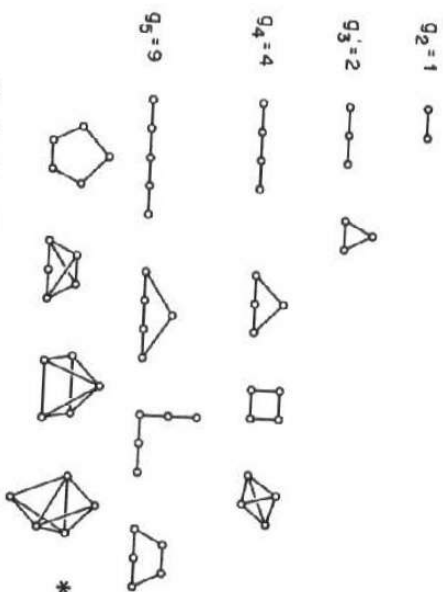


Fig. 9. Seq. 462, geometries (for \* see text).

### 3.4 COMBINATIONS AND FIGURATE NUMBERS

The most basic combinatorial number is the *binomial coefficient*

$$\binom{n}{r} = \frac{n(n-1)(n-2)\cdots(n-r+1)}{1 \cdot 2 \cdot 3 \cdots r} = \frac{n!}{r!(n-r)!}$$

which is the number of selections, or combinations, of  $n$  unlike things taken  $r$  at a time, has gf

$$(1+x)^n = \sum_{r=0}^n \binom{n}{r} x^r,$$

and is the  $(r+1)$ th term in the  $(n+1)$ th row of Pascal's triangle

				1					
			1	2	1				
		1	3	3	1				
	1	4	6	4	1				
1	6	15	10	10	5	1			
							15	6	1

These are also called *figurate numbers* since they are the numbers of points in certain figures. For example,  $\binom{n}{2}$  and  $\binom{n}{3}$  are the *triangular* and *tetrahedral* numbers (Fig. 10, seqs. 1002, 1363).

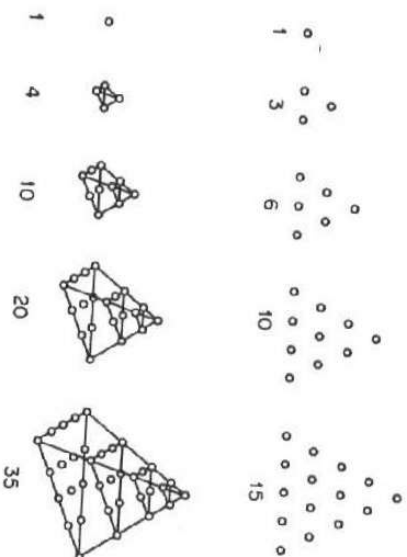


Fig. 10. Seqs. 1002 and 1363, the triangular and tetrahedral numbers.

Other examples of figurate numbers are the *polygonal numbers*  $P(r, s) = \frac{1}{2}r(rs - s + 2)$ . Figure 11 shows seq. 1350, the *square numbers*  $P(r, 2) = r^2$ ; and seq. 1562, the *pentagonal numbers*  $P(r, 3) = \frac{1}{2}r(3r - 1)$ . Many other figurate numbers, including cubes, fourth powers, etc., will be found in the table. For further pictures see Hogben [HO3].

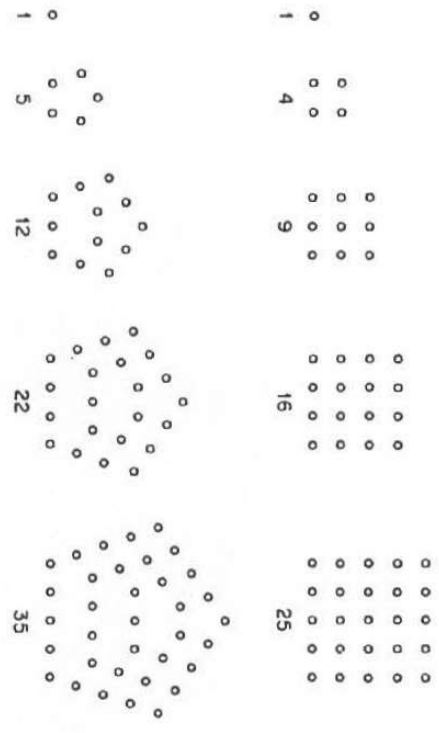


Fig. 11. Seqs. 1350 and 1562, the square and pentagonal numbers.

### 3.5 CATALAN NUMBERS AND DISSECTIONS

Next to the figurate numbers, the Catalan numbers are the most frequently occurring combinatorial numbers. (Gould [GO4] lists over 240 references.) They are defined by

$$c_n = \frac{1}{n+1} \binom{2n}{n},$$

and form seq. 557: 1, 2, 5, 14, 42, 132, 429, 1430, 4862, 16796, ... A gif is  $1 + x + 2x^2 + 5x^3 + \dots = (2x)^{-1}[1 - (1 - 4x)^{1/2}]$ .

Some of the interpretations of  $c_n$  are:

- (1) The number of ways of dissecting a convex polygon of  $n + 2$  sides into  $n$  triangles by drawing nonintersecting diagonals (Fig. 12a).
- (2) The number of ways of completely parenthesizing a product of  $n + 1$  letters (so that there are two factors inside each set of parentheses). The examples for  $n = 1, 2, 3$  (arranged to show the correspondence with the dissections of Fig. 12a) are:

$$\begin{aligned} n = 1 & \quad (ab); & n = 2 & \quad a(bc), (ab)c; \\ n = 3 & \quad (ab)(cd), a((bc)d), ((ab)c)d, a(b(cd)), (a(bc))d. \end{aligned}$$

- (3) The number of bifurcated rooted planar trees with  $n + 1$  endpoints. (A planar tree is one which has been drawn on a plane, and bifurcated means that each edge splits in two at each node. See Fig. 12b. The trees are drawn to show the correspondence with the dissections and the parentheses.)
- (4) In an election with two candidates A and B, each receiving  $n$  votes,  $c_n$  is the number of ways the votes can come in so that A is never behind B (Feller [FE1 1 71] and Comtet [CO1 1 94]).

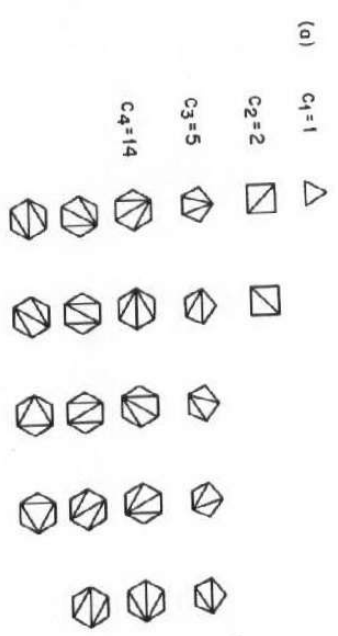


Fig. 12. Seq. 577, the Catalan numbers.

Figure 13 illustrates seq. 942, the number of different dissections of a polygon when two dissections are considered to be the same if a rotation or reflection sends one into the other. Figure 14 illustrates seq. 391, giving  $\frac{1}{2}n(n + 1) + 1$ , the maximum number of pieces obtained by slicing a pancake with  $n$  slices. The numbers



of  $n$ -sided polygons in the  $n$ th diagram of Fig. 14 form seq. 1181: 0, 0, 1, 3, 12, 70, 465, 3507, 30016, ... (Robinson [AMM 58 462 51]). Seq. 491: 2, 4, 8, 15, 26, 42, 64, 93, 130, 176, ... gives  $(n+2)(n+3)/6$ , the maximum number of pieces obtained with  $n$  slices of a cake.

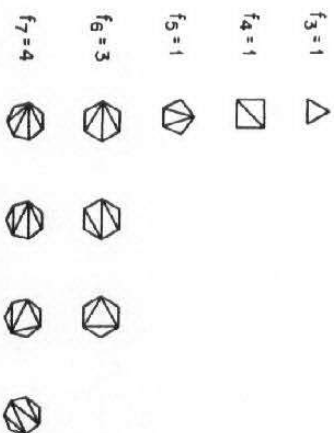


Fig. 13. Seq. 942, dissections of a polygon.

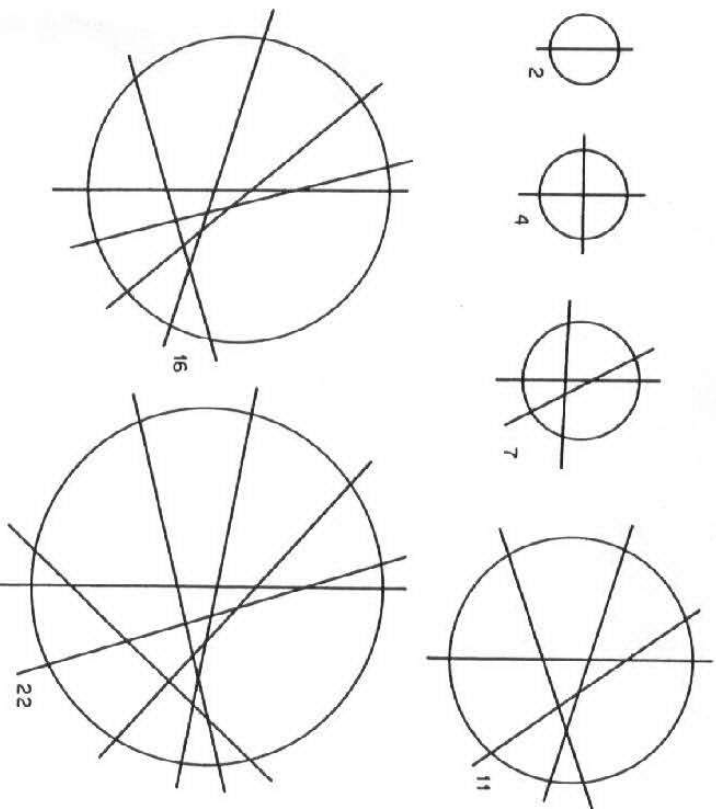


Fig. 14. Seq. 391, slicing a pancake.

3.6 NECKLACES AND IRREDUCIBLE POLYNOMIALS

Figure 15 illustrates seq. 203,  $T_n$ , the number of different necklaces that can be made from beads of two colors, when the necklaces can be rotated but not turned over. This is also the number of irreducible binary polynomials whose degree divides  $n$ , an important sequence in digital circuitry; and has the formula  $T_n = \sum \phi(d)2^{n/d}$ , where  $\phi(d)$  is the Euler totient function (seq. 111, Section 3.14) and the sum is over all divisors  $d$  of  $n$ . (See Berlekamp [BE2 70] and Golomb [CMA 1 358 69].) If turning over is allowed, the number of different necklaces is given by seq. 202: 2, 3, 4, 6, 8, 13, 18, 30, 46, 78, ... (See Gilbert and Riordan [JJM 5 657 61].)

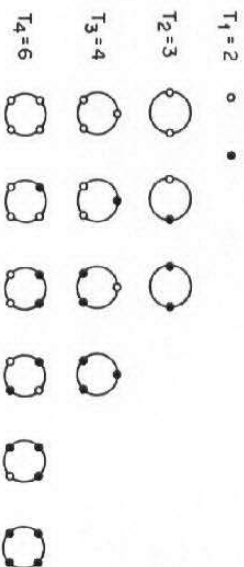


Fig. 15. Seq. 203, necklaces.

3.7 KNOTS

Figure 16 shows seq. 322: 0, 0, 1, 1, 2, 3, 7, 18, 41, 123, 367, ?, the number of knots with  $n$  crossings, in which the crossings alternate. (See Tait [TA1 1 334] and Conway [JL2 343].)

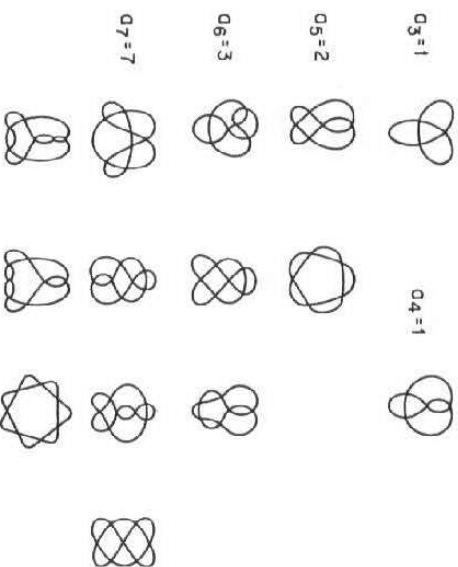


Fig. 16. Seq. 322, knots.

3.8 STAMPS

Figure 17 shows seq. 576: 1, 1, 2, 5, 14, 39, 120, 358, 1176, 3527, ... (six more terms are known), the number of ways of folding a strip of stamps.

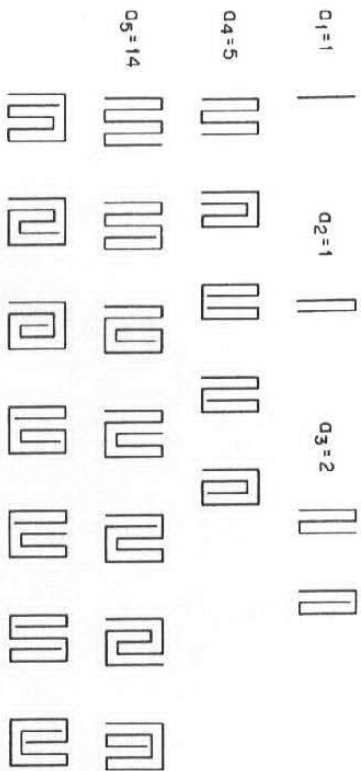


Fig. 17. Seq. 576, folding a strip of stamps.

3.9 POLYOMINOES

A polyomino with  $p$  squares is a connected set of  $p$  squares from a chessboard pattern. Polyominoes are *free* if they can be rotated and turned over (Fig. 18), and *fixed* otherwise. Unless otherwise stated, all polyominoes are free. Polyominoes may also be formed from triangles, rectangles, cubes (Fig. 19), etc. In no case is a formula known for the general term. (See Golomb [GO21])

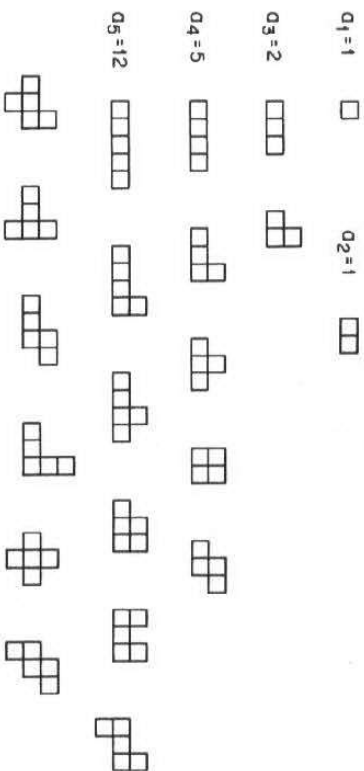


Fig. 18. Seq. 561, square polyominoes.

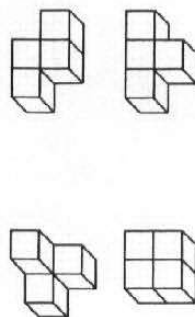
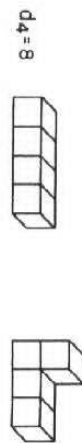
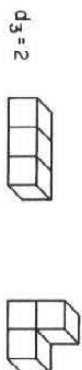


Fig. 19. Seq. 731, polyominoes made from cubes.

3.10 BOOLEAN FUNCTIONS

A Boolean (or switching) function is a function  $f(x_1, \dots, x_n)$ , where each variable  $x_i$  is 0 or 1, and  $f$  takes on the values 0 or 1.

These arise in the design of logical circuits, when the names of the variables do not matter. So it is natural to say that two such functions are equivalent if they differ only in the names of the variables (so that  $x_1 + x_2x_3$  is equivalent to  $x_2 + x_1x_3$ ), and to ask for the number of inequivalent functions. The answers to this (which is seq. 1405: 4, 12, 80, 3984, ...) and to many similar questions (allowing complementation of the variables, etc.) are given by the Pólya counting theory (Section 3.11).

Two generalizations that will be found in the table are (i) Post functions, which are functions  $f(x_1, \dots, x_n)$ , where each  $x_i$  and  $f$  can take any value from 0 to  $m - 1$ ; and (ii) switching networks, which are  $n$ -input,  $k$ -output networks such that each of the outputs is a Boolean function of the  $n$  inputs. For details see Harrison [HA2, MU3 85].

3.11 PÓLYA COUNTING THEORY

A large number of counting problems involving graphs, necklaces, Boolean functions, and patterns of various kinds have been solved by the

theorems of Redfield, Pólya, and De Bruijn. (See Riordan [R1 131], De Bruijn [BE6 144], Harrison [HA2 127, MU3 85], and Harary [HA5 178].)

3.12 PARTITIONS

The following are the most important sequences of partitions.

The main such sequence is number 244: 1, 2, 3, 5, 7, 11, . . . , giving the number of partitions of  $n$  into integer parts (Fig. 20). A gf is

$$1 + x + 2x^2 + 3x^3 + 5x^4 + \dots = \prod_{i=1}^{\infty} (1 - x^i)^{-1}.$$

(See Gupta [RSS2] and David *et al.* [DKB 273].)

Those partitions of  $n$  in which all parts are distinct form seq. 100: 1, 1, 2, 2, 3, 4, 5, . . . with gf

$$1 + x + x^2 + 2x^3 + 2x^4 + \dots = \prod_{i=1}^{\infty} (1 + x^i).$$

The partitions of the even numbers into parts which are powers of two form the binary partition function  $b(n)$ , seq. 378: 1, 2, 4, 6, 10, 14, 20, 26, 36, 46, . . . , with recurrence  $b(n) = b(n-1) + b(\lfloor \frac{1}{2}n \rfloor)$ .

$p(1) = 1$	1
$p(2) = 2$	$2, 1^2$
$p(3) = 3$	$3, 21, 1^3$
$p(4) = 5$	$4, 31, 2^2, 21^2, 1^4$
$p(5) = 7$	$5, 41, 32, 31^2, 2^21, 21^3, 1^5$
$p(6) = 11$	$6, 51, 42, 41^2, 3^2, 321, 31^3, 2^2, 2^21^2, 21^4, 1^6$
$p(7) = 15$	$7, 61, 52, 51^2, 43, 421, 41^3, 3^21, 3^22, 321^2, 31^4, 2^21, 2^21^3, 21^5, 1^7$

Fig. 20. Seq. 244, the number of partitions of  $n$ .

Figure 21 illustrates the number of planar partitions of  $n$ , seq. 1016, with gf

$$1 + x + 3x^2 + 6x^3 + \dots = \prod_{i=1}^{\infty} (1 - x^i)^{-1}.$$

Figure 22 shows  $S(n, k)$ , the Stirling numbers of the second kind, or the number of partitions of a set of  $n$  labeled objects into  $k$  parts.

$r(1) = 1$	1	1					
$r(2) = 3$	2	11	1				
$r(3) = 6$	3	21	2	111	11	1	
$r(4) = 13$	4	31	3	1	11	11	1

Fig. 21. Seq. 1016, planar partitions.

n \ k	1	2	3	4	Total
1	1				1
2	12	1, 2			2
3	123	1, 23 2, 13 3, 12	1, 2, 3		5
4	1234	1, 234 3, 124 4, 123 12, 34 14, 23	1, 2, 34 1, 4, 23 2, 4, 13 3, 4, 12	1, 2, 3, 4	15

Fig. 22.  $S(n, k)$ , the Stirling numbers of the second kind, and seq. 585, the Bell numbers.

The numbers continue:

1	1					1
1	1	1				2
1	3	1				5
1	7	6	1			15
1	15	25	10	1		52
1	31	90	65	15	1	203
1	63	301	350	140	21	877
						...

row sums  
 $B(n)$

A gf for  $S(n, k)$  is

$$x^n = \sum_{k=0}^n S(n, k) x(x-1) \cdots (x-k+1).$$

Both the columns and diagonals of this array will be found in the main table.

The row sums are the *Bell numbers*  $B(n)$ , seq. 585.  $B(n)$  is also the number of equivalence relations on a set of  $n$  objects (Section 3.2) and as gf

$$1 + x + 2\frac{x^2}{2!} + 5\frac{x^3}{3!} + \cdots = e^{e^x - 1}.$$

See Abramowitz and Stegun [AS1 835]. David *et al.* [DKB 223], and Comtet [CO1 2 381.]

### 3.13 PERMUTATIONS

A *permutation* of  $n$  objects is any rearrangement of them, and is specified either by a table:

1	2	3	4	5
3	5	4	1	2

or by a product of cycles: (134)(25), both of which mean replace 1 by 3, 2 by 4, 4 by 1, 2 by 5, and 5 by 2.

Figure 23 shows  $s(n, k)$ , the *Stirling numbers of the first kind*, or the numbers of permutations of  $n$  objects containing  $k$  cycles. The numbers continue:

						row sums
1	1					$n!$
2	3	1				1
6	11	6	1			2
24	50	35	10	1		6
120	274	225	85	15	1	24
720	1764	1624	735	175	21	120
						720
						5040
						:
						:
						:

A gf for  $s(n, k)$  is

$$x(x-1) \cdots (x-n+1) = \sum_{k=0}^n (-1)^{n-k} s(n, k) x^k.$$

Both the columns and diagonals of this array will be found in the main table. The row sums are the *factorial* numbers  $n!$ , seq. 659, the total num-

ber of permutations of  $n$  objects. References are as given above for the Stirling numbers of the second kind.

Factorial  $n$  is the product  $1 \cdot 2 \cdot 3 \cdots n$  of the first  $n$  numbers. The products of the first  $n$  even numbers,  $(2n)!! = 2 \cdot 4 \cdot 6 \cdots (2n) = 2^n \cdot n!$ , seq. 742: 2, 8, 48, 384, 3840, 46080, ..., and of the first  $n$  odd numbers,  $(2n-1)!! = 1 \cdot 3 \cdot 5 \cdots (2n-1) = (2n)! / (2^n \cdot n!)$ , seq. 1217: 1, 3, 15, 105, 945, 10395, ... are called *double factorials*.

$n \backslash k$	1	2	3	4	Total
1	(1)				1
2	(12)	(1)(2)			2
3	(123) (132)	(1)(23) (2)(13) (3)(12)	(1)(2)(3)		6
4	(1234) (1243) (1324) (1342) (1423) (1432)	(1)(234) (2)(134) (3)(124) (4)(123) (12)(34) (14)(23)	(1)(2)(34) (1)(3)(24) (1)(4)(23) (2)(3)(14) (2)(4)(13) (3)(4)(12)	(1)(2)(3)(4)	24

Fig. 23.  $s(n, k)$ , the Stirling numbers of the first kind; and seq. 659, the factorial numbers.

$D_2=1$	1 2				
	2 1				
$D_3=2$	1 2 3	1 2 3			
	2 3 1	3 1 2			
$D_4=9$	1 2 3 4	1 2 3 4	1 2 3 4	1 2 3 4	
	2 1 4 3	2 3 4 1	2 4 1 3	2 4 1 3	
	1 2 3 4	1 2 3 4	1 2 3 4	1 2 3 4	
	3 1 4 2	3 4 1 2	3 4 2 1	3 4 2 1	
	1 2 3 4	1 2 3 4	1 2 3 4	1 2 3 4	
	4 1 2 3	4 3 1 2	4 3 2 1	4 3 2 1	

Fig. 24. Seq. 766, derangements.

Figure 24 shows  $D_n$ , the number of *derangements* of  $n$  objects, or the permutations in which every object is moved from its original position (seq. 766). These are also called *subfactorial* or *rencontres* numbers, and have the recurrence  $D_n = nD_{n-1} + (-1)^n$ . (See Riordan [R1 57].)

Figure 25 illustrates seq. 587, the Euler numbers  $E_n$ , or the number of permutations of  $n$  objects which first rise and then alternately fall and rise. (Only the second rows of the permutations are shown.)

The even numbered Euler numbers form seq. 1667: 1, 5, 61, 1385, 521, . . . , and have gf

$$1 + 1 \frac{x^2}{2!} + 5 \frac{x^4}{4!} + 61 \frac{x^6}{6!} + \dots = \sec x.$$

Often these are called the Euler numbers instead of seq. 587.)

The odd numbered Euler numbers form seq. 829: 1, 2, 16, 272, 7936, 53792, . . . , and are called the *tangent numbers*  $T_n = E_{2n-1}$ . They have

$$x + 2 \frac{x^3}{3!} + 16 \frac{x^5}{5!} + \dots = \tan x.$$

$E_1 = 1$	1								
$E_2 = 1$		1 2							
$E_3 = 2$		1 3 2	2 3 1						
$E_4 = 5$	1 3 2 4	1 4 2 3	2 3 1 4	2 4 1 3	3 4 1 2				
$E_5 = 16$	1 3 2 5 4	1 4 2 5 3	1 4 3 5 2	1 5 2 4 3	1 5 3 4 2	2 3 1 5 4	2 4 1 5 3	2 4 3 5 1	2 5 1 4 3
	2 5 1 4 3	2 5 3 4 1	3 4 1 5 2	3 4 2 5 1	3 5 1 4 2	3 5 2 4 1	4 5 1 3 2	4 5 2 3 1	

Fig. 25. Seq. 587, the Euler numbers.

The Bernoulli numbers  $B_n$  are defined by

$$B_n = \frac{2n E_{2n-1}}{2^{2n} (2^{2n} - 1)},$$

and form the sequence

$$\frac{1}{6}, \frac{1}{30}, \frac{1}{42}, \frac{1}{30}, \frac{1}{66}, \frac{5}{2730}, \frac{7}{6}, \frac{3617}{510}, \dots$$

th gf

$$1 - \frac{x}{2} + \frac{1}{6} \frac{x^2}{2!} - \frac{1}{30} \frac{x^4}{4!} + \frac{1}{42} \frac{x^6}{6!} - \dots = \frac{x}{e^x - 1}.$$

The numerators and denominators form seqs. 1677 and 1746.

Finally the Genocchi numbers are defined by  $G_n = 2^{2-n} n E_{2n-1}$ , and form seq. 1233: 1, 1, 3, 17, 155, 2073, 38227, . . . , with gf

$$1 \frac{x}{2!} + 1 \frac{x^3}{4!} + 3 \frac{x^5}{6!} + 17 \frac{x^7}{8!} + \dots = \tan \frac{1}{2} x.$$

The Euler, tangent, Bernoulli, and Genocchi numbers arise in all branches of mathematics. For applications and properties see Jordan [JO2], David and Barton [DB1], Comtet [CO1] and Gould [AMM 79 44 72]; for tables see Fletcher *et al.* [FMR 1 65] and Knuth and Buchholz [MTAC 21 663 67].

### 3.14 SEQUENCES FROM NUMBER THEORY

The table contains many number-theoretic sequences, of which the following are typical:

- (1) The prime numbers, lucky numbers, and other sequences generated by sieves (seqs. 241, 377, 1035, 1048);
- (2) the Euler totient function  $\phi(n)$ : the number of integers not exceeding and relatively prime to  $n$  (seq. III);
- (3) from the Goldbach conjecture: the number of ways of writing  $2n$  as a sum of two primes (various sequences—see index);
- (4) quadratic partitions of primes: a prime of the form  $4n + 1$  has a unique representation as  $a^2 + b^2$  with  $a \geq b$ . Sequences 169 and 33 give  $a$  and  $b$ ;
- (5) the number of integers less than or equal to  $2^n$  expressible in the form  $u^2 + v^2$ , where  $u$  and  $v$  are integers (seq. 265);
- (6) Mersenne primes: the numbers  $n$  such that  $2^n - 1$  is prime (seq. 248);
- (7) from Euler's proof that there are an infinity of primes: let  $p_1 = 2, p_2, \dots, p_n$  be primes, and define  $p_{n+1}$  to be the smallest (largest) prime factor of  $p_1 p_2 \dots p_n + 1$  (seqs. 329, 330);
- (8) Beatty sequences: if  $\alpha, \beta$  are positive irrational numbers such that  $(1/\alpha) + (1/\beta) = 1$ , then the Beatty sequences

$$[\alpha], [2\alpha], [3\alpha], \dots \quad \text{and} \quad [\beta], [2\beta], [3\beta], \dots$$

together contain all the positive integers without repetition, where  $[x]$  denotes the greatest integer less than or equal to  $x$ . (See Honsberger [HO2].) For example,  $\alpha = \frac{1}{2}(1 + \sqrt{5}) = 1.61803 \dots$  gives seqs. 917: 1, 3, 4, 6, 8, 9, . . . and 509: 2, 5, 7, 10, 13, 15, . . .

The following test for Beatty sequences is due to R. L. Graham. If  $a_1, a_2, \dots$  is a Beatty sequence, then the values of  $a_1, \dots, a_{n-1}$  determine

to within 1. Look at the sums  $a_1 + a_{n-1}$ ,  $a_2 + a_{n-2}$ , ...,  $a_{n-1} + a_1$ . All these sums have the same value,  $V$  say, then  $a_n$  must equal  $V$  or  $V + 1$ ; but if they take on the two values  $V$  and  $V + 1$ , and no others, then must equal  $V + 1$ . If anything else happens, it is not a Beatty sequence. For example, in seq. 917,  $a_1 + a_1 = 2$  so  $a_2$  must be 2 or 3 (it is 3);  $a_1 + a_2 = 4$  so  $a_3$  must be 4 or 5 (it is 4);  $a_1 + a_3 = 5$  and  $a_2 + a_3 = 6$ , so  $a_4$  must be 6 (it is); and so on.

For further information about number-theoretic sequences see the comprehensive works of Dickson [DI2] and Lehmer [LE1].

3.15 PUZZLE SEQUENCES

This section describes some sequences with simple yet unexpected generating principles. They have all been given as puzzles at one time or other. Of course all of the sequences given in Chapters II and III make good puzzles.

- (1) Sequences related to well-known constants (e.g., seq. 1291: 1, 4, 4, 2, 1, 3, 5, 6, 2, 3, ..., the decimal expansion of  $\sqrt{2}$ ) or to other combinatorial sequences (seq. 2127: 1, 15, 29, 12, 26, 12, 26, 9, ... is related to the Euler—guess!). See also seqs. 684, 880, 1679, 1812, etc.
- (2) Sequences depending on the binary expansions of numbers (e.g., 1.41: 1, 2, 1, 2, 2, 3, 1, 2, 2, ... gives the number of 1's in the binary expansion of  $n + 1$ ; see also seqs. 360, 388).
- (3) Sequences depending on the English words or Arabic numerals used to describe them (e.g., seq. 2218: 1, 21, 21000, 101, 121, 1101, ...; smallest number requiring  $n$  words in English; see also seqs. 1818, 97).
- (4) The terms not in some well-known sequence (e.g., seq. 1319: 4, 7, 9, 10, 11, 12, 14, 15, ... the non-Fibonacci numbers).
- (5) Sequences obtained by bisecting (i.e., taking every other term of) well-known sequences (e.g., seq. 1067: 1, 3, 7, 18, 47, 123, 322, ...; a section of seq. 924, the Lucas numbers; see also seqs. 569, 1101).
- (6) Sequences obtained by alternating the terms of two sequences (e.g., 889: 3, 2, 1, 7, 4, 1, 1, 8, 5, 2, 9, ...; mixing  $\pi$  and  $e$ , is the only simple given).

The following pleasing puzzles are not in the table because they are quite or are not integers.

- (7)  $\frac{1}{2}, \frac{1}{3}, \frac{1}{6}, 12, 24, 30, 120, 240, 1200, 2400$ , English money in 50.
- (8) 3, 8, 8, 4, 89, 75, 30, 28, ?, planetary diameters in thousands of statute miles.

- (9) 8, 5, 4, 9, 1, 7, 6, 3, 2, 0; or 8, 8000000000, ..., 18, 18000000000, ..., 18000000, ..., 18000, ..., 80, ..., 88, ..., 85, ..., 84, ..., 11, ..., 15, ..., 5, ..., 4, ..., the numbers arranged in alphabetical order (in English).
- (10) 12, 13, 14, 15, 20, 22, 30, 110, 1100, the number 12 written to the bases 10, 9, 8, ..., 2.
- (11) 14, 18, 23, 28, 34, 42, 50, 59, 66, 72, 79, 86, 96, 103, 110, 116, 125, 137, 145, 157, 168, 181, 191, 207, 215, 225, 231, 238, 242, the local stops on the New York IRT subway.
- (12) 1714, 1727, 1760, 1820, 1910, 1936, dates of the accessions of the Georges to the English throne.
- (13) 1732, 1735, 1743, 1751, 1758, 1767, 1767, 1782, 1773, 1790, 1795, 1784, 1800, 1804, 1791, 1809, 1808, 1822, 1822, 1831, 1830, 1837, 1833, 1837, 1843, 1858, 1857, 1856, 1865, 1872, 1874, 1882, 1884, 1890, 1917, 1908, 1913, dates of birth of presidents of the U.S.A.
- (14) The integers 1, 2, 3, ... drawn next to a mirror. (See Fig. 26.)
- (15) O, T, T, F, F, S, S, E, N, T, E, T, T, F, F, S, S, E, N, T, T, T, T, ..., the initial letters of the English names for the numbers.

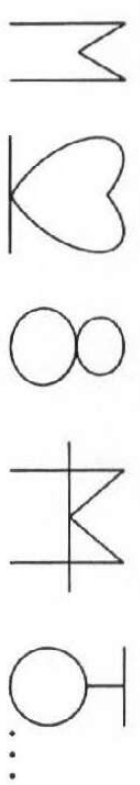


Fig. 26. A puzzle.

3.16 SEQUENCES FROM LATTICE STUDIES IN PHYSICS

In the last twenty years physicists have studied a number of basic combinatorial problems related to crystal lattices. Typical problems are to find the number of self-avoiding paths of length  $n$  on a given lattice, or the number of ways a particular graph can be drawn on the lattice. A number of such sequences will be found in the main table. For further information see Montroll [BE6 96], Sykes *et al.* [JMP 7 1557 66], Kasteleyn [HA1 43], Percus [PE3], and Domb [ACP 15 229 69].











- 94 1, 2, 2, 3, 3, 5, 6, 8, 8, 12, 13, 17, 19, 26, 28, 37, 40, 52, 58, 73, 79, 102, 113, 139, 154, 191, 210, 258, 284, 345, 384, 462, 509, 614, 679, 805, 893, 1060, 1171, 1382  
PARTITIONS INTO NON-PRIME PARTS. REF JNSM 9 91 69.
- 95 1, 2, 2, 3, 3, 5, 6, 8, 9, 11, 14, 19, 22  
ROTATABLE PARTITIONS. REF JLMS 43 504 68.
- 96 1, 2, 2, 3, 4, 1, 8, 1, 10, 9, 16, 18, 12, 42, 4, 58, 38, 82, 88, 54, 188, 18, 248, 151, 334, 338, 260, 760, 120  
FROM SYMMETRIC FUNCTIONS. REF PLMS 23 309 23.
- 97 1, 2, 2, 3, 4, 3, 4, 4, 3, 4, 4, 5, 5, 4, 6, 5, 6, 6, 6, 4, 6, 7, 6, 6, 5, 7, 6, 10, 4, 7, 8, 5, 5, 6, 7, 6, 6, 6, 6, 5, 5, 6, 7, 7, 6, 5, 7, 6, 5, 7, 6, 7, 9, 7, 7, 9, 5, 7, 10, 7, 7  
CONSECUTIVE QUADRATIC NONRESIDUES. REF BAMS 32 284 26.
- 98 1, 2, 2, 3, 4, 5, 6, 7, 8, 10, 11, 13, 14, 16, 18, 20, 22, 26, 29  
DENUMERANTS. REF R1 152.
- 99 1, 2, 2, 3, 4, 5, 6, 7, 8, 11, 12, 15, 16, 19, 22, 25, 28, 34, 40  
DENUMERANTS. REF R1 152.
- 100 1, 2, 2, 3, 4, 5, 6, 8, 10, 12, 15, 18, 22, 27, 32, 38, 46, 54, 64, 76, 89, 104, 122, 142, 165, 192, 222, 256, 296, 340, 390, 448, 512, 585, 668, 760, 864, 982, 1113, 1260, 1426  
PARTITIONS INTO DISTINCT PARTS. REF ASI 836.
- 101 1, 2, 2, 3, 4, 5, 7, 9, 11, 15, 18, 23, 30, 37, 47, 58, 71, 90, 110, 136, 164, 201, 248, 300, 364, 436, 525, 638, 764, 919, 1090, 1297, 1549, 1845, 2194, 2592, 3060, 3590  
MAXIMUM OF A PARTITION FUNCTION. REF JIMS 6 112 42. PSPM 19 172 71.
- 102 1, 2, 2, 3, 4, 5, 7, 9, 12, 16, 21, 28, 37, 49, 65, 86, 114, 151, 200, 265, 351, 465, 616, 816, 1081, 1432, 1897, 2513, 3329, 4410, 5842, 7739, 10252, 13581, 17991, 23833  
 $A(N) = A(N-2) + A(N-3)$ . REF JAZ 90. MMAAG 41 17 68.
- 103 1, 2, 2, 3, 4, 6, 9, 14, 22, 35, 56, 90, 145, 234, 378, 611, 988, 1598, 2585, 4182, 6766, 10947, 17712, 28658, 46369, 75026, 121394, 196419, 317812, 514230, 832041  
FIBONACCI NUMBERS + 1. REF JAZ 97.
- 104 1, 2, 2, 3, 5, 6, 9, 13, 14, 15, 20  
RELATED TO ZARANKIEWICZS PROBLEM. REF TI 126.
- 105 1, 2, 2, 3, 6, 0, 6, 7, 9, 7, 7, 4, 9, 9, 7, 8, 9, 6, 9, 6, 4, 0, 9, 1, 7, 3, 6, 6, 8, 7, 3, 1, 2, 7, 6, 2, 3, 5, 4, 4, 0, 6, 1, 8, 3, 5, 9, 6, 1, 1, 5, 2, 5, 7, 2, 4, 2, 7, 0, 8, 9, 7, 2, 4, 5, 4, 1, 0, 5  
SQUARE ROOT OF 5. REF RSA XVIII. MTAC 22 234 68.
- 106 1, 2, 2, 3, 7, 15, 12, 30, 8, 32, 162, 21  
FROM SEDLACEKS PROBLEM ON SOLUTIONS OF  $X + Y = Z$ . REF GUR.
- 107 1, 2, 2, 3, 7, 25, 121, 721, 5041, 40321, 362881, 3628801, 39916801, 479001601, 6227020801, 87178291201, 1307674368001, 20922789888001, 355687428096001  
FACTORIAL  $N + 1$ . REF ASI 833.
- 108 1, 2, 2, 4, 2, 4, 2, 4, 6, 2, 6, 4, 2, 4, 6, 5, 2, 6, 4, 2, 6, 4, 8, 4, 2, 4, 2, 4, 14, 4, 6, 2, 10, 2, 6, 4, 6, 2, 10, 2, 4, 2, 12, 12, 4, 2, 4, 6, 2, 10, 6, 6, 2, 6, 4, 2, 10, 14, 4, 2, 4  
DIFFERENCES BETWEEN CONSECUTIVE PRIMES. REF ASI 870.

- 109 1, 2, 2, 4, 2, 4, 4, 8, 2, 4, 4, 8, 4, 8, 8, 16, 2, 4, 4, 8, 4, 8, 8, 16, 4, 8, 8, 16, 8, 16, 1  
32, 2, 4, 4, 8, 4, 8, 16, 4, 8, 8, 16, 8, 16, 16, 32, 4, 8, 8, 16, 8, 16, 16, 32, 8, 16, 16, 32  
RELATED TO BINARY EXPANSION OF  $N$ . REF G03.
- 110 1, 2, 2, 4, 2, 6, 2, 6, 4, 10, 2, 12, 6, 4, 16, 6, 18, 4, 6, 10, 22, 2, 20, 12, 18, 6, 28, 4, 30, 8, 10, 16, 12, 6, 36, 18, 12, 4, 40, 6, 42, 10, 12, 22, 46, 4, 42, 20, 16, 12, 52, 18, 20  
REDUCED TOTIENT FUNCTION. REF CAU (2) 12 43. LEI 7.
- 111 1, 2, 2, 4, 2, 6, 4, 6, 4, 10, 4, 12, 6, 8, 8, 16, 6, 18, 8, 12, 10, 22, 8, 20, 12, 18, 12, 28, 8, 30, 16, 20, 16, 24, 12, 36, 18, 24, 16, 40, 12, 42, 20, 24, 22, 46, 16, 42, 20, 32, 24  
EULER TOTIENT FUNCTION. REF ASI 840. MTAC 23 682 69.
- 112 1, 2, 2, 4, 4, 6, 7, 10, 11, 16, 17, 23, 26, 33, 37, 47, 52, 64, 72, 86, 96, 115, 127, 149, 166, 192, 212, 245, 269, 307, 338, 382, 419, 472, 515, 576, 629, 699, 760, 843, 913  
EXPANSION OF A GENERATING FUNCTION. REF CAY 10 415.
- 113 1, 2, 2, 4, 4, 7, 8, 12, 14, 21, 24, 34, 41, 55, 66, 88, 105, 137, 165, 210, 253, 320, 383, 478, 574, 708, 847, 1039, 1238, 1507, 1794, 2167, 2573, 3094, 3660, 4378, 5170  
PARTITIONS WITH NO PART OF SIZE 1. REF TAI 1 334. ASI 836.
- 114 1, 2, 2, 4, 4, 8, 9, 18, 23, 44, 63, 122, 180, 362, 612, 1162, 2056, 3912, 7155, 1364  
NECKLACES. REF JIM 5 662 61.
- 115 1, 2, 2, 4, 4, 8, 10, 20, 30, 56, 94, 180, 316, 596, 1096, 2068, 3856, 7316, 13798, 26272  
NECKLACES. REF JIM 5 662 61.
- 116 1, 2, 2, 4, 5, 7, 9, 13, 16, 22, 27, 36, 44, 57, 70, 89, 108, 135, 163, 202, 243, 297, 355, 431, 513, 617, 731, 874, 1031, 1225, 1439, 1701, 1991, 2341, 2731, 3197, 3717  
PARTITIONS INTO PARTS PRIME TO 3. REF PSPM 8 145 65.
- 117 1, 2, 2, 4, 5, 9, 12, 21, 30, 51, 76, 127, 195, 322, 504, 826, 1309, 2135, 3410, 5545, 8900, 14445, 23256, 37701, 60813, 98514, 159094, 257608, 416325, 673933, 1089648  
PACKING A BOX WITH DOMINOES. REF AMM 69 61 62.
- 118 1, 2, 2, 4, 5, 10, 14, 26, 42, 78, 132, 249, 445, 842, 1561, 2988, 5671, 10981, 21209, 41472, 81181, 160176, 316749, 629333, 1256070, 2515169, 5049816, 10172638  
SERIES-REDUCED TREES. REF AM1 101 150 59. HAS 232. CA3.
- 119 1, 2, 2, 4, 6, 6, 11, 16, 20, 28, 41, 51, 70, 93, 122  
PLANAR PARTITIONS. REF MA2 2 332.
- 120 1, 2, 2, 4, 6, 8, 18, 20, 56, 48, 178, 132, 574, 348, 1870, 1008  
FOLDING A STRIP OF STAMPS. REF JCT 5 151 68.
- 121 1, 2, 2, 4, 6, 10, 16, 30, 52, 94  
SHIFT REGISTERS. REF G01 172.
- 122 1, 2, 2, 4, 6, 11, 18, 37, 66, 135, 265  
BORON TREES. REF CAY 9 451.
- 123 1, 2, 2, 4, 6, 12, 20, 39, 71, 137, 261, 511, 995, 1974, 3915, 7841, 15749, 31835, 64540, 131453, 268498, 550324, 1130899, 2330381, 4813031, 9963288, 20665781  
SERIES-REDUCED PLANTED TREES. AM1 101 150 59. CA3.
- 124 1, 2, 2, 4, 7, 12, 16, 32  
COVERING NUMBERS. REF JLMS 44 80 69.

- 125 1, 2, 2, 4, 8, 4, 16, 12, 48, 80, 136, 420, 1240, 2872, 7652, 18104, 50184  
QUEENS PROBLEM. REF PSAM 10 93 60.
- 126 1, 2, 2, 4, 8, 13, 25, 44, 83, 152, 286, 538, 1020, 1942, 3725, 7145, 13781, 26627,  
51572, 100099, 194633, 379037, 739250, 1443573, 2822186, 5522889  
POPULATION OF  $U^{*+2} + 16V^{*+2}$ . REF MTAC 20 567 66.
- 127 1, 2, 2, 4, 10, 16, 28, 48, 76, 110, 144, 182, 222, 264, 310, 356, 408, 458, 536, 610,  
664, 762, 842, 924, 1010, 1096, 1188, 1288, 1396, 1510, 1624, 1742, 1862  
PERIODIC DIFFERENCES. REF TCPS 2 220 1827.
- 128 1, 2, 2, 4, 10, 28, 84, 264, 858, 2860, 9724, 33592, 117572, 416024, 1485800,  
5348860, 19389690, 70715340  
FROM BINOMIAL COEFFICIENTS. REF TH1 164, FMR 1 55.
- 129 1, 2, 2, 4, 12, 22, 58, 158, 448, 1342, 4199, 13384  
POLYTOPES BY NUMBER OF EDGES. REF JCT 7 157 69.
- 130 1, 2, 2, 5, 4, 7, 7, 11, 9, 8, 6, 9, 4, 6, 22, 10, 4, 8, 4  
PRIMITIVE GROUPS. REF JL2 178.
- 131 1, 2, 2, 5, 5, 12, 12, 27, 28, 64, 67, 147, 158, 348, 373, 799, 879, 1886, 2069, 4335,  
4864  
SQUARE FILAMENTS. REF PL2 1 337 70.
- 132 1, 2, 2, 6, 6, 18, 16, 48, 60, 176, 144, 630, 756, 1800, 2048, 7710, 7776, 27594,  
24000, 84672  
RELATED TO EULERS TOTIENT FUNCTION. REF BE1 296.
- 133 1, 2, 2, 6, 8, 18, 30, 67, 127  
BORON TREES. REF CAY 9 451.
- 134 1, 2, 2, 6, 9, 17, 30, 54, 98, 183, 341, 645, 1220, 2327, 4451, 8555, 16489, 31859,  
61717, 119779, 232919, 453584, 884544, 1727213, 3376505, 6607371  
POPULATION OF  $U^{*+2} + 12V^{*+2}$ . REF MTAC 20 567 66.
- 135 1, 2, 2, 6, 9, 20, 37, 86, 183, 419  
HYDROCARBONS. REF BS1 201.
- 136 1, 2, 2, 6, 14, 34, 82, 198, 478, 1154, 2786, 6726, 16238, 39202, 94642, 229486,  
551814, 1331714, 3215042, 7761798, 18738638, 45239074, 109216786, 263672646  
 $A(N) = 2A(N-1) + A(N-2)$ . REF AJM 1 187 1878.
- 137 1, 2, 2, 6, 16, 20, 132, 28, 1216, 936, 23540, 34782, 138048, 469456, 1601264,  
9112660, 18108928, 182135008, 161934624, 3804634784, 404007680, 83297957568  
FROM PERMUTATIONS OF ORDER 2. REF CJM 7 168 55.
- 138 1, 2, 2, 6, 38, 390, 6062, 134526  
COLORED GRAPHS. REF CJM 22 596 70.
- 139 1, 2, 2, 7, 10, 20, 36, 65, 118, 221, 409, 776, 1463, 2788, 5328, 10222, 19714,  
38054, 73685, 142944, 277838, 540889, 1054535, 2058537, 4023278  
POPULATION OF  $U^{*+2} + 10V^{*+2}$ . REF MTAC 20 563 66.
- 140 1, 2, 2, 8, 8, 112, 656, 5504, 49024, 491264  
RELATED TO LATIN RECTANGLES. REF R1 210.
- 141 1, 2, 2, 8, 12, 88, 176, 2752, 8784  
SELF-COMPLEMENTARY ORIENTED GRAPHS. REF KNAW 73 443 70.
- 142 1, 2, 2, 8, 72, 1536, 86080, 14487040, 8274797440, 17494930604032  
THRESHOLD FUNCTIONS. REF PGEC 19 821 70.
- 143 1, 2, 2, 9, 11, 37, 79, 249, 671, 2182, 6692  
POLYTOPES BY NUMBER OF EDGES. REF JCT 7 157 69.
- 144 1, 2, 2, 10, 28, 207, 1288, 10366, 91266  
HIT POLYNOMIALS. REF R13.
- 145 1, 2, 2, 10, 218, 64594, 4294642034, 18446744047940725978,  
3402823669209384463334247399005993378250  
NONDEGENERATE BOOLEAN FUNCTIONS. REF HA2 170.
- 146 1, 2, 2, 10, 52246, 2631645209645100680142  
INVERTIBLE BOOLEAN FUNCTIONS. REF PGEC 13 350 64.
- 147 1, 2, 2, 17, 1, 91  
QUEENS PROBLEM. REF SL1 49.
- 148 1, 2, 2, 18, 66, 374, 1694, 9822, 51698  
BAXTER PERMUTATIONS. REF MA4 2 25 67.
- 149 1, 2, 2, 20, 38, 146, 368, 1070, 2824, 7680, 19996  
SUSCEPTIBILITY FOR SQUARE LATTICE. REF PHA 28 924 62.
- 150 1, 2, 2, 22, 563, 1676257  
TYPES OF LATIN SQUARES. REF R1 210. FY1 22. JCT 5 177 68.

## SEQUENCES BEGINNING 1, 2, 3

- 151 1, 2, 3, 0, 2, 5, 8, 5, 0, 9, 2, 9, 4, 0, 4, 5, 6, 8, 4, 0, 1, 7, 9, 9, 1, 4, 5, 4, 6, 8, 4, 3  
6, 4, 2, 0, 7, 6, 0, 1, 1, 0, 1, 4, 8, 8, 6, 2, 8, 7, 7, 2, 9, 7, 6, 0, 3, 3, 2, 7, 9, 0, 0, 9, 6, 7, 1  
NATURAL LOGARITHM OF 10. REF RS4 2.
- 152 1, 2, 3, 0, 11, 0, 17, 15, 14, 51  
A PARTITION FUNCTION. REF JNSM 9 103 69.
- 153 1, 2, 3, 0, 25, 152, 1350, 12644, 131391, 1489568, 18329481, 243365514,  
3468969962, 52848096274, 857073295427, 14744289690560, 268202790690465  
FROM DISCORDANT PERMUTATIONS. REF KYU 10 13 56.
- 154 1, 2, 3, 1, 4, 5, 1, 3, 1, 3, 1, 1, 8, 15, 3, 7, 4, 5, 2, 3, 3, 6, 2, 3, 2, 3, 1, 1, 16, 19,  
10, 5, 15, 4, 5, 7, 15, 3, 7, 4, 5, 2, 3, 5, 13, 3, 5, 4, 7, 1, 3, 3, 5, 2, 3, 1, 3, 1, 1, 32, 47, 11  
A PROBLEM IN PARITY. REF IJ1 11 163 69.
- 155 1, 2, 3, 1, 2, 3, 4, 2, 1, 2, 3, 3, 2, 3, 4, 1, 2, 2, 3, 3, 4, 3, 1, 2, 3, 4, 2, 3, 4, 2, 3,  
2, 3, 1, 2, 3, 4, 2, 2, 3, 3, 3, 2, 3, 4, 3, 1, 2, 3, 2, 2, 3, 4, 3, 3, 2, 3, 4, 2, 3, 4, 1, 2, 3, 3, 2, 3,  
LEAST NUMBER OF SQUARES TO REPRESENT N.

- 156 1, 2, 3, 1, 4, 3, 2, 1, 4, 2, 6, 4, 1, 2, 7, 1, 4, 3, 2, 1, 4, 6, 7, 4, 1, 2, 8, 5, 4, 7, 2, 1, 8, 6, 7, 4, 1, 2, 3, 1, 4, 7, 2, 1, 8, 2, 7, 4, 1, 2, 8, 1, 4, 7, 2, 1, 4, 2, 7, 4, 1, 2, 8, 1, 4, 7, 2, 1, 4, 2, 8, 1, 4, 7, 2, 1, 8  
THE GAME OF KAYLES. REF PCPS 52 516 56.
- 157 1, 2, 3, 1, 5, 4, 3, 3, 9, 2, 11, 5  
A NUMBER-THEORETIC FUNCTION. REF MTS 67 11 58.
- 158 1, 2, 3, 2, 0, 1, 7, 2, 6, 8, 22, 7, 0, 33, 3, 14, 51, 46, 19, 12, 94, 42, 23, 113, 150, 54, 48, 345, 116, 109, 403, 498, 140, 219, 1057, 326, 259, 1271, 1641, 308, 656, 3396  
FROM SYMMETRIC FUNCTIONS. REF PLMS 23 297 23.
- 159 1, 2, 3, 2, 1, 2, 2, 4, 2, 2, 1, 0, 4, 2, 3, 2, 2, 4, 0, 2, 2, 0, 4, 2, 3, 0, 2, 6, 2, 2, 1, 2, 0, 2, 2, 2, 4, 2, 0, 4, 0, 1, 2, 0, 4, 2, 0, 2, 2, 5, 2, 0, 2, 2, 4, 4, 2, 0, 2, 4, 2, 0, 2, 4, 0, 0  
THE SQUARE OF EULERS PRODUCT. REF PLMS 21 190 1889.
- 160 1, 2, 3, 2, 2, 4, 4, 4, 4, 4, 3, 5, 4, 3, 5, 5, 6, 6, 4, 6, 7, 4, 4, 7, 7, 6, 5, 5, 7, 8, 6, 5, 4, 7, 6, 6, 6, 6, 6, 4, 7, 6, 7, 7, 5, 6, 6, 7, 6, 7, 8, 7, 10, 7, 9, 9, 7, 10, 5, 5  
CONSECUTIVE QUADRATIC RESIDUES. REF BAMS 32 284 26.
- 161 1, 2, 3, 2, 3, 4, 4, 4, 5, 6, 5, 4, 6, 4, 7, 8, 3, 6, 8, 6, 7, 10, 8, 6, 10, 6, 7, 12, 5, 10, 12, 4, 10, 12, 9, 10, 14, 8, 9, 16, 9, 8, 18, 8, 9, 14, 6, 12, 16, 10, 11, 16, 12, 14, 20, 12, 11, 24  
DECOMPOSITIONS OF  $2N$  INTO SUM OF 2 ODD PRIMES. REF FVS 4(4) 7 27. LE1 80.
- 162 1, 2, 3, 2, 5, 2, 3, 7, 2, 11, 13, 2, 3, 5, 17, 19, 2, 23, 7, 29, 3, 31, 2, 37, 41, 43, 47, 5, 53, 59, 2, 11, 61, 3, 67, 71, 73, 79, 13, 83, 89, 2, 97, 101, 103, 107, 7, 109, 113, 17, 127  
RELATED TO HIGHLY COMPOSITE NUMBERS. REF RAM 115.
- 163 1, 2, 3, 2, 5, 5, 7, 10, 12, 17, 22, 29, 39, 51, 68, 90, 118, 158, 209, 277, 367, 486, 644, 853, 1130, 1497, 1983, 2627, 3480, 4610, 6107, 8090, 10717, 14197, 18807, 24914  
 $A(N) = A(N-2) + A(N-3)$ . REF AMM 15 209 08. JAZ 90. FQ 6(3) 68 66.
- 164 1, 2, 3, 3, 3, 5, 7, 6, 6, 10, 12, 11, 13, 17, 20, 21, 21, 27, 34, 33, 36, 46, 51, 53, 58, 68, 78, 82, 89, 104, 118, 123, 131, 154, 171, 179, 197, 221, 245, 262, 279, 314, 349, 369  
MOCK THETA NUMBERS. REF TAMS 72 495 52.
- 165 1, 2, 3, 3, 5, 9, 16, 28, 50, 89, 159, 285, 510, 914, 1639, 2938, 5269, 9451, 16952, 30410, 54555, 97871, 175588, 315016, 565168, 1013976, 1819198, 3263875, 5855833  
BINARY CODES. REF PGIT 17 309 71.
- 166 1, 2, 3, 4, 3, 5, 3, 6, 1, 2, 6, 7, 4, 5, 8, 3, 9, 7, 6, 9, 1, 2, 6, 11, 4, 10, 9, 3, 12, 9, 12, 13, 8, 3, 14, 12, 13, 6, 1, 2, 12, 11, 5, 15, 16, 9, 3, 13, 8, 15, 12, 17, 16, 6, 14, 15, 10, 3, 17  
QUADRATIC PARTITIONS OF PRIMES. REF CU2 1. LE1 55. MITAC 23 459 89.
- 167 1, 2, 3, 4, 5, 3, 7, 4, 6, 5, 11, 4, 13, 7, 5, 6, 17, 6, 19, 5, 7, 11, 23, 4  
N DIVIDES FACTORIAL  $A(N)$ . REF AMM 25 210 18.
- 168 1, 2, 3, 4, 5, 5, 7, 6, 6, 7, 11, 7, 13, 9, 8, 8, 17, 8, 19, 9, 10, 13, 23, 9, 10, 15, 9, 11, 29, 10, 31, 10, 14, 19, 12, 10, 37, 21, 16, 11, 41, 12, 43, 15, 11, 25, 47, 11, 14, 12, 20, 17  
SUM OF PRIMES DIVIDING  $N$ . REF MTAC 23 181 69.
- 169 1, 2, 3, 4, 5, 6, 5, 7, 6, 8, 8, 9, 10, 10, 8, 11, 10, 11, 13, 10, 12, 14, 15, 13, 15, 16, 13, 14, 16, 17, 13, 14, 16, 18, 17, 18, 17, 19, 20, 20, 15, 17, 20, 21, 19, 22, 20, 21, 19, 20  
QUADRATIC PARTITIONS OF PRIMES. REF CU2 1. AMM 56 526 49.
- 170 1, 2, 3, 4, 5, 6, 7, 1, 2, 3, 4, 5, 6, 7, 8, 2, 3, 4, 5, 6, 7, 8, 9, 3, 4, 5, 1, 2, 3, 4, 5, 4, 5, 6, 2, 3, 4, 5, 6, 5, 6, 7, 3, 4, 5, 6, 7, 8, 4, 5, 6, 2, 3, 4, 5, 6, 5, 6, 7, 3, 4, 1, 2, 3, 4, 5, 6  
LEAST NUMBER OF CUBES TO REPRESENT  $N$ . REF JRAM 14 279 1835. LE1 81.
- 171 1, 2, 3, 4, 5, 6, 7, 6, 6, 10, 11, 12, 13, 14, 15, 8, 17, 12, 19, 20, 21, 22, 23, 18, 10, 26, 9, 28, 29, 30, 31, 10, 33, 34, 35, 24, 37, 38, 39, 30, 41, 42, 43, 44, 30, 46, 47, 24, 14  
MOSAIC NUMBERS. REF BAMS 69 446 63. CUM 17 1010 65.
- 172 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 3, 4, 5, 6, 7, 8, 9, 10, 11, 1  
LEAST NUMBER OF FOURTH POWERS TO REPRESENT  $N$ . REF JRAM 46 3 1853. LE1 82.
- 173 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 4  
THE NATURAL NUMBERS.
- 174 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 42, 43, 45, 46, 47, 48, 50, 51, 52, 5  
 $N+2 + N + 41$  IS PRIME
- 175 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 15, 20, 22, 24, 30, 33, 36, 40, 44, 48, 50, 55, 60, 66, 70, 77, 80, 88, 90, 99, 100, 101, 102, 104, 105, 110, 111, 112, 115, 120, 122, 124, 1  
DIVISIBLE BY EACH DIGIT. REF JRM 1 217 68.
- 176 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 12, 13, 15, 16, 18, 21, 22, 24, 25, 28, 30, 33, 37, 40, 42, 45, 48, 57, 58, 60, 70, 72, 78, 85, 88, 93, 102, 105, 112, 120, 130, 133, 165, 168, 177, 1  
THE SUITABLE NUMBERS OF EULER. REF BO1 427.
- 177 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 12, 14, 15, 16, 18, 20, 21, 24, 25, 27, 28, 30, 32, 35, 36, 40, 42, 45, 48, 49, 50, 54, 56, 60, 63, 64, 70, 72, 75, 80, 81, 84, 90, 96, 98, 100, 105, 10  
CONTAIN NO PRIME FACTOR GREATER THAN 7.
- 178 1, 2, 3, 4, 5, 6, 7, 8, 9, 11, 22, 33, 44, 55, 66, 77, 88, 99, 101, 111, 121, 131, 141, 151, 161, 171, 181, 191, 202, 212, 222, 232, 242, 252, 262, 272, 282, 292, 303, 313, 32  
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- 179 1, 2, 3, 4, 5, 6, 7, 9, 10, 11, 12, 14, 16, 17, 20, 21, 22, 25, 27, 29, 31, 32, 36, 39, 4, 42, 45, 46, 47, 49, 51, 54, 55, 56, 57, 60, 61, 65, 66, 67, 69, 71, 77, 84, 86, 87, 90, 94  
 $N(N-1) - 1$  IS PRIME. REF PO1 249. LE1 46.
- 180 1, 2, 3, 4, 5, 6, 8, 9, 10, 11, 12, 14, 15, 16, 18, 20, 21, 22, 23, 24, 26, 27, 28, 29, 3, 32, 33, 35, 36, 39, 40, 41, 42, 44, 46, 48, 50, 51, 52, 53, 54, 55, 56, 58, 60, 63, 64, 65  
VALUES OF EULER TOTIENT FUNCTION. REF BA2 64 (DIVIDED BY 2).
- 181 1, 2, 3, 4, 5, 6, 8, 9, 14, 15, 16, 22, 28, 29, 36, 37, 54, 59, 85, 93, 117, 119, 161, 189, 193, 256, 308, 322, 327, 411, 466, 577, 591, 902, 928, 946  
 $45 \cdot 2 \cdot N - 1$  IS PRIME. REF MTAC 23 874 69.
- 182 1, 2, 3, 4, 5, 6, 8, 10, 11, 13, 16, 18, 20, 23, 26, 29, 32, 35, 39, 43, 46, 50, 55, 59, 63, 68, 73, 78, 83, 88, 94, 100, 105, 111, 118, 124, 130, 137, 144, 151, 158, 165, 173, 1  
GENUS OF COMPLETE GRAPH. REF PNAS 60 438 88.
- 183 1, 2, 3, 4, 5, 7, 8, 9, 10, 11, 12, 14, 15, 16, 18, 19, 21, 22, 23, 24, 25, 26, 29, 30, 3, 32, 33, 35, 37, 38, 40, 42, 43, 44, 45, 46, 47, 49, 51, 52, 53, 54, 56, 57, 58, 60, 63, 64  
A NUMBER-THEORETIC FUNCTION. REF IAS 5 382 37.
- 184 1, 2, 3, 4, 5, 7, 8, 9, 10, 12, 13, 14, 15  
WYTHOFF GAME. REF CMB 2 189 59.

- 185 1, 2, 3, 4, 5, 7, 8, 9, 11, 13, 16, 17, 19, 23, 25, 27, 29, 31, 32, 37, 41, 43, 47, 49, 53, 59, 61, 64, 67, 71, 73, 79, 81, 83, 89, 97, 101, 103, 107, 109, 113, 121, 125, 127, 128  
PRIME POWERS. REF ASI 870.
- 186 1, 2, 3, 4, 5, 7, 8, 10, 12, 14, 16, 19, 21, 24, 27, 30, 33, 37, 40, 44, 48, 52, 56, 61, 65, 70, 75, 80, 85, 91, 96, 102, 108, 114, 120, 127, 133, 140, 147, 154, 161, 169, 176, 184  
PARTITIONS INTO AT MOST 3 PARTS. REF RS2 2.
- 187 1, 2, 3, 4, 5, 7, 9, 11, 13, 16, 17, 19, 23, 24, 25, 29, 30  
A TWO-WAY CLASSIFICATION OF INTEGERS. REF CMB 2 89 59.
- 188 1, 2, 3, 4, 5, 7, 10, 11, 12, 14, 15, 18, 24, 25, 26, 28, 29, 31, 33, 35, 38, 39, 42, 43, 46, 49, 50, 53, 56, 59, 63, 64, 67, 68, 75, 81, 82, 87, 89, 91, 92, 94, 96, 106, 109, 120, 124  
FROM CUBAN PRIMES. REF MES 41 144 12.
- 189 1, 2, 3, 4, 6, 10, 19, 27, 33, 39, 157, 183, 386, 664, 687, 969, 1281, 1332, 2917, 2993, 3376, 6002  
SIZE OF MERSENNE PRIMES. REF BE3 19. NAMS 18 608 71.
- 190 1, 2, 3, 4, 6, 12, 15, 20, 30, 60, 60, 84, 105, 140, 210, 210, 420, 420, 420, 420, 840, 840, 1260, 1540, 2310, 2520, 4620, 4620, 5460, 5460, 9240, 9240, 13860  
LARGEST ORDER OF PERMUTATION OF N SYMBOLS. REF BSMF 97 187 69.
- 191 1, 2, 3, 4, 6, 13, 10, 24, 22, 45, 30, 158, 74, 245, 368, 693, 522, 2637, 1610, 7341  
NECKLACES. REF IJM 8 269 64.
- 192 1, 2, 3, 4, 6, 7, 8, 9, 11, 12, 13, 14, 16, 17, 18, 19, 21, 22, 23, 24, 25, 27, 28, 29, 30, 32, 33, 34, 35, 37, 38, 39, 40, 42, 43, 44, 45, 46, 48, 49, 50, 51, 53, 54, 55, 56, 58, 59  
A BEATTY SEQUENCE. REF CMB 2 189 58.
- 193 1, 2, 3, 4, 6, 7, 9, 10, 12, 13, 14, 16, 17  
WYTHOFF GAME. REF CMB 2 188 59.
- 194 1, 2, 3, 4, 6, 7, 9, 15, 22, 28, 30, 46, 60, 63, 127, 153, 172, 303, 471, 532, 865, 900, 1366, 2380, 3310, 4495, 6321, 7447, 10198, 11425, 21846, 24369, 27286, 28713  
 $X^{*n} + X + 1$  IS IRREDUCIBLE OVER  $GF(2)$ . REF IC 16 502 70.
- 195 1, 2, 3, 4, 6, 7, 11, 18, 34, 38, 43, 55, 64, 76, 94, 103, 143, 206, 216, 306, 324, 391, 458, 470, 827  
 $3 \cdot 2^{*n} - 1$  IS PRIME. REF MTAC 23 874 69.
- 196 1, 2, 3, 4, 6, 8, 9, 10, 12, 16, 18, 20, 24, 30, 32, 36, 40, 48, 60, 64, 72, 80, 84, 90, 96, 100, 108, 120, 128, 144, 160, 168, 180, 192, 200, 216, 224, 240, 256, 288, 320, 336  
RELATED TO HIGHLY COMPOSITE NUMBERS. REF RAM 87.
- 197 1, 2, 3, 4, 6, 8, 9, 11, 12, 16, 17, 18, 19, 22, 24, 25, 27, 32, 33, 34, 36, 38, 41, 43, 44, 48, 49, 50  
OF THE FORM  $X^{*2} + 2Y^{*2}$ . REF EUL (1) 1 421 11. LEI 59.
- 198 1, 2, 3, 4, 6, 8, 9, 12, 15, 16, 21, 24, 24, 32, 36, 36, 45, 48, 48, 60, 66, 64, 75, 84, 81, 96, 105, 96, 120, 128, 120, 144, 144, 171, 180, 168, 192, 210, 192, 231, 240, 216  
DEGREES OF RATIONAL POLYNOMS. REF BU2 39 103 47.
- 199 1, 2, 3, 4, 6, 8, 10, 12, 15, 18, 21, 24, 28, 32, 36, 40, 45, 50  
RESTRICTED PARTITIONS. REF CAY 2 277.
- 200 1, 2, 3, 4, 6, 8, 10, 12, 16, 18, 20, 24, 30, 36, 42, 48, 60, 72, 84, 90, 96, 108, 120, 144, 168, 180, 210, 216, 240, 288, 300, 336, 360, 420, 480, 504, 540, 600, 630, 660, 720  
HIGHLY ABUNDANT NUMBERS. REF TAMS 56 467 44.
- 201 1, 2, 3, 4, 6, 8, 11, 13, 16, 18, 26, 28, 36, 38, 47, 48, 53, 57, 62, 69, 72, 77, 82, 87, 97, 99, 102, 106, 114, 126, 131, 138, 145, 148, 155, 175, 177, 180, 182, 189, 197, 206  
A SELF-GENERATING SEQUENCE. REF UL I X. ATI 249.
- 202 1, 2, 3, 4, 6, 8, 13, 18, 30, 46, 78, 126, 224, 380, 687, 1224, 2250, 4112, 7685, 14310, 27012  
NECKLACES. REF IJM 5 662 61.
- 203 1, 2, 3, 4, 6, 8, 14, 20, 36, 60, 108, 188, 352, 632, 1182, 2192, 4116, 7712, 14602, 27596, 52488, 99880, 190746, 364724, 689252, 1342184, 2581428, 4971068, 9587580  
NECKLACES OF 2 COLORS. REF IJM 5 662 61. GOI 172.
- 204 1, 2, 3, 4, 6, 9, 12, 16, 22, 29, 38, 50, 64, 82, 105, 132, 166, 208, 258, 320, 395, 484, 592, 722, 876, 1060  
COEFFICIENTS OF AN ELLIPTIC FUNCTION. REF CAY 9 128.
- 205 1, 2, 3, 4, 6, 9, 12, 18, 27, 36, 54, 81, 108, 162, 243, 324, 486, 729, 972, 1458, 2187, 2916, 4374, 6561, 8748, 13122, 19683, 26244, 39366, 59049, 78732, 118098  
SUBGROUPS OF SYMMETRIC GROUP. REF CMB 8 627 65. JRM 4 168 71.
- 206 1, 2, 3, 4, 6, 9, 13, 19, 27, 38, 54, 77, 109, 154, 218, 309, 437, 618, 874, 1236, 1748, 2472, 3496, 4944, 6992, 9888, 13984, 19777, 27969, 39554, 55938, 79108, 111871  
A NONLINEAR RECURRENCE. REF MMAG 43 143 70.
- 207 1, 2, 3, 4, 6, 9, 13, 19, 28, 41, 60, 88, 129, 189, 277, 406, 595, 872, 1278, 1873, 2745, 4023, 5896, 8641, 12664, 18560, 27201, 39865, 58425, 85626, 125491, 183916  
 $A(N) = A(N-1) + A(N-3)$ . REF LA3 13. FQ 2 225 64. JAZ 91. MMAG 41 15 68.
- 208 1, 2, 3, 4, 6, 9, 14, 22, 35, 56, 90, 145, 234, 378, 611, 988, 1598, 2585, 4182, 6766, 10947, 17712, 28658, 46369, 75026, 121394, 196419, 317812, 514230, 832041  
RESTRICTED PERMUTATIONS. REF CMB 4 32 61 (DIVIDED BY 3).
- 209 1, 2, 3, 4, 6, 9, 14, 23, 38  
PAIRWISE PRIME POLYNOMIALS. REF IC 13 615 68.
- 210 1, 2, 3, 4, 6, 12, 15, 20, 30, 60, 84, 105, 140, 210, 420, 840, 1260, 1540, 2310, 2520, 4620, 5460, 9240, 13860, 16380, 27720, 30030, 32760, 60060, 120120, 180180  
LARGEST ORDER OF PERMUTATION OF N SYMBOLS. REF BSMF 97 187 69.
- 211 1, 2, 3, 4, 6, 16, 16, 30  
POINT-SYMMETRIC TOURNAMENTS. REF CMB 13 322 70.
- 212 1, 2, 3, 4, 7, 13, 24, 44, 83, 157, 297, 567, 1085, 2086, 4019, 7766, 15039, 29181, 56717, 110408, 215225, 420076, 820836, 1605587, 3143562, 6160098, 12080946  
LANDAUS APPROXIMATION. REF MTAC 18 79 84.
- 213 1, 2, 3, 4, 8, 10, 12, 19, 37, 54, 65, 77, 314, 366, 770, 1327, 1373, 1937, 2561, 2663, 5834, 5985, 6751, 12003  
SIZE OF EVEN PERFECT NUMBERS. REF BE3 19. NAMS 18 608 71.
- 214 1, 2, 3, 4, 9, 27, 512, 134217728, (NEXT TERM HAS 155 DIGITS)  
AN EXPONENTIAL FUNCTION ON PARTITIONS. REF AMM 76 830 69.

- 715 1, 2, 3, 4, 11, 17, 29, 49, 65, 144  
PARTITION FUNCTION. REF JNSM 9 103 69.
- 716 1, 2, 3, 4, 40, 210, 1477, 11672, 104256, 1036050  
FROM MENAGE POLYNOMIALS. REF R1 197.
- 717 1, 2, 3, 5, 1, 13, 7, 17, 11, 89, 1, 233, 29, 61, 47, 1597, 19, 4181, 41  
PRIMITIVE DIVISORS OF FIBONACCI NUMBERS. REF FO 1(3) 15 63.
- 718 1, 2, 3, 5, 4, 7, 6, 9, 13, 8, 10, 19, 14, 12, 29, 16, 21, 22, 37, 18, 27, 20, 43, 33, 34, 8, 49, 24, 61, 32, 67, 30, 73, 45, 57, 44, 40, 36, 50, 42, 52, 101, 63, 85, 109, 91, 74, 54  
VERSE OF A DIVISOR FUNCTION. REF BA2 85.
- 719 1, 2, 3, 5, 5, 7, 7, 7, 11, 9, 9, 11, 13, 11, 11, 15, 13, 13, 13, 17, 15, 19, 15, 19, 17, 11, 17, 19, 17, 17, 19, 21, 25, 19, 19, 23, 25, 23, 21, 23, 21, 21, 29, 23, 25, 23, 27, 29, 23  
QUADRATIC PARTITIONS OF PRIMES. REF CU2 1, LE1 55.
- 720 1, 2, 3, 5, 6, 5, 8, 9, 11, 10, 7, 15, 15, 14, 17, 24, 24, 21, 13, 19, 27, 25, 29, 26, 44, 4, 29, 46, 39, 46, 27, 42, 47, 47, 54, 35, 41, 60, 51, 37, 48, 45, 49, 50, 49, 53  
NUMBERS WITH INTEGRAL HARMONIC MEAN. REF AMM 61 95 54.
- 721 1, 2, 3, 5, 6, 6, 7, 8, 10, 13, 13, 14, 17, 17, 18, 19, 20, 22, 23, 27, 29, 29, 19, 31, 32, 35, 36, 37, 40, 43, 46, 48, 50, 53, 55, 57, 60, 60, 61, 63, 66, 66, 68, 71, 74, 77  
ATTICE POINTS IN CIRCLES. REF MTAC 20 306 66.
- 722 1, 2, 3, 5, 6, 7, 2, 10, 11, 3, 13, 14, 15, 17, 2, 19, 5, 21, 22, 23, 6, 26, 3, 7, 29, 30, 11, 2, 33, 34, 35, 37, 38, 39, 10, 41, 42, 43, 11, 5, 46, 47, 3, 2, 51, 13, 53, 6, 55, 14, 57, 58  
REMOVE SQUARES FROM N. REF NCM 4 168 1878.
- 723 1, 2, 3, 5, 6, 7, 8, 10, 11, 12, 13, 14, 15, 17, 18, 19, 20, 21, 22, 23, 24, 26, 27, 28, 19, 30, 31, 32, 33, 34, 35, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 50, 51, 52, 53, 54  
10 SQUARES. REF HO2 97.
- 724 1, 2, 3, 5, 6, 7, 8, 10, 11, 13, 14, 15, 17, 19, 21, 22, 23, 24, 26, 27, 29, 30, 31, 32, 13, 34, 35, 37, 38, 39, 40, 41, 42, 43, 46, 47, 51, 53, 54, 55, 56, 57, 58, 59, 61, 62, 65, 66  
CONTAIN ODD POWERS ONLY. REF AMM 73 139 66.
- 725 1, 2, 3, 5, 6, 7, 9, 10, 11, 13, 14, 15, 16, 18, 19, 20, 22, 23, 24, 26, 27, 28, 29, 31, 32, 33, 35, 36, 37, 39, 40, 41, 42, 44, 45, 46, 48, 49, 50, 52, 53, 54, 56, 57, 58, 59, 61, 62  
A BEATTY SEQUENCE. REF CMB 2 188 59.
- 726 1, 2, 3, 5, 6, 7, 19, 21, 23, 31, 37, 38, 44, 69, 73  
EAST POSITIVE PRIMITIVE ROOTS. REF RSS XLIV.
- 727 1, 2, 3, 5, 6, 8, 10, 13, 15, 18, 21, 25, 28, 32, 36, 41, 45, 50  
RESTRICTED PARTITIONS. REF CAY 2 277.
- 728 1, 2, 3, 5, 6, 8, 12, 14, 15, 17, 20, 21, 24, 27, 33, 38, 41, 50, 54, 57, 59, 62, 66, 69, 71, 75, 77, 78, 80, 89, 90, 99, 101, 105, 110, 111, 117, 119, 131, 138, 141, 143, 147, 150  
(N + 1) + 1 IS PRIME. REF CUI 1 245, LIN 3 209 29, LE1 46.
- 729 1, 2, 3, 5, 6, 9, 11, 15, 18, 23, 27, 34, 39, 47, 54, 64, 72, 84, 94, 108, 120, 136, 150, 169, 185, 206, 225, 249, 270, 297, 321, 351, 378, 411, 441, 478, 511, 551, 588, 632, 672  
PARTITIONS INTO AT MOST 4 PARTS. REF RS2 2.
- 730 1, 2, 3, 5, 6, 10, 11, 17, 21, 27, 33, 46, 53, 68, 82, 104, 123, 154, 179, 221, 262, 314, 369, 446, 515, 614, 715, 845, 977, 1148, 1321, 1544, 1778, 2050, 2361, 2736, 3121  
MOCK THETA NUMBERS. REF TAMS 72 495 52.
- 731 1, 2, 3, 5, 7, 8, 9, 13, 14, 18, 19, 24, 25, 29, 30, 35, 36, 40, 41, 46, 51, 56, 63, 68, 72, 73, 78, 79, 83, 84, 89, 94, 115, 117, 126, 153, 160, 165, 169, 170, 175, 176, 181, 186  
A SELF-GENERATING SEQUENCE. REF ULI IX.
- 732 1, 2, 3, 5, 7, 8, 10, 12, 13, 18, 20, 27, 28, 33, 37, 42, 45, 47, 55, 58, 60, 62, 63, 65, 67, 73, 75, 78, 80, 85, 88, 90, 92, 102, 103, 105, 112, 115, 118, 120, 125, 128, 130, 132  
(2N)\*\*2 + 1 IS PRIME. REF KR1 1 11.
- 733 1, 2, 3, 5, 7, 9, 12, 15, 18, 22, 26, 30, 35, 40, 45, 51, 57, 63, 70, 77, 84, 92, 100, 108, 117, 126, 135, 145, 155, 165, 176, 187, 198, 210, 222, 234, 247, 260, 273, 287, 301  
RELATED TO ZARANKIEWICZS PROBLEM. REF TH 126 (DIVIDED BY 2).
- 734 1, 2, 3, 5, 7, 10, 11, 13, 14, 18, 21, 22, 31, 42, 67, 70, 71, 73, 251, 370, 375, 389, 407  
39.2\*\*N + 1 IS PRIME. REF PAMS 9 674 58.
- 735 1, 2, 3, 5, 7, 10, 12, 17, 18, 23, 25, 30, 32, 33, 38, 40, 45, 47, 52, 58, 70, 72, 77, 87, 95, 100, 103, 107, 110, 135, 137, 138, 143, 147, 170, 172, 175, 177, 182, 192, 205, 213  
6A - 1, 6A + 1 ARE TWIN PRIMES. REF LE3 69.
- 736 1, 2, 3, 5, 7, 10, 13, 18, 23, 30, 37, 47, 57, 70, 83, 101, 119, 142, 165, 195, 225, 262, 299, 346, 393, 450, 507, 577, 647, 730, 813, 914, 1015, 1134, 1253, 1395, 1537  
A LINEAR RECURRENCE. REF FO 9 135 71.
- 737 1, 2, 3, 5, 7, 10, 13, 18, 23, 30, 37, 47, 57, 70, 84, 101, 119, 141, 164, 192, 221, 255, 291, 333, 377, 427, 480, 540, 603, 674, 748, 831, 918, 1014, 1115, 1226, 1342, 1469  
PARTITIONS INTO AT MOST 5 PARTS. REF RS2 2.
- 738 1, 2, 3, 5, 7, 10, 14, 19, 26, 35, 47, 62, 82, 107, 139, 179, 230, 293  
PLANAR PARTITIONS. REF PCPS 47 686 51.
- 739 1, 2, 3, 5, 7, 10, 14, 20, 27, 37, 49, 66, 86, 113, 146, 190, 242, 310, 392, 497, 623, 782, 973, 1212, 1498, 1851, 2274, 2793, 3411, 4163, 5059, 6142, 7427, 8972, 10801  
REPRESENTATIONS OF THE SYMMETRIC GROUP. REF CJM 4 383 52.
- 740 1, 2, 3, 5, 7, 10, 14, 20, 29, 43, 65, 100, 156, 246, 391, 625, 1003, 1614, 2602, 4200, 6785, 10967, 17733, 28680, 46392, 75050, 121419, 196445, 317839, 514258  
NTH FIBONACCI NUMBER + N. REF HO2 96.
- 741 1, 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97, 101, 103, 107, 109, 113, 127, 131, 137, 139, 149, 151, 157, 163, 167, 173, 179  
PRIMES. REF ASI 870.
- 742 1, 2, 3, 5, 7, 11, 13, 17, 19, 31, 37, 41, 61, 73, 97, 101, 109, 151, 163, 181, 193, 241, 251, 257, 271, 401, 433, 487, 541, 577, 601, 641, 751, 769, 811, 1153, 1201, 1297  
A RESTRICTED CLASS OF PRIMES. REF KR1 1 53.
- 743 1, 2, 3, 5, 7, 11, 14, 20, 26, 35, 44, 58, 71, 90, 110, 136, 163, 199, 235, 282, 331, 391, 454, 532, 612, 709, 811, 931, 1057, 1206, 1360, 1540, 1729, 1945, 2172, 2432  
PARTITIONS INTO AT MOST 6 PARTS. REF CAY 10 415, RS2 2.

- 244 1, 2, 3, 5, 7, 11, 15, 22, 30, 42, 56, 77, 101, 135, 176, 231, 297, 385, 490, 627, 792, 1002, 1255, 1575, 1958, 2436, 3010, 3718, 4565, 5604, 6842, 8349, 10143, 12310, 14883  
NUMBER OF PARTITIONS OF N. REF RS2 90, R1 122, ASI 836.
- 245 1, 2, 3, 5, 7, 11, 17, 25, 38, 57, 86, 129, 194, 291, 437, 656, 985, 1477, 2216, 3325, 4987, 7481, 11222, 16834, 25251, 37876, 56815, 85222, 127834, 191751, 287626  
QUOTIENT OF 3\*\*N / 2\*\*N. REF JIMS 2 40 36, LE1 82.
- 246 1, 2, 3, 5, 7, 11, 19, 43, 53, 79, 107, 149  
LEAST POSITIVE PRIME PRIMITIVE ROOTS. REF RS5 XLV.
- 247 1, 2, 3, 5, 7, 11, 101, 131, 151, 181, 191, 313, 353, 373, 383, 727, 757, 787, 797, 919, 929, 10301, 10501, 10601, 11311, 11411, 12421, 12721, 12821, 13331, 13831  
PALINDROMIC PRIMES. REF BE3 228.
- 248 1, 2, 3, 5, 7, 13, 17, 19, 31, 61, 89, 107, 127, 521, 607, 1279, 2203, 2281, 3217, 4253, 4423, 9689, 9941, 11213, 19937  
MERSENNE PRIMES. REF MTAC 18 93 64, NAMS 18 608 71.
- 249 1, 2, 3, 5, 7, 13, 20, 35, 55, 96, 156, 267, 433, 747, 1239, 2089, 3498, 5912  
PARAFINNS. REF JACS 54 1544 32.
- 250 1, 2, 3, 5, 7, 17, 31, 89, 127, 521, 607, 1279, 2281, 3217, 4423, 9689  
IRREDUCIBLE MERSENNE TRINOMIALS. REF IC 15 68 69.
- 251 1, 2, 3, 5, 8, 9, 10, 11, 12, 18, 19, 22, 26, 28, 30, 31, 33, 35, 36, 38, 39, 40, 41, 44, 46, 47, 48, 50, 52, 54, 55, 56, 58, 61, 62, 66, 67, 68, 69, 71, 72, 74, 76, 77, 80, 82, 83, 91  
ELLIPTIC CURVES. REF JRAM 212 23 63.
- 252 1, 2, 3, 5, 8, 11, 12, 14, 18, 20, 21, 27, 29, 30, 32, 35, 44, 45, 48, 50  
OF THE FORM 2X\*\*2 + 3Y\*\*2. REF EUL (1) 1 425 11.
- 253 1, 2, 3, 5, 8, 12, 18, 26, 38, 53, 75, 103, 142, 192, 260, 346, 461, 605, 796  
PLANAR PARTITIONS. REF PCPS 47 686 51.
- 254 1, 2, 3, 5, 8, 13, 17, 26, 34, 45, 54, 67, 81, 97, 115, 132, 153, 171, 198, 228, 256, 288, 323, 357, 400, 439, 488, 530, 581, 627, 681, 732, 790, 843, 908, 963, 1029, 1085  
A SELF-GENERATING SEQUENCE. REF AMM 75 80 68, RLG.
- 255 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 232, 375, 606, 979, 1582, 2556, 4130, 6673, 10782, 17421, 28148, 45480, 73484, 118732, 191841, 309967, 500829, 809214, 1307487  
DYING RABBITS. REF FQ 2 108 64.
- 256 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, 610, 987, 1597, 2584, 4181, 6765, 10946, 17711, 28657, 46368, 75025, 121393, 196418, 317811, 514229, 832040, 1346269  
FIBONACCI NUMBERS A(N) = A(N-1) + A(N-2). REF HW1 148, REC 11 20 62, HO1.
- 257 1, 2, 3, 5, 8, 14, 21, 39, 62, 112, 189, 352, 607, 1144, 2055, 3883, 7154, 13602  
NECKLACES. REF JUM 5 663 61.
- 258 1, 2, 3, 5, 8, 14, 23, 39, 65, 110, 184, 310, 520, 876, 1471, 2475, 4159, 6996, 11759  
PARAFINNS. REF JACS 54 1105 32.
- 259 1, 2, 3, 5, 8, 15, 26, 48, 87, 161, 299, 563, 1066, 2030, 3885, 7464, 14384, 27779, 53782, 104359, 202838, 394860, 769777, 1502603, 2936519, 5744932  
POPULATION OF U\*\*2 - 2V\*\*2. REF MTAC 20 560 66.

- 260 1, 2, 3, 5, 8, 21, 29, 79, 661, 740, 19161, 19901, 118666, 138567, 3167140, 3305707, 29612796, 32918503, 62531299, 595700194, 6582231493, 1253931687  
CONVERGENTS TO FIFTH ROOT OF 5. REF AMP 46 116 1866, LE1 67, HPR.
- 261 1, 2, 3, 5, 9, 16, 28, 50, 89, 159, 285, 510, 914, 1639, 2938, 5269, 9451, 16952, 30410, 54555, 97871, 175596, 315016, 565168, 1013976, 1819198, 3263875, 5855833  
PARTITIONS INTO POWERS OF 1/2. REF EMS 11 224 59, ST3.
- 262 1, 2, 3, 5, 9, 16, 28, 51, 93, 170, 315, 585, 1091, 2048, 3855, 7280, 13797, 26214  
NECKLACES. REF JUM 5 663 61, ME1.
- 263 1, 2, 3, 5, 9, 16, 29, 52, 94, 175, 327, 616, 1169, 2231, 4273, 8215, 15832, 30628, 59345, 115208, 224040, 436343, 850981, 1661663, 3248231, 6356076, 12448925  
RAMANUJANS APPROXIMATION. REF MTAC 18 79 64.
- 264 1, 2, 3, 5, 9, 16, 29, 53, 98, 181, 341, 640, 1218, 2321, 4449, 8546, 16482, 31845, 61707, 118760, 232865, 453511, 884493, 1727125, 3376376, 6607207  
POPULATION OF 3U\*\*2 + 4V\*\*2. REF MTAC 20 567 66.
- 265 1, 2, 3, 5, 9, 16, 29, 54, 97, 180, 337, 633, 1197, 2280, 4357, 8363, 16096, 31064, 60108, 116555, 226419, 440616, 858696, 1675603, 3273643, 6402706, 12534812  
POPULATION OF U\*\*2 + V\*\*2. REF MTAC 20 560 66.
- 266 1, 2, 3, 5, 9, 17, 33, 65, 129, 257, 513, 1025, 2049, 4097, 8193, 16385, 32769, 65537, 131073, 262145, 524289, 1048577, 2097153, 4194305, 8388609, 16777217  
2\*\*N + 1. REF BA1.
- 267 1, 2, 3, 5, 9, 18, 35, 75, 159, 355, 802, 1858, 4347, 10359, 24894, 60523, 148284, 366319, 910726, 2278658, 5731580, 14490245, 93839412, 240215803, 617105614  
PARAFINNS. REF JACS 54 2919 32.
- 268 1, 2, 3, 5, 9, 18, 35, 75, 159, 357, 799  
HYDROCARBONS. REF BS1 201.
- 269 1, 2, 3, 5, 10, 11, 26, 32, 39, 92, 116, 134, 170, 224  
25.4\*\*N + 1 IS PRIME. REF PAMS 9 674 58.
- 270 1, 2, 3, 5, 10, 18, 35, 63, 126, 231  
FROM RADONS THEOREM. REF MFM 73 12 66.
- 271 1, 2, 3, 5, 10, 24, 69, 384  
LINEAR SPACES. REF BSM 19 424 67.
- 272 1, 2, 3, 5, 10, 27, 119, 1113, 29375, 2730166  
THRESHOLD FUNCTIONS. REF PGEC 19 821 70.
- 273 1, 2, 3, 5, 11, 16, 38, 54, 130, 184, 444, 628, 1516, 2144, 5176, 7320, 17672, 24992, 60336, 85328, 206000, 291328, 703328, 994656, 2401312, 3395968, 8198592  
A LINEAR RECURRENCE. REF AMM 72 1024 65.
- 274 1, 2, 3, 5, 11, 24, 55, 136, 345, 900, 2412, 6563, 18122, 50699, 143255, 408419, 1172854, 3395964  
PARAFINNS. REF JACS 54 1544 32.
- 275 1, 2, 3, 5, 11, 47, 923, 409619, 83763206255, 3508125906290858798171, 6153473687096578758448522809275077520433167  
HAMILTON NUMBERS. REF RS3 178 288 1887, LU1 496.



- 276 1, 2, 3, 5, 12, 14, 11, 13, 20, 72, 19, 42, 132, 84, 114, 29, 30, 110, 156, 37, 156, 420, 210, 156, 552, 462, 72, 53, 420, 342, 59  
SHUFFLING CARDS. REF SIAMR 3 296 61.
- 277 1, 2, 3, 5, 12, 36, 110, 326, 963, 2964, 9797, 34818, 130585, 506996, 2018454, 8238737, 34627390, 150485325, 677033911, 3147372610, 15066340824, 74025698886  
FROM A DIFFERENTIAL EQUATION. REF AMM 67 766 80.
- 278 1, 2, 3, 5, 13, 83, 2503, 976253, 31601312113, 2560404986164794683, 202523113189037952478722304798003  
FROM A CONTINUED FRACTION. REF AMM 63 711 56.
- 279 1, 2, 3, 5, 16, 231, 53105, 2820087664, 7952894429824835871, 63248529811938901240357985099443351745  
 $A(N) = A(N-1)^{**2} - A(N-2)^{**2}$ . REF EUR 27 6 64.
- 280 1, 2, 3, 6, 2, 0, 1, 10, 0, 2, 10, 6, 7, 14, 0, 10, 12, 0, 6, 0, 9, 4, 10, 0, 18, 2, 0, 6, 14, 18, 11, 12, 0, 0, 22, 0, 20, 14, 6, 22, 0, 0, 23, 26, 0, 18, 4, 0, 14, 2, 0, 20, 0, 0, 0, 12, 3, 30  
GLAISHERS CHI FUNCTION. REF QJM 20 151 1884.
- 281 1, 2, 3, 6, 5, 11, 14, 22, 30, 47, 66, 99, 143, 212, 308, 454, 663, 974, 1425, 2091, 3062, 4490, 6578, 9643, 14130, 20711, 30351, 44484, 65192, 95546, 140027, 205222  
 $A(N) = A(N-2) + A(N-3) + A(N-4)$ . REF IDM 8 64 01. FQ 6(3) 68 68.
- 282 1, 2, 3, 6, 7, 10, 14, 15, 21, 30, 35, 42, 70, 105, 210, 221, 230, 231, 238, 247, 253, 255, 266, 273, 285, 286, 299, 322, 323, 330, 345, 357, 374, 385, 390, 391, 399, 418, 429  
A SPECIALLY CONSTRUCTED SEQUENCE. REF AMM 74 874 67.
- 283 1, 2, 3, 6, 7, 11, 14, 17, 33, 42, 43, 63, 65, 67, 81, 134, 162, 206, 211, 366  
 $9.2^{**N} + 1$  IS PRIME. REF PAMS 9 674 58.
- 284 1, 2, 3, 6, 8, 10, 22, 35, 42, 43, 46, 56, 91, 102, 106, 142, 190, 208, 266, 330, 360, 382, 462, 503, 815  
 $3.3^{**N} - 1$  IS PRIME. REF MITAC 23 874 69.
- 285 1, 2, 3, 6, 8, 16, 24, 42, 69, 124, 208, 378, 668, 1214, 2220, 4110, 7630, 14308, 26931  
NECKLACES. REF JIM 5 663 61.
- 286 1, 2, 3, 6, 9, 14, 20, 29, 42, 58, 79, 108, 145, 191, 252, 329, 427, 549, 704, 894, 1136, 1427, 1793, 2237, 2789, 3450, 4268, 5248, 6447, 7880, 9619, 11691, 14199  
MIXED PARTITIONS. REF JNSM 9 91 69.
- 287 1, 2, 3, 6, 9, 18, 30, 56, 99, 186, 335, 630, 1161, 2182, 4080, 7710, 14532, 27594, 52377, 99858, 190557, 364722, 698870, 1342176, 2580795, 4971008, 9586395  
IRREDUCIBLE POLYNOMIALS, OR NECKLACES. REF JIM 5 663 61. JSIAM 12 288 64.
- 288 1, 2, 3, 6, 9, 26, 53, 146, 369, 1002  
NECKLACES. REF JIM 2 302 58.
- 289 1, 2, 3, 6, 10, 11, 21, 30, 48, 72, 110, 171, 260, 401, 613, 942, 1445, 2216, 3401, 5216, 8004, 12278, 18837, 28899, 44335, 68018, 104349, 160089, 245601, 376791  
A FIELDER SEQUENCE. REF FQ 6(3) 68 68.
- 290 1, 2, 3, 6, 10, 17, 21, 38, 57, 92, 143, 225, 351, 555, 868, 1366, 2142, 3365, 5282, 8296, 13023, 20451, 32108, 50417, 79160, 124295, 195159, 306431, 481139, 755462  
A FIELDER SEQUENCE. REF FQ 6(3) 68 68.
- 291 1, 2, 3, 6, 10, 17, 28, 46, 75, 122, 198, 321, 520, 842, 1363, 2206, 3570, 5777, 9348, 15126, 24475, 39602, 64078, 103681, 167760, 271442, 439203, 710646, 1149850  
 $A(N) = A(N-1) + A(N-2) + 1$ . REF JAZ 95.
- 292 1, 2, 3, 6, 10, 19, 35, 62, 118, 219, 414, 783, 1497, 2860, 5503, 10593, 20471, 39637, 76918, 149501, 291115, 567581, 1108022, 2165621, 4237085, 8297727  
POPULATION OF  $U^{*+2} + 4V^{*+2}$ . REF MITAC 18 84 64.
- 293 1, 2, 3, 6, 10, 19, 35, 67, 127, 248, 482, 952, 1885, 3765, 7546, 15221, 30802, 62620, 127702, 261335, 536278, 1103600, 2276499, 4706985, 9752585, 20247053  
SERIES-REDUCED PLANTED TREES. AM1 101 150 59. CAS.
- 294 1, 2, 3, 6, 10, 20, 35, 70, 126, 252, 462, 924, 1716, 3432, 6435, 12870, 24310, 48620, 92378, 184756, 352716, 705432, 1352078, 2704156, 5200300, 10400600  
CENTRAL BINOMIAL COEFFICIENTS  $C(N, N/2)$ . REF RS1. ASI 828.
- 295 1, 2, 3, 6, 10, 20, 36, 72, 137, 274, 543  
RESTRICTED HEXAGONAL POLYOMINOES. REF EMS 17 11 70.
- 296 1, 2, 3, 6, 11, 20, 37, 68, 125, 230, 423, 778, 1431, 2632, 4841, 8904, 16377, 30122, 55403, 101902, 187427, 344732, 634061, 1166220, 2145013, 3945294, 7565227  
TRIBONACCI NUMBERS  $A(N) = A(N-1) + A(N-2) + A(N-3)$ . REF FQ 5 211 67.
- 297 1, 2, 3, 6, 11, 22, 42, 84, 165  
RANDOM TOURNAMENTS. REF CMB 13 108 70.
- 298 1, 2, 3, 6, 11, 23, 46, 98, 207, 451, 983, 2179, 4850, 10905, 24631, 56011, 127912, 293547, 676157, 1563372, 3626149, 8436379, 19680277, 46026618  
WEDDERBURN-ETHERINGTON NUMBERS. REF CO1 1 68.
- 299 1, 2, 3, 6, 11, 23, 47, 106, 295, 551, 1301, 3159, 7741, 19320, 48629, 123867, 317955, 823065, 2144505, 5623756, 14828074, 39299897, 104636890, 279793450  
UNLABELED TREES. REF R1 138. HAS 232.
- 300 1, 2, 3, 6, 11, 24, 47, 103, 214, 481, 1030, 2337, 5131, 11813, 26329, 60958, 137821, 321690, 734428, 1721998, 3966556, 9352353, 21683445, 51296030, 119663812  
STRUCTURE OF RAYLEIGH POLYNOMIAL. REF DMJ 31 517 64.
- 301 1, 2, 3, 6, 12, 23, 44, 85, 164, 316, 609, 1174, 2263, 4362, 8408, 16207, 31240, 60217, 116072, 223736, 431265, 831290, 1602363, 3088654, 5953572, 11475879  
TETRAPACCI NUMBERS. REF FQ 8 7 70.
- 302 1, 2, 3, 6, 12, 26, 59, 146, 368  
SERIES-PARALLEL NUMBERS. REF ICM 1 646 50.
- 303 1, 2, 3, 6, 13, 28, 62  
ALKYLS. REF ZFK 93 437 36.
- 304 1, 2, 3, 6, 13, 35, 116  
CONNECTED WEIGHTED LINEAR SPACES. REF BSM 22 234 70.
- 305 1, 2, 3, 6, 14, 36, 94, 250, 675, 1838, 5053, 14016, 39169, 110194, 311751, 886160, 2529260  
FIXED TRIANGULAR POLYOMINOES. REF LUS.
- 306 1, 2, 3, 6, 15, 63, 567, 14755, 1366318  
THRESHOLD FUNCTIONS. REF PGEC 19 821 70.

- 07 1, 2, 3, 6, 18, 206, 7888299  
 COLEMAN FUNCTIONS. REF JSIAM 12 294 64.
- 08 1, 2, 3, 6, 20, 150, 3287, 244158, 66291591, 68863243522  
 THRESHOLD FUNCTIONS. REF PGEC 19 821 70.
- 09 1, 2, 3, 6, 20, 168, 7581, 7828354, 2414682040998  
 ONE-TONE BOOLEAN FUNCTIONS, OR DEDEKINDS PROBLEM. REF HA2 188. BIT 63. CO1 2  
 16. WE1 181.
- 10 1, 2, 3, 6, 22, 402, 1228158, 400507806643728  
 COLEMAN FUNCTIONS. REF JSIAM 11 827 63.
- 11 1, 2, 3, 6, 30, 75, 81  
 2<sup>++N</sup> - 1 IS PRIME. REF MTAC 23 875 69.
- 12 1, 2, 3, 7, 5, 11, 103, 71, 661, 269, 329891, 39916801, 2834329, 75024347,  
 790360487, 46271341, 1059511, 1000357, 123610951, 171331127363831  
 ARGEST FACTOR OF FACTORIAL (N) + 1. REF SMA 14 25 48.
- 13 1, 2, 3, 7, 8, 10, 16, 18, 19, 40, 48, 55, 90, 96, 98, 190, 398, 456, 502  
 7.2<sup>++N</sup> + 1 IS PRIME. REF PAMS 9 675 58.
- 14 1, 2, 3, 7, 10, 13, 18, 27, 37, 51, 74, 157, 271, 458, 530, 891  
 1.2<sup>++N</sup> - 1 IS PRIME. REF MTAC 23 874 69.
- 15 1, 2, 3, 7, 10, 13, 25, 26, 46, 60, 87, 90, 95, 145, 160, 195, 216, 308, 415  
 4<sup>++N</sup> + 1 IS PRIME. REF PAMS 9 674 58.
- 16 1, 2, 3, 7, 12, 27, 55, 127, 284, 682  
 ENTERED TREES. REF CAV 9 438.
- 17 1, 2, 3, 7, 13, 31, 65, 154, 347, 824, 1905, 4512, 10546, 24935, 58476, 138002,  
 23894, 763172, 1790585, 4213061, 9878541  
 QUARE FILAMENTS. REF PL2 1 337 70.
- 18 1, 2, 3, 7, 14, 32, 72, 171, 405, 989, 2426, 6045, 15167, 38422, 97925, 251275,  
 48061, 1679869, 4372872, 11428365, 29972078, 78859809, 208094977, 550603722  
 HYDROCARBONS. REF JACS 55 253 33.
- 19 1, 2, 3, 7, 15, 34, 78, 182, 429, 1019, 2433, 5830, 14004, 33694, 81159, 195635,  
 71819, 1138286, 2746794, 6629290, 16001193, 38624911, 93240069, 225087338  
 UM OF FIBONACCI AND PELL NUMBERS.
- 20 1, 2, 3, 7, 15, 43, 131, 468, 1776, 7559, 34022, 166749, 853823, 4682358  
 FINEMENTS OF PARTITIONS. REF GUS.
- 21 1, 2, 3, 7, 16, 54  
 IFFERENT GRAPHS, ALLOWING COMPLEMENTATION. REF KNAW 69 339 66.
- 22 1, 2, 3, 7, 18, 41, 123, 367  
 LTERNATING KNOTS. REF TAI 1 345. JL2 343.
- 23 1, 2, 3, 7, 21, 49, 166, 549  
 LTERNATING AND NONALTERNATING KNOTS. REF TAI 1 345. JL2 343.
- 24 1, 2, 3, 7, 21, 135, 2470, 175428  
 HRESHOLD FUNCTIONS. REF PGEC 19 821 70.

- 325 1, 2, 3, 7, 23, 41, 71, 191, 409, 2161, 5881, 36721, 55441, 71761, 110881, 760321  
 LEAST POSITIVE PRIMITIVE ROOTS. REF RSS XLIV.
- 326 1, 2, 3, 7, 23, 43, 67, 83, 103, 127, 163, 167, 223, 227, 283, 367, 383, 443, 463,  
 467, 487, 503, 523, 547, 587, 607, 643, 647, 683, 727, 787, 823, 827, 863, 883, 897, 907  
 PRIMES DIVIDING ALL FIBONACCI SEQUENCES. REF FQ 2 38 64.
- 327 1, 2, 3, 7, 23, 89, 113, 523, 887, 1129, 1327, 9551, 15683, 19609, 31397, 155921,  
 360653, 370261, 492113, 134953, 1357201, 2010733, 465253, 17051707  
 INCREASING GAPS BETWEEN PRIMES. REF KRI 1 14. MTAC 18 649 64.
- 328 1, 2, 3, 7, 23, 164, 3779, 619779, 2342145005, 1451612289057674,  
 3399886472013047318638149, 4935316984175079105557291745555191750431  
 $A(N) = A(N - 1)A(N - 2) + A(N - 3)$ . REF GUS.
- 329 1, 2, 3, 7, 43, 13, 53, 5, 6221671, 38709183810571  
 FROM EUCLIDS PROOF. REF BAMS 69 737 63.
- 330 1, 2, 3, 7, 43, 139, 50207, 340999, 3202139, 410353  
 FROM EUCLIDS PROOF. REF NAMS 11 376 64.
- 331 1, 2, 3, 7, 43, 1807, 3263443, 10650056950807, 113423713055421844361000443,  
 12864938693278671740537145898360961546653259485195807  
 $A(N + 1) = A(N)^{++2} - A(N) + 1$ . REF CJM 15 475 63. AMM 70 403 63.
- 332 1, 2, 3, 8, 10, 12, 14, 17, 23, 24, 27, 28, 37, 40, 41, 44, 45, 53, 59, 66, 70, 71, 77,  
 80, 82, 87, 90, 97, 99, 102, 105, 110, 114, 119, 121, 124, 127, 133, 136, 138, 139, 144  
 $(2N)^{++4} + 1$  IS PRIME. REF MTAC 21 246 67.
- 333 1, 2, 3, 8, 13, 20, 31, 32, 53, 76, 79, 80, 117, 176, 181, 182, 193, 200, 283, 284,  
 285, 286, 293, 440, 443, 468, 661, 678, 683, 684, 1075, 1076, 1087, 1088, 1091, 1092  
 RELATED TO LIOUVILLES FUNCTION. REF IAS 12 408 40.
- 334 1, 2, 3, 8, 18, 44, 115, 294, 783  
 RECTANGULAR POLYMINOES. REF SPH 7 203 37.
- 335 1, 2, 3, 8, 19, 27, 100, 227, 781, 1008, 3805, 4813, 148195, 153008, 760227,  
 913235, 2586697, 24193508, 147747745, 615184488, 762932233, 1378116721  
 CONVERGENTS TO CUBE ROOT OF 4. REF AMP 46 106 1866. LE1 67. HPR.
- 336 1, 2, 3, 8, 24, 108, 640, 4492, 36336, 329900, 3326788  
 PATTERNS. REF MES 37 61 07.
- 337 1, 2, 3, 8, 30, 144, 840, 5760, 45360, 403200, 3991680, 43545600, 518918400,  
 6706022400, 93405312000, 1394852659200, 22230464256000, 376610217984000  
 SUMS OF FACTORIAL NUMBERS. REF CJM 22 26 70.
- 338 1, 2, 3, 8, 51, 1538, 599871, 19417825808, 1573273218577214751,  
 124442887685693556895657990772138  
 FROM A CONTINUED FRACTION. REF AMM 63 711 56.
- 339 1, 2, 3, 9, 20, 73  
 PARTITIONS OF A POLYGON. REF BAMS 54 359 48.
- 340 1, 2, 3, 10, 27, 98  
 SIGNED TREES. REF AM1 101 154 59.

- 341 1, 2, 3, 10, 1382, 420, 10851, 4398670, 7333662, 51270780, 7090922730, 2155381956, 94997844116, 68926730208040  
NUMERATORS OF BERNOULLI NUMBERS. REF DA2 2 208.
- 342 1, 2, 3, 11, 22, 26, 101, 111, 121, 202, 212, 264, 307, 836, 1001, 1111, 2002, 2285, 2636, 10001, 10101, 10201, 11011, 11111, 11211, 20002, 20102, 22865, 24846, 30693  
SQUARE IS A PALINDROME. REF JRM 3 94 70.
- 343 1, 2, 3, 11, 69, 701, 10584, 222965, 6253604, 225352709, 10147125509, 558317255704, 36859086001973, 2875667025409598, 261713458398275391  
 $A(N) = N(N - 1)/2 + A(N - 2)$ .
- 344 1, 2, 3, 12, 10, 60, 105, 280, 252, 2520, 2310, 27720, 25740, 24024, 45045, 720720, 680680, 12252240, 11638628, 11085360, 10581480, 232792560, 223092870  
L. C. M. OF BINOMIAL COEFFICIENTS  $C(N, 1), C(N, 2), \dots, C(N, N)$ .
- 345 1, 2, 3, 12, 52, 456, 6873, 191532, 9733032, 903753248, 154108311046  
NONTRANSITIVE PRIME TOURNAMENTS. REF DMJ 37 332 70.
- 346 1, 2, 3, 24, 5, 720, 105, 2240, 189, 3628800, 385, 479001600, 19305, 896896, 2027025, 20922789889000, 85085, 6402373705728000, 8729721, 47297536000  
N-PH-TORIAL. REF AMM 60 422 53.
- 347 1, 2, 3, 26, 13, 1074, 1457, 61802, 7929, 4218722  
SUMS OF LOGARITHMIC NUMBERS. REF MST 31 78 63.
- 348 1, 2, 3, 56, 43265728  
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- 349 1, 2, 4, 1, 3, 6, 5, 2, 8, 4, 10, 9, 1, 8, 5, 11, 12, 10, 2, 4, 9, 13, 6, 11, 8, 16, 5, 13, 17, 18, 15, 2, 4, 11, 6, 19, 17, 13, 16, 10, 1, 3, 20, 12, 22, 18, 17, 22, 23, 11, 2, 16, 19, 13, 8  
QUADRATIC PARTITIONS OF PRIMES. REF CU2 1, LE1 55.
- 350 1, 2, 4, 3, 6, 10, 12, 4, 8, 18, 6, 11, 20, 18, 28, 5, 10, 12, 36, 12, 20, 14, 12, 23, 21, 8, 52, 20, 18, 58, 60, 6, 12, 66, 22, 35, 9, 20, 30, 39, 54, 82, 8, 28, 11, 12, 10, 36, 48, 30  
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- 351 1, 2, 4, 4, 6, 8, 8, 8, 13, 12, 12, 16, 14, 16, 24, 16  
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- 352 1, 2, 4, 4, 6, 8, 8, 12, 14  
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- 353 1, 2, 4, 4, 6, 16, 16, 30, 88  
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- 354 1, 2, 4, 5, 6, 7, 8, 10, 11, 12, 13, 15  
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- 355 1, 2, 4, 5, 6, 8, 9, 11, 12, 13, 15, 16, 18, 19, 20, 22, 23, 25, 26, 27, 29, 30, 32, 33, 34, 36, 37, 38, 40, 41, 43, 44, 45, 47, 48, 50, 51, 52, 54, 55, 57, 58, 59, 61, 62, 64, 65, 66  
A BEATTY SEQUENCE. REF CMB 3 21 60.
- 356 1, 2, 4, 5, 7, 8, 9, 11, 12, 14, 15, 16  
A BEATTY SEQUENCE. REF CMB 2 188 59.
- 357 1, 2, 4, 5, 7, 8, 10, 11, 13, 14, 16, 17, 19, 20, 22, 23, 25, 26, 28, 29, 31, 32, 34, 37, 38, 40, 41, 43, 44, 46, 47, 49, 50, 52, 53, 55, 56, 58, 59, 61, 62, 64, 65, 67, 68, 70, SEMI-TRIBONACCI NUMBERS. REF FQ 6(3) 261 68.
- 358 1, 2, 4, 5, 7, 8, 11, 13, 16, 17, 19, 31, 37, 41, 47, 53, 61, 71, 79, 113, 313, 353  
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- 359 1, 2, 4, 5, 7, 9, 10, 12, 14, 16, 17, 19, 21, 23, 25, 26, 28, 30, 32, 34, 36, 37, 39, 43, 45, 47, 49, 50, 52, 54, 56, 58, 60, 62, 64, 65, 67, 69, 71, 73, 75, 77, 79, 81, 82, 84, 1 ODD, 2 EVEN, 3 ODD, ... REF AMM 67 380 60.
- 360 1, 2, 4, 5, 7, 9, 12, 13, 15, 17, 20, 22, 25, 28, 32, 33, 35, 37, 40, 42, 45, 48, 52, 57, 60, 64, 67, 71, 75, 80, 81, 83, 85, 88, 90, 93, 96, 100, 102, 105, 108, 112, 115, 119  
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- 361 1, 2, 4, 5, 8, 9, 10, 13, 16, 17, 18, 20, 25, 26, 29, 32, 34, 36, 37, 40, 41, 45, 49, 52, 53, 58, 61, 64, 65, 68, 72, 73, 74, 80, 81, 82, 85, 89, 90, 97, 99, 100, 101, 104, 106  
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- 362 1, 2, 4, 5, 8, 9, 12, 14, 17, 18, 23, 24, 27, 30, 34, 35, 40, 41, 46, 49, 52, 53, 60, 65, 68, 73, 74, 81, 82, 87, 90, 93, 96, 104, 105, 108, 111, 118, 119, 126, 127, 132, 137  
A NUMBER-THEORETIC FUNCTION. REF DVSS 2 281 1884.
- 363 1, 2, 4, 5, 8, 10, 14, 15, 16, 21, 22, 25, 26, 28, 33, 34, 35, 36, 38, 40, 42, 46, 46, 50, 53, 57, 60, 62, 64, 65, 70, 77, 80, 81, 83, 85, 86, 90, 91, 92, 100  
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- 364 1, 2, 4, 5, 8, 12, 19, 30, 48, 77, 124, 200, 323, 522, 844, 1365, 2208, 3572, 5779, 9350, 15128, 24477, 39604, 64080, 103683, 167762, 271444, 439205, 710648, 114984  
 $A(N) = A(N - 1) + A(N - 2) - 1$ . REF JAZ 97.
- 365 1, 2, 4, 5, 10, 14, 17, 31, 41, 73, 80, 82, 116, 125, 145, 157, 172, 202, 224, 266, 289, 293, 463  
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- 366 1, 2, 4, 5, 10, 19, 36, 68, 138  
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- 367 1, 2, 4, 5, 14, 14, 39, 42, 132, 132, 424, 429, 1428, 1430, 4848, 4862, 16796, 16796, 58739, 58786, 208012, 208012, 742768, 742900, 2674426, 2674440, 9694416  
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- 368 1, 2, 4, 6, 3, 10, 25, 12, 42, 8, 40, 202, 21  
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- 369 1, 2, 4, 6, 7, 10, 11, 12, 22, 23, 25, 26, 27, 30, 36, 38, 42, 43, 44, 45, 50, 52, 54, 59, 70, 71, 72, 74, 75, 76, 78, 86, 87, 91, 102, 103, 106, 107, 108, 110, 116, 118, 119  
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- 370 1, 2, 4, 6, 8, 10, 12, 16, 18, 20, 22, 24, 28, 30, 32, 36, 40, 42, 44, 46, 48, 52, 54, 58, 60, 64, 66, 70, 72, 78, 80, 82, 84, 88, 90, 92, 96, 100  
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71 1, 2, 4, 6, 8, 10, 12, 16, 18, 20, 22, 24, 28, 30, 32, 36, 40, 42, 44, 46, 48, 52, 54, 56, 60, 64, 66, 70, 72, 78, 80, 82, 84, 88, 92, 96, 100, 102, 104, 106, 108, 110, 112, 116  
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72 1, 2, 4, 6, 8, 12, 16, 24, 32, 36, 48, 64, 72, 96, 120, 128, 144, 192, 216, 240, 256, 384, 432, 480, 512, 576, 720, 768, 864, 960, 1024, 1152, 1296, 1440, 1536, 1728  
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73 1, 2, 4, 6, 8, 14, 26

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74 1, 2, 4, 6, 9, 12, 16, 20, 25, 30, 36, 42, 49, 56, 64, 72, 81, 90, 100, 110, 121, 132, 156, 169, 182, 196, 210, 225, 240, 256, 272, 289, 306, 324, 342, 361, 380, 400, 420  
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75 1, 2, 4, 6, 9, 13, 18, 24, 31, 39, 50, 62, 77, 93, 112, 134, 159, 187, 252, 292  
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76 1, 2, 4, 6, 10, 12, 18, 22, 28, 32, 42, 46, 58, 64, 72, 80, 96, 102, 120, 128, 140, 150, 180, 200, 212, 230, 242, 270, 278, 308, 324, 344, 360, 384, 396, 432, 450, 474, 490  
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77 1, 2, 4, 6, 10, 12, 18, 22, 30, 34, 42, 48, 58, 60, 78, 82, 102, 108, 118, 132, 150, 174, 192, 210, 214, 240, 258, 274, 282, 322, 330, 360, 372, 402, 418, 442, 454, 498  
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78 1, 2, 4, 6, 10, 14, 20, 26, 36, 46, 60, 74, 94, 114, 140, 166, 202, 238, 284, 330, 390, 524, 598, 692, 786, 900, 1014, 1154, 1294, 1460, 1626, 1828, 2030, 2266, 2506  
IE BINARY PARTITION FUNCTION. REF FQ 4 117 66. PCPS 66 376 69. AT1 400.

79 1, 2, 4, 6, 10, 14, 21, 29, 41, 55, 76, 100, 134, 175, 230, 296, 384, 489, 626, 791, 101, 1254, 1574, 1957, 2435, 3009, 3717, 4564, 5603, 6841, 8348, 10142, 12309  
IEES OF HEIGHT 2. REF IBMJ 4 475 60. KU1.

80 1, 2, 4, 6, 10, 14, 24, 30

ZE OF MINIMAL GRAPHS. REF SA1 94.

81 1, 2, 4, 6, 10, 16, 26, 44, 76, 132, 234, 420, 761, 1391, 2561, 4745, 8841, 16551, 114, 58708, 111136, 211000, 401650, 766372, 1465422, 2807599, 5388709, 10359735  
IM OF (2\*\*N)/N

82 1, 2, 4, 6, 10, 18, 33, 60, 111, 205, 385, 725, 1374, 2610, 4993, 9578, 18426, 568, 68806, 133411, 259145, 504222, 982538, 1917190, 3745385, 7324822  
POPULATION OF U\*\*2 + 2V\*\*2. REF MTAC 20 560 66.

83 1, 2, 4, 6, 11, 19, 33, 55, 95, 158, 267, 442, 731, 1193, 1947  
ANAR PARTITIONS. REF MA2 2 332.

84 1, 2, 4, 6, 11, 19, 34, 63, 117, 218, 411, 780, 1487, 2849, 5477, 10555, 20419, 563, 76805, 149360, 290896, 567321, 1107775, 2165487, 4237384, 8299283  
MANUJANS APPROXIMATION. REF MTAC 18 84 64.

85 1, 2, 4, 6, 12, 24, 36, 48, 60, 120, 180, 240, 360, 720, 840, 1260, 1680, 2520, 5040, 60, 10080, 15120, 20160, 25200, 27720, 45360, 50400, 55440, 83160, 110880, 166320  
GHLY COMPOSITE NUMBERS. REF RAM 87.

386 1, 2, 4, 6, 16, 20, 24, 28, 34, 46, 48, 54, 56, 74, 80, 82, 88, 90, 106, 118, 132, 140, 142, 154, 160, 164, 174, 180, 194, 198, 204, 210, 220, 228, 238, 242, 248, 254, 266, 272  
N\*\*4 + 1 IS PRIME. REF MTAC 21 246 67.

387 1, 2, 4, 6, 16, 20, 36, 54, 60, 96, 124, 150, 252, 356, 460, 612, 654, 664, 698, 702, 972  
172\*\*N - 1 IS PRIME. REF MTAC 22 421 68.

388 1, 2, 4, 7, 8, 11, 13, 14, 16, 19, 21, 22, 25, 26, 28, 31, 32, 35, 37, 38, 41, 42, 44, 47, 49, 50, 52, 55, 56, 59, 61, 62, 64, 67, 69, 70, 73, 74, 76, 79, 81, 82, 84, 87, 88, 91, 93  
ODD NUMBER OF ONES IN BINARY EXPANSION. REF CMB 2 86 59.

389 1, 2, 4, 7, 8, 12, 13, 17, 20, 26, 28, 35, 37, 44, 48, 57, 60, 70, 73, 83, 88, 100, 104, 117, 121, 134, 140, 155, 160, 176, 181, 197, 204, 222, 228, 247, 253, 272, 280, 301, 308  
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390 1, 2, 4, 7, 11, 16, 21, 28, 35  
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391 1, 2, 4, 7, 11, 16, 22, 29, 37, 46, 56, 67, 79, 92, 106, 121, 137, 154, 172, 191, 211, 232, 254, 277, 301, 326, 352, 379, 407, 436, 468, 497, 529, 562, 596, 631, 667, 704, 742  
CENTRAL POLYGONAL NUMBERS (N(N-1)/2 + 1, OR SLICING A PANCAKE, REF MAG 30 150 46, H03 22, FQ 3 296 65.

392 1, 2, 4, 7, 11, 16, 23, 31, 41, 53, 67, 83, 102, 123, 147, 174, 204, 237, 274, 314, 358, 406, 458, 514, 575, 640, 710, 785, 865, 950, 1041, 1137, 1239, 1347, 1461, 1581  
A PARTITION FUNCTION. REF CAY 2 278. JACS 53 3084 31. AMS 26 304 55.

393 1, 2, 4, 7, 12, 8, 80, 84, 820  
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394 1, 2, 4, 7, 12, 18, 27, 38, 53, 71, 94, 121, 155, 194, 241, 295, 359, 431, 515, 609, 717, 837, 973, 1123, 1292, 1477, 1683, 1908, 2157, 2427, 2724, 3045, 3396, 3774, 4185  
A PARTITION FUNCTION. REF AMS 26 304 55.

395 1, 2, 4, 7, 12, 19, 29, 42, 60, 83, 113, 150, 197, 254, 324, 408, 509, 628, 769, 933, 1125, 1346, 1601, 1892, 2225, 2602, 3029, 3509, 4049, 4652, 5326, 6074, 6905, 7823  
A PARTITION FUNCTION. REF AMS 26 304 55.

396 1, 2, 4, 7, 12, 19, 30, 45, 67, 97, 139, 195, 272, 373, 508, 684, 915, 1212, 1597, 2087, 2714, 3506, 4508, 5763, 7338, 9296, 11732, 14742, 18460, 23025, 28629, 35471  
PARTITIONS INTO PARTS OF 2 KINDS. REF RS2 90. RCI 199. FQ 9 332 71.

397 1, 2, 4, 7, 12, 20, 33, 54, 88, 143, 232, 376, 609, 966, 1596, 2583, 4180, 6764, 10945, 17710, 28656, 46367, 75024, 121392, 196417, 317810, 514228, 832039, 1346268  
FIBONACCI NUMBERS - 1. REF R1 155. AENS 79 203 62, FQ 3 295 65.

398 1, 2, 4, 7, 12, 21, 38, 68, 124, 229, 428, 806, 1530, 2919, 5591, 10750, 20717, 40077, 77653, 150752, 293161, 570963, 1113524, 2174915, 4250367, 8317036  
RAMANUJANS INTEGRAL. REF MTAC 18 85 64.

399 1, 2, 4, 7, 12, 22, 39, 70, 126, 225, 404, 725  
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400 1, 2, 4, 7, 12, 22, 41, 72, 137, 254, 476, 901, 1716, 3274, 6286, 12090, 23331, 45140, 87511, 169972, 330752, 644499, 1257523, 2456736, 4804666, 9405749  
POPULATION OF U\*\*2 + 4V\*\*2. REF MTAC 20 560 66.

- 101 1, 2, 4, 7, 13, 15, 18, 19, 20, 21, 22, 23, 25, 28, 29, 30, 35, 38, 40, 43, 44, 45, 48, 9, 50, 51, 54, 55, 56, 57, 58, 59, 60, 63, 65, 66, 71, 72, 74, 75, 79, 81, 84, 85, 87, 91, 93  
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- 102 1, 2, 4, 7, 13, 17, 30, 60, 107, 197, 257, 454, 908, 1619  
JUMPING PROBLEM. REF DOI 259.
- 103 1, 2, 4, 7, 13, 22, 40, 70, 126, 225, 411, 746, 1376, 2537, 4719, 8799, 16509, 1041, 58635, 111012, 210870, 401427, 766149, 1465019, 2807195, 5387990, 10358998  
REDUCIBLE POLYNOMIALS, OR NECKLACES. REF JSIAM 12 288 64.
- 104 1, 2, 4, 7, 13, 24, 42, 76, 137, 245, 441  
RESTRICTED PARTITIONS. REF EMS 11 224 59.
- 105 1, 2, 4, 7, 13, 24, 43, 78, 141, 253, 456  
RESTRICTED PARTITIONS. REF EMS 11 224 59.
- 106 1, 2, 4, 7, 13, 24, 44, 81, 149, 274, 504, 927, 1705, 3136, 5768, 10609, 19513, 5890, 66012, 121415, 223317, 410744, 755476, 1389537, 2555757, 4700770, 8646064  
RIBONACCI NUMBERS  $A(N) = A(N-1) + A(N-2) + A(N-3)$ . REF FQ 1(3) 71 63, 5 211 7.
- 107 1, 2, 4, 7, 13, 25, 43, 83, 157, 296, 564, 1083, 2077, 4006, 7733, 14968, 29044, 6447, 109864, 214197, 418080, 816907, 1598040, 3129063, 6132106  
IDD POPULATION OF  $U^{*+2} + V^{*+2}$ . REF MTAC 18 84 64.
- 108 1, 2, 4, 7, 14, 23, 42, 76, 139, 258, 482, 907, 1717, 3269, 6257, 12020, 23171, 4762, 86683, 168233, 327053, 636837, 1241723, 2424228, 4738426  
POPULATION OF  $2U^{*+2} + 3V^{*+2}$ . REF MTAC 20 563 66.
- 109 1, 2, 4, 7, 14, 24, 43, 82, 149, 284, 534, 1015, 1937, 3713, 7136, 13759, 26597, 1537, 100045, 194586, 378987, 739161, 1443465, 2821923, 5522689  
POPULATION OF  $4U^{*+2} + 4UV + 5V^{*+2}$ . REF MTAC 20 567 66.
- 110 1, 2, 4, 7, 14, 27, 52, 100, 193, 372, 717, 1382, 2664, 5135, 9898, 19079, 36776, 0988, 136641, 263384, 507699, 978602, 1886316, 3635991, 7008598, 13509507  
ETRANACCI NUMBERS. REF FQ 8 7 70.
- 111 1, 2, 4, 7, 14, 29, 60, 127, 275, 598, 1320, 2936  
IRON TREES. REF CAV 9 450.
- 112 1, 2, 4, 7, 39, 202, 1219, 9468, 83425, 80017  
IRAPHS COMPOSED OF TWO CIRCUITS. REF REA.
- 113 1, 2, 4, 8, 1, 6, 3, 2, 6, 4, 1, 2, 8, 2, 5, 6, 5, 1, 2, 1, 0, 2, 4, 2, 0, 4, 8, 4, 0, 9, 6, 8, 1, 2, 1, 6, 3, 8, 4, 3, 2, 7, 6, 8, 6, 5, 5, 3, 6, 1, 3, 1, 0, 7, 2, 2, 6, 2, 1, 4, 4, 5, 2, 4, 2, 8, 8, 1  
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- 114 1, 2, 4, 8, 7, 5, 10, 11, 13, 8, 7, 14, 19, 20, 22, 26, 25, 14, 19, 29, 31, 26, 25, 41, 37, 9, 40, 35, 43, 41, 37, 47, 58, 62, 61, 59, 64, 56, 67, 71, 61, 50, 46, 56, 58, 62, 70, 68  
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- 115 1, 2, 4, 8, 10, 12, 14, 18, 32, 48, 54, 72, 148, 184, 248, 270, 274, 420  
 $2^{*+N} - 1$  IS PRIME. REF MTAC 22 421 68.
- 416 1, 2, 4, 8, 13, 21, 31, 45, 60, 76, 97, 119, 144, 170, 198, 231, 265, 300, 336, 374, 414, 456, 502, 550, 599, 649, 702, 759, 819, 881, 945, 1010, 1080, 1157, 1237, 1318  
A SELF-GENERATING SEQUENCE. REF AMM 75 80 68. RLG.
- 417 1, 2, 4, 8, 13, 24, 42, 76, 140, 257, 483, 907, 1717, 3272, 6261, 12027, 23172, 44769, 86708, 168245, 327073, 636849, 1241720, 24242290, 4738450  
POPULATION OF  $U^{*+2} + 6V^{*+2}$ . REF MTAC 20 563 66.
- 418 1, 2, 4, 8, 14, 18, 28, 40, 52, 70, 88, 104, 140  
GENERALIZED DIVISOR FUNCTION. REF PLMS 19 111 19.
- 419 1, 2, 4, 8, 15, 26, 42, 64, 93, 130, 176, 232, 299, 378, 470, 576, 697, 834, 988, 1160, 1351, 1562, 1794, 2048, 2325, 2626, 2952, 3304, 3683, 4090, 4526, 4992, 5489  
SLICING A CAKE. REF MAG 30 150 46. FQ 3 296 65.
- 420 1, 2, 4, 8, 15, 27, 47, 79, 130, 209, 330, 512, 784, 1183, 1765, 2604, 3804, 5504, 7898, 11240  
PLANAR PARTITIONS. REF PCPS 47 686 51.
- 421 1, 2, 4, 8, 15, 27, 47, 80, 134, 222, 365, 597, 973, 1582, 2568, 4164, 6747, 10927, 17691, 28636, 46346, 75002, 121369, 196393, 317785, 514202, 832012, 1346240  
A NONLINEAR BINOMIAL SUM. REF FQ 3 295 65.
- 422 1, 2, 4, 8, 15, 29, 53, 98, 177, 319, 565, 1001, 1749, 3047, 5264, 9054, 15467, 26320, 44532, 75054, 125904, 210413, 350215, 580901, 960035, 1581534, 2596913  
TREES OF HEIGHT AT MOST 3. REF IBMJ 4 475 60. KUI.
- 423 1, 2, 4, 8, 15, 29, 56, 108, 208, 401, 773, 1490, 2872, 5536, 10671, 20569, 39648, 76424, 147312, 283953, 547337, 1055026, 2033628, 3919944, 7556935, 14564533  
TETRANACCI NUMBERS. REF AMM 33 232 26. FQ 1(3) 74 63.
- 424 1, 2, 4, 8, 15, 240, 15120, 672, 8400, 100800, 69300, 4950, 17199000, 22422400, 33633600, 201801600, 467812800, 102918816000  
COEFFICIENTS FOR NUMERICAL DIFFERENTIATION. REF PHM 33 11 42. BAMS 48 922 42.
- 425 1, 2, 4, 8, 16, 21, 42, 51, 102, 112, 224, 235, 470, 486, 972, 990, 1980, 2002, 4004, 4027, 8054, 8078, 16156, 16181, 32362, 32389, 64778, 64806, 129612, 129641, 259282  
A SELF-GENERATING SEQUENCE. REF AMM 75 80 68.
- 426 1, 2, 4, 8, 16, 22, 24, 28, 36, 42, 44, 48, 56, 62, 64, 68, 76, 82  
PERIODIC DIFFERENCES. REF TCPS 2 219 1827.
- 427 1, 2, 4, 8, 16, 31, 57, 99, 163, 256, 386, 562, 794, 1093, 1471, 1941, 2517, 3214, 4048, 5036, 6196, 7547, 9109, 10903, 12951, 15276, 17902, 20854, 24158, 27841, 31931  
BINOMIAL COEFFICIENT SUMS. REF MAG 30 150 46. FQ 3 296 65.
- 428 1, 2, 4, 8, 16, 31, 58, 105, 185, 319, 541, 906, 1503, 2476, 4058, 6626, 10790, 17537, 28464, 46155, 74791, 121137, 196139, 317508, 513901, 831686, 1345888  
A NONLINEAR BINOMIAL SUM. REF FQ 3 295 65.
- 429 1, 2, 4, 8, 16, 31, 61, 120, 236, 464, 912, 1793, 3525, 6930, 13624, 26784, 52656, 103519, 203513, 400096, 786568, 1546352, 3040048, 5976577, 11749641  
PENTANACCI NUMBERS. REF FQ 5 280 67.
- 430 1, 2, 4, 8, 16, 32, 63, 124, 244, 480, 944, 1856, 3649, 7174, 14104, 27728, 54512, 107168, 210687, 414200, 814296, 1600864, 3147216, 6187264, 12163841  
A PROBABILITY DIFFERENCE EQUATION. REF AMM 32 389 25.

- 71 1, 2, 4, 8, 16, 32, 63, 125, 248, 492, 976, 1936, 3840, 7617, 15109, 29970, 59448, 7920, 233904, 463968, 920319, 1825529, 3621088, 7182728, 14247536  
XANACCI NUMBERS. REF FQ 5 260 67.
- 72 1, 2, 4, 8, 16, 32, 64, 128, 256, 512, 1024, 2048, 4096, 8192, 16384, 32768, 65536, 1072, 262144, 524288, 1048576, 2097152, 4194304, 8388608, 16777216  
IWBERS OF TWO. REF BA1. MTAC 23 456 69.
- 73 1, 2, 4, 8, 16, 36, 85, 239  
EIGHTED LINEAR SPACES. REF BSM 22 234 70.
- 74 1, 2, 4, 8, 17, 39, 71, 152, 314, 628, 1357, 2725, 5551, 12212, 24424, 48848, 8807, 218715, 433531, 878162, 1867334, 3845668, 7802447, 16705005  
IWBERS OF TWO WRITTEN IN BASE 9. REF EUR 14 13 51.
- 75 1, 2, 4, 8, 17, 36, 78, 171, 379  
STINCT VALUES TAKEN BY  $2^{n-2} \dots + 2$ . REF GUT.
- 76 1, 2, 4, 8, 17, 39, 89, 211, 507, 1238, 3057, 7639, 19241, 48865, 124906, 321198, 0219, 2156010, 5622109, 14715813, 38649152, 101821927, 269010485  
COHOLS OR ROOTED TREES OF DEGREE AT MOST 4. REF JACS 54 2919 32. F12 41.397.
- 77 1, 2, 4, 8, 18, 40, 91, 210, 492, 1185, 2786, 6710, 16267, 39650, 97108, 238824, 9521  
IMS OF FERMAT COEFFICIENTS. REF MMAG 27 143 54.
- 78 1, 2, 4, 8, 18, 44, 122, 362, 1162, 3914, 13648  
CKLACES. REF IJM 5 664 61.
- 79 1, 2, 4, 8, 20, 52, 152, 472, 1520, 5044, 17112, 59008, 206260, 729096, 2601640, 59844, 33904324, 123580884, 452902072, 1667837680, 6168510256  
PRESENTATIONS OF ZERO. REF CMB 11 292 68.
- 70 1, 2, 4, 8, 20, 56, 180, 596, 2068, 7316, 26272  
CKLACES. REF IJM 5 664 61.
- 7 1, 2, 4, 8, 20, 100, 2116, 1114244, 68723671300, 1180735735906024030724, 0141183460507917357914971986913657860  
E SUM OF  $2^{n-1}C(n, k)$ . REF GO3.
- 2 1, 2, 4, 8, 21, 52, 131, 316, 765, 1846, 4494  
LATED TO PARTITIONS OF A NUMBER. REF AMM 76 1036 69.
- 3 1, 2, 4, 8, 22, 52, 140, 366, 992  
CKLACES. REF IJM 2 302 58.
- 4 1, 2, 4, 8, 24, 84, 328, 1372, 6024  
ERGY FUNCTION FOR SQUARE LATTICE. REF PHA 28 925 62.
- 5 1, 2, 4, 9, 10, 12, 27, 37, 38, 44, 48, 78, 112, 168, 229, 297, 339  
 $2^{n+1}$  IS PRIME. REF PAMS 9 674 58.
- 6 1, 2, 4, 9, 11, 23, 32, 39, 44, 51, 53, 60, 65, 72, 86, 93, 95, 114, 123, 156, 170, 179, 3, 200, 207, 212, 219, 228, 233, 240, 249, 261, 270, 303, 317, 333, 338, 345, 375, 389  
 $N + 1) + 1/7$  IS PRIME. REF CUI 1 250.
- 447 1, 2, 4, 9, 16, 29, 47, 77, 118, 181, 267, 392, 560, 797, 1111, 1541, 2106, 2863, 3846, 5142, 6808, 8973, 11733, 15275, 19753, 25443, 32582, 41569, 52770, 66757  
BIPARTITE PARTITIONS. REF PCPS 49 72 53. N11 1.
- 448 1, 2, 4, 9, 18, 42, 96, 229, 549, 1346, 3326, 8329  
CARBON TREES. REF CAV 9 454. ZFK 93 437 36.
- 449 1, 2, 4, 9, 19, 42, 89, 191, 402, 847, 1763, 3667, 7564, 15564, 31851, 64987, 132031, 267471, 539949, 1087004, 2181796, 4367927, 8721533, 17372967, 34524291  
TREES OF HEIGHT AT MOST 4. REF IBMJ 4 475 60. KUI1.
- 450 1, 2, 4, 9, 19, 48, 117, 307, 821, 2277  
MINIMAL TRIANGLE GRAPHS. REF MTAC 21 249 67.
- 451 1, 2, 4, 9, 20, 45, 105, 249, 599  
ESTERS. REF JACS 56 157 34.
- 452 1, 2, 4, 9, 20, 46, 105, 246, 583, 1393, 3355, 8133, 19825, 48554, 119412, 294761  
SUMS OF FERMAT COEFFICIENTS. REF MMAG 27 143 54.
- 453 1, 2, 4, 9, 20, 47, 108, 252, 582, 1345, 3086, 7072, 16121, 36667, 83099, 187865, 423610, 953033, 2139158, 4792126, 10714105, 23911794, 53273599, 118497834  
TREES OF HEIGHT AT MOST 5. REF IBMJ 4 475 60. KUI1.
- 454 1, 2, 4, 9, 20, 48, 115, 286, 719, 1842, 4766, 12496, 32973, 87811, 235381, 634847, 1721159, 4688676, 12826228, 35221832, 97055181, 268282855, 743724984  
ROOTED UNLABELED TREES. REF R1 138. HAS 232.
- 455 1, 2, 4, 9, 20, 51, 125, 329, 862, 2311, 6217, 16949, 46350, 127714, 353272, 981753  
CONNECTED GRAPHS WITH AT MOST ONE CYCLE. REF F12 41.399.
- 456 1, 2, 4, 9, 21, 51, 127, 323, 835, 2188, 5798, 15511, 41835, 113634, 310572, 853467, 2356779, 6536382, 18199284, 50852019, 142547559, 400763223, 1129760415  
GENERALIZED BALLOT NUMBERS. REF BAMS 54 359 48. JSIAM 17 254 69.
- 457 1, 2, 4, 9, 21, 52, 129, 332, 859, 2261  
PARAFINIS. REF JACS 56 157 34.
- 458 1, 2, 4, 9, 21, 56, 148, 428, 1305, 4191, 14140, 50159, 185987, 720298, 2905512, 12180208  
GRAPHS BY POINTS AND LINES. REF R1 146. ST1.
- 459 1, 2, 4, 9, 22, 59, 167, 490, 1486, 4639, 14805, 48107, 158808, 531469, 1799659, 6157068, 21258104, 73996100, 259451116, 951695102, 3251073303  
TOURNAMENT SCORES. REF CMB 7 135 64. MOI 68.
- 460 1, 2, 4, 9, 23, 63, 177, 514, 1527, 4625, 14230, 44357, 139779, 444558, 1425151, 4600339, 14939649, 48778197, 160019885, 527200711  
RELATED TO SERIES-PARALLEL NUMBERS. REF JM2 21 92 42.
- 461 1, 2, 4, 9, 23, 63, 188  
MIXED HUSIMI TREES. REF PNAS 42 535 56.
- 462 1, 2, 4, 9, 26, 101, 950  
GEOMETRIES. REF JM2 49 127 70.

463 1, 2, 4, 10, 24, 55, 128, 300, 700, 1632, 3809, 8890, 20744, 48408  
RESTRICTED PERMUTATIONS. REF AENS 79 207 62.

464 1, 2, 4, 10, 24, 66, 174, 504, 1406, 4210, 12198, 37378, 111278, 346846, 1053874  
FOLDING A STRIP OF STAMPS. REF CJM 2 397 50. JCT 5 151 68.

465 1, 2, 4, 10, 24, 66, 176, 493, 1361  
FOLDING A LINE. REF AMM 44 51 37.

466 1, 2, 4, 10, 24, 66, 180, 522, 1532, 4624, 14136, 43930, 137908, 437502, 1399068,  
4507382, 14611576, 47633486, 156047204, 513477502, 1696305720, 5623993944  
SERIES-PARALLEL NETWORKS. REF JIM2 21 87 42. R1 142.

467 1, 2, 4, 10, 25, 64, 166  
ALKYLs. REF ZFK 93 437 36.

468 1, 2, 4, 10, 25, 70, 196, 574, 1681  
FOLDING A LINE. REF AMM 44 51 37.

469 1, 2, 4, 10, 26, 76, 232, 764, 2620, 9496, 35696, 140152, 568504, 2390480,  
10349536, 46206736, 211799312, 997313824, 4809701440, 23758664096  
 $A(N) = A(N-1) + (N-1)A(N-2)$ . REF LUI 1 221. R1 86. MU2 6. DMJ 35 659 68.

470 1, 2, 4, 10, 27, 74, 202, 548, 1490, 4052, 11013, 29937, 81377, 221207, 601302,  
1634509, 4443055, 12077476, 32829965, 89241150, 242582598, 659407867  
COSH(N). REF AMP 3 33 1843. MNAS 14(5) 14 25. H44. LF1 93.

471 1, 2, 4, 10, 29, 90, 295, 1030, 3838, 15168, 63117, 275252, 1254801, 5968046,  
29551768, 152005634, 810518729, 4472244574, 25497104007, 149993156234  
FROM A DIFFERENTIAL EQUATION. REF AMM 67 766 60.

472 1, 2, 4, 10, 32, 122, 544, 2770, 15872, 101042, 707584, 5405530, 44736512,  
398721962, 3907514624, 38783024290, 419730685952, 4809759350882  
RELATED TO EULER NUMBERS. REF AMM 65 534 58. DKB 262.

473 1, 2, 4, 10, 36, 202  
CHANGING MONEY. REF NMT 10 65 62.

474 1, 2, 4, 10, 37, 138  
ROOTED PLANAR MAPS. REF CJM 15 542 63.

475 1, 2, 4, 10, 46, 1372, 475499108  
BOOLEAN FUNCTIONS. REF JSIAM 12 294 64.

476 1, 2, 4, 11, 15, 19, 23, 37, 44, 57, 78, 88, 95, 106, 134, 156, 205, 221, 232, 249,  
310, 323, 414, 429, 452, 550, 576, 639, 667, 715, 785, 816, 837, 946, 1003, 1038, 1122  
OF THE FORM  $(P^{**2} - 49)/120$  WHERE P IS PRIME. REF IAS 5 382 37.

477 1, 2, 4, 11, 19, 56, 96, 296, 554, 1593, 3093  
PERMUTATION GROUPS. REF JPC 33 1069 29.

478 1, 2, 4, 11, 33, 116, 435, 1832, 8167, 39700, 201785, 1099449, 6237505  
REFINEMENTS OF PARTITIONS. REF GUS.

479 1, 2, 4, 11, 34, 156, 1044, 12346, 274668, 12005168, 1018997864, 165091172592,  
50502031367952, 29054155657235488, 31426485969804308768  
GRAPHS OR REFLEXIVE SYMMETRIC RELATIONS. REF MI 17 22 55. MAN 174 68 67. HAS  
214.

480 1, 2, 4, 12, 32, 108, 336, 1036, 3120, 9540, 29244  
RESTRICTED PERMUTATIONS. REF AENS 79 213 62.

481 1, 2, 4, 12, 34, 111, 360, 1226  
ROOTED PLANAR 2-TREES. REF MAT 15 121 68.

482 1, 2, 4, 12, 39, 202, 1219, 9468, 83435, 836017, 9223092, 111255228,  
1453132944, 20433309147, 307690667072, 4940118795869, 84241805734539  
POLYGONS. REF AMM 67 349 60.

483 1, 2, 4, 12, 48, 200, 1040, 5600, 33600  
SORTING NUMBERS. REF PSPM 19 173 71.

484 1, 2, 4, 12, 56, 456, 6880, 191536, 9733056, 903753248, 154108311168  
TOURNAMENTS. REF MO1 87.

485 1, 2, 4, 12, 81, 1694, 123565, 33207256, 34448225389  
THRESHOLD FUNCTIONS. REF PGEC 19 821 70.

486 1, 2, 4, 12, 81, 2646  
SELF-DUAL MONOTONE BOOLEAN FUNCTIONS. REF WE1 181.

487 1, 2, 4, 12, 108, 10476, 108625644, 117983922690793836,  
139202068568601568785946949658348  
A NONLINEAR RECURRENCE. REF SA2.

488 1, 2, 4, 13, 41, 226, 1072, 9374, 60958, 723916, 5892536, 86402812, 83764188  
14512333928, 162925851376, 3252104882056, 41477207604872  
TERMS IN A SKEW DETERMINANT. REF PRSE 21 354 1896.

489 1, 2, 4, 13, 42, 308  
CONNECTED LINEAR SPACES. REF BSM 19 424 67.

490 1, 2, 4, 14, 34, 98, 270, 768, 2192, 6360, 18576, 54780, 162658, 486154, 14611  
4413968, 13393816, 40807290  
PARAFFINS. REF JACS 54 1105 32.

491 1, 2, 4, 14, 54, 332, 2246, 18264, 164950, 1664354, 18423144, 222406776,  
2905943328, 40865005494, 615376173184, 9880209206458, 168483518571798  
POLYGONS. REF AMM 67 349 60.

492 1, 2, 4, 14, 104, 1882, 94572, 15028134, 8378070864, 17561539552946  
THRESHOLD FUNCTIONS. REF PGEC 19 821 70.

493 1, 2, 4, 14, 128, 3882, 412736, 151223522, 189581406208, 820064805806914,  
12419748847290729472, 668590083306794321516802  
BINOMIAL COEFFICIENT SUMS. REF PGEC 14 322 65.

494 1, 2, 4, 14, 222, 616126, 200233952527184  
BOOLEAN FUNCTIONS. REF HA2 153.

495 1, 2, 4, 16, 56, 256, 1072, 6224, 33616, 218656, 1326656, 9893632, 70186624,  
574017536, 4454046976, 40073925376, 347165733632, 3370414011904  
PERMUTATIONS OF ORDER 4. REF CJM 7 159 55.

496 1, 2, 4, 16, 80, 520, 3640, 29120  
PERMUTATIONS BY NUMBER OF CYCLES. REF RI 85.

- 497 1, 2, 4, 16, 256, 65536, 4294967296, 18446744073709551616,  
34028236692093846374607431768211456  
2\*\*(2\*\*N). REF MTAC 23 456 69.
- 498 1, 2, 4, 24, 128, 880, 7440  
SORTING NUMBERS. REF PSPM 19 173 71.
- 499 1, 2, 4, 24, 1104, 2435424, 118625752248704, 281441383062305809756861824,  
15841850420004711075386369241884118003210485743490304  
A SLOWLY CONVERGING SERIES. REF AMM 54 138 47.
- 500 1, 2, 4, 60, 1276, 41888, 1916064, 116522048, 9069595840, 878460379392  
RELATED TO LATIN RECTANGLES. REF BU2 33 125 41.
- 501 1, 2, 4, 104, 272, 3104, 79808  
EXPANSION OF  $\sin x / \sin x$ . REF MMAG 31 189 58.

## SEQUENCES BEGINNING 1, 2, 5

- 502 1, 2, 5, 3, 15, 140, 5, 56  
QUEENS PROBLEM. REF SL1 49.
- 503 1, 2, 5, 4, 12, 6, 9, 23, 11, 27, 34, 22, 10, 33, 15, 37, 44, 28, 80, 19, 81, 14, 107, 89,  
64, 16, 82, 60, 53, 138, 25, 114, 148, 136, 42, 104, 115, 63, 20, 143, 29, 179, 67, 109  
A NUMBER-THEORETIC FUNCTION. REF AMM 58 526 49.
- 504 1, 2, 5, 5, 16, 7, 50  
TRANSITIVE GROUPS. REF BAMS 2 143 1896.
- 505 1, 2, 5, 6, 7, 10, 12, 14, 15, 20, 21, 22, 23, 25, 26, 30, 31, 34, 36, 37, 38, 39, 41, 42,  
45, 46, 47, 49, 50, 52, 53, 54, 55, 57, 58, 60, 62, 66, 69, 70, 71, 72, 73, 74, 76, 78, 79  
ELLIPTIC CURVES. REF JRAM 212 24 83.
- 506 1, 2, 5, 6, 8, 12, 18, 30, 36, 41, 66, 189, 201, 209, 276, 353, 408, 438, 534  
3.2\*\*N + 1 IS PRIME. REF PAMS 9 674 58.
- 507 1, 2, 5, 6, 11, 13, 17, 22, 27, 29, 37, 44, 44, 55  
GENERALIZED DIVISOR FUNCTION. REF PLMS 19 112 19.
- 508 1, 2, 5, 6, 14, 21, 29, 30, 54, 90, 134, 155, 174, 230, 234, 251, 270, 342, 374, 461,  
494, 550, 666, 750, 810, 990, 1890, 2070, 2486, 2757, 2966, 3150, 3566, 3630, 4554  
LATTICE POINTS IN SPHERES. REF MTAC 20 306 66.
- 509 1, 2, 5, 7, 10, 13, 15, 18, 20, 23, 26, 28, 31, 34, 36, 39, 41, 44, 47, 49, 52, 54, 57,  
60, 62, 65, 68, 70, 73, 75, 78, 81, 83, 86, 89, 91, 94, 96, 99, 102, 104, 107, 109, 112, 115  
A BEATTY SEQUENCE. REF CMB 2 191 59. AMM 72 1144 65.
- 510 1, 2, 5, 7, 11, 14, 20, 24, 30, 35  
CONSISTENT ARCS IN A TOURNAMENT. REF CMB 12 263 69. REI.
- 511 1, 2, 5, 7, 12, 15, 22, 26, 35, 40, 51, 57, 70, 77, 92, 100, 117, 126, 145, 155, 176,  
187, 210, 222, 247, 260, 287, 301, 330, 345, 376, 392, 425, 442, 477, 495, 532, 551, 590  
GENERALIZED PENTAGONAL NUMBERS. REF AMM 76 884 69. HO2 119.
- 512 1, 2, 5, 7, 12, 19, 31, 50, 81, 131, 212, 343, 555, 898, 1453, 2351, 3804, 6155,  
9959, 16114, 26073, 42187, 68260, 110447, 178707, 289154, 467861, 757015, 122448.  
 $A(N) = A(N-1) + A(N-2)$ . REF FQ 3 129 65.
- 513 1, 2, 5, 7, 19, 26, 71, 97, 265, 362, 989, 1351, 3691, 5042, 13775, 18317, 51409  
70226, 191861, 262087, 716035, 978122, 2672279, 3650401, 9973081, 13623482  
 $A(2N) = A(2N-1) + A(2N-2)$ ,  $A(2N+1) = 2A(2N) + A(2N-1)$ . REF MOET 1 10 16. N2  
181.
- 514 1, 2, 5, 7, 26, 265, 1351, 5042, 13775, 18817, 70226, 716035, 3650401  
RELATED TO GENOCCHI NUMBERS. REF AMM 36 645 35.
- 515 1, 2, 5, 8, 11, 14, 18, 22, 27, 31  
PARTITIONS INTO NON-INTEGRAL POWERS. REF PCPS 47 214 51.
- 516 1, 2, 5, 8, 13, 16, 21, 26, 35  
RAMSEY NUMBERS. REF CMB 8 579 65.
- 517 1, 2, 5, 8, 13, 18, 25, 32, 41, 50, 61, 72, 85, 98, 113, 128, 145, 162, 181, 200, 22  
242, 265, 288, 313, 338, 365, 392, 421, 450, 481, 512, 545, 578, 613, 648, 685, 722, 7  
NEAREST INTEGER TO  $(N+2 + 1/2)$ .
- 518 1, 2, 5, 8, 14, 21, 32, 45, 65, 88, 121, 161, 215, 280, 367, 471, 607, 771, 980, 12  
1551, 1933, 2410, 2983, 3690, 4536, 5674, 6811, 8317, 10110, 12276  
TREES OF DIAMETER 4. REF IBMJ 4 476 60. KU1.
- 519 1, 2, 5, 8, 18, 29, 57, 96, 183, 318, 603, 1080, 2047, 3762  
POLYTOPES. REF GR2 424.
- 520 1, 2, 5, 8, 21, 42, 96, 222, 495, 1177, 2717, 6435, 15288, 36374, 87516, 210494  
509694, 1237736, 3014882, 7370860, 18059899, 44379535, 109298070, 269766655  
PARTITIONS OF POINTS ON A CIRCLE. REF BAMS 54 359 48.
- 521 1, 2, 5, 9, 2, 1, 0, 4, 9, 8, 9, 4, 8, 7, 3, 1, 6, 4, 7, 6, 7, 2, 1, 0, 6, 0, 7, 2, 7, 8, 2,  
8, 3, 5, 0, 5, 7, 0, 2, 5, 1, 4, 6, 4, 7, 0, 1, 5, 0, 7, 9, 8, 0, 0, 8, 1, 9, 7, 5, 1, 1, 2, 1, 5, 5, 2  
CUBE ROOT OF 2. REF SMA 18 175 52.
- 522 1, 2, 5, 9, 14, 20, 27, 35, 44, 54, 65, 77, 90, 104, 119, 135, 152, 170, 189, 209, 2  
252, 275, 299, 324, 350, 377, 405, 434, 464, 495, 527, 560, 594, 629, 665, 702, 740, 7  
N(N + 3)/2.
- 523 1, 2, 5, 9, 15, 23, 34, 47, 64, 84, 108, 136, 169, 206, 249, 297, 351, 411, 478, 55  
HYDROCARBONS. REF JACS 55 684 33.
- 524 1, 2, 5, 9, 17, 27, 40, 55, 73, 117, 143  
RATIONAL POINTS IN A QUADRILATERAL. REF JRAM 227 47 67.
- 525 1, 2, 5, 9, 17, 28, 47, 73, 114, 170, 253, 365, 525, 738, 1033, 1422, 1948, 2634,  
3545, 4721, 6259, 8227, 10767, 13990, 18105, 23286, 29837, 38028, 48297, 61053  
PARTITIONS INTO PARTS OF 2 KINDS. REF RS2 90. RCI 199.
- 526 1, 2, 5, 9, 18, 35, 57  
POLYHEDRA. REF JRM 4 123 71.
- 527 1, 2, 5, 9, 21, 44, 103, 232, 571, 1368, 3441  
TOTAL DIAMETER OF UNLABELED TREES. REF IBMJ 4 476 60.



- 528 1, 2, 5, 9, 22, 62, 177, 560, 1939  
SERIES-REDUCED STAR GRAPHS. REF JMP 7 1585 66.
- 529 1, 2, 5, 10, 13, 17, 26, 29, 37, 41, 50, 53, 58, 61, 65, 73, 74, 82, 85, 89, 97, 101, 106, 109, 113, 122, 125, 130, 137, 145, 149, 157, 170, 173, 181, 185, 193, 197, 202, 218  
SOLUBLE PELLIANS. REF AMP 52 48 1871. KR1 1 46. LE1 56.
- 530 1, 2, 5, 10, 15, 25, 37, 52, 67, 97, 117  
GENERALIZED DIVISOR FUNCTION. REF PLMS 19 112 19.
- 531 1, 2, 5, 10, 16, 24, 33, 44, 56, 70, 85, 102, 120, 140, 161, 184, 208, 234, 261, 290, 320  
SERIES-REDUCED PLANTED TREES. REF R11.
- 532 1, 2, 5, 10, 18, 32, 55, 90, 144, 226, 346, 522, 777, 1138, 1648, 2362, 3348, 4704, 6554, 9056, 12425, 16932  
COEFFICIENTS OF AN ELLIPTIC FUNCTION. REF CAY 9 128.
- 533 1, 2, 5, 10, 19, 33, 57, 92, 147, 227, 345, 512, 752, 1083, 1545, 2174, 3031, 4179, 5719, 7752, 10438, 13946, 18519, 24428, 32051, 41805, 54265, 70079, 90102, 115318  
PARTITIONS INTO PARTS OF 2 KINDS. REF RS2 90. RCI 199.
- 534 1, 2, 5, 10, 20, 24, 26, 41, 53, 130, 149, 205, 234, 287, 340, 410, 425, 480, 586, 840, 850, 986, 1680, 1843, 2260, 2591, 3023, 3024, 3400, 3959, 3960, 5182, 5183, 7920  
LATTICE POINTS IN CIRCLES. REF MTAC 20 306 66.
- 535 1, 2, 5, 10, 20, 35, 62, 102, 167, 262, 407, 614, 919, 1345, 1952, 2798, 3950, 5524, 7671, 10540, 14388, 19470, 26190, 34968, 46439, 61275, 80455, 105047, 136541  
PARTITIONS INTO PARTS OF 2 KINDS. REF RS2 90. RCI 199.
- 536 1, 2, 5, 10, 20, 36, 65, 110, 185, 300, 481, 752, 1165, 1770, 2665, 3956, 5822, 8470, 12230, 17490, 24842, 35002, 49010, 68150, 94235, 129512, 177087, 240840  
PARTITIONS INTO PARTS OF 2 KINDS. REF RS2 90. RCI 199.
- 537 1, 2, 5, 10, 20, 38, 71, 130, 235, 420, 744, 1308, 2285, 3970, 6865, 11822, 20284, 34690, 59155, 100610, 170711, 289032, 488400, 823800, 1387225, 2332418, 3916061  
CONVOLVED FIBONACCI NUMBERS. REF RCI 101. FQ 3 51 65, 8 163 70.
- 538 1, 2, 5, 10, 20, 40, 86, 192, 440, 1038, 2492, 6071, 14960, 37198, 93193, 234956, 595561, 1516638, 3877904, 9950907, 25615653, 66127186, 171144671  
EXPONENTIAL INTEGRAL OF N. REF RS3 160 384 1870. PHM 33 757 42. FMR 1 267.
- 539 1, 2, 5, 10, 22, 40, 75, 130, 230, 382, 636, 1016, 1633, 2540, 3942, 5978, 9057  
COEFFICIENTS OF MODULAR FUNCTIONS. REF PLMS 9 386 59.
- 540 1, 2, 5, 10, 24, 63, 165, 467, 1405, 4435, 14775, 51814, 190443, 732472, 2939612  
GRAPHS BY POINTS AND LINES. REF R1 146. ST1.
- 541 1, 2, 5, 10, 25, 56, 139, 338, 852  
ALCOHOLS. REF BER 8 1545 1875.
- 542 1, 2, 5, 11, 21, 39, 73, 129, 226, 388, 659, 1100, 1821  
PLANAR PARTITIONS. REF MA2 2 332.
- 543 1, 2, 5, 11, 23, 47, 94, 185  
COMPOSITIONS. REF R1 155.
- 544 1, 2, 5, 11, 25, 66, 172, 485, 1446, 4541, 15036, 52496, 192218, 737248  
GRAPHS BY POINTS AND LINES. REF R1 146. ST1.
- 545 1, 2, 5, 11, 26, 68, 177, 497, 1476  
GRAPHS BY NUMBER OF LINES. REF R1 146. ST1. MAN 174 68 67.
- 546 1, 2, 5, 11, 28, 74, 199, 551, 1553, 4436, 12832, 37496, 110500, 328092, 98049, 294689, 8901891, 27011286, 82299275  
PARAFFINS. REF JACS 54 1105 32.
- 547 1, 2, 5, 11, 31, 77, 214, 576, 1592, 4375, 12183, 33864, 94741, 265461, 746372  
CONNECTED GRAPHS WITH 1 CYCLE. REF F12 41.399.
- 548 1, 2, 5, 11, 38, 174, 984, 6600, 51120, 448560, 4394880  
BINOMIAL COEFFICIENT SUMS. REF CJM 22 26 70.
- 549 1, 2, 5, 12, 17, 63, 143, 492, 635, 2397, 3032, 93357, 96389, 478913, 575302, 1629517, 15240955, 93075247, 387541943, 480617190, 868159133, 2216935456  
CONVERGENTS TO CUBE ROOT OF 4. REF AMP 46 106 1866. LE1 67. HPR.
- 550 1, 2, 5, 12, 24, 56, 113, 248, 503, 1043, 2080, 4169, 8145, 15897, 30545, 58402, 110461, 207802, 387561, 718875, 1324038, 2425473, 4416193, 7999516, 14411507  
RELATED TO SOLID PARTITIONS. REF MTAC 24 956 70.
- 551 1, 2, 5, 12, 27, 59, 127  
COMPOSITIONS. REF R1 155.
- 552 1, 2, 5, 12, 29, 70, 169, 408, 985, 2378, 5741, 13860, 33461, 80782, 195025, 470832, 1136689, 2744210, 6625109, 15994428, 38613965, 93222358, 225058681  
PELL NUMBERS  $A(N) = 2A(N-1) + A(N-2)$ . REF FQ 4 373 66.
- 553 1, 2, 5, 12, 30, 74, 188, 478, 1235, 3214, 8450, 22370, 59676, 160140, 432237, 1172436, 3194870, 8741442, 24007045, 66154654, 182864692, 506909562, 14088549  
POWERS OF ROOTED TREE ENUMERATOR. REF R1 150.
- 554 1, 2, 5, 12, 30, 76, 196  
GENERALIZED BALLOT NUMBERS. REF JSIAM 17 254 69.
- 555 1, 2, 5, 12, 31, 80, 210, 555, 1479, 3959  
PARAFFINS. REF JACS 56 157 34.
- 556 1, 2, 5, 12, 32, 94, 289, 910, 2934, 9686, 32540, 110780  
BALANCING WEIGHTS. REF JCT 7 132 69.
- 557 1, 2, 5, 12, 33, 87, 252, 703, 2105, 6099, 18689, 55639, 173423, 526937, 166405, 5137233, 16393315, 51256709, 164951529, 521198861, 1668959630, 5382512216  
FOLDING A LINE. REF MTAC 22 198 68.
- 558 1, 2, 5, 12, 33, 90, 261, 766, 2312, 7068, 21965, 66954, 218751, 699534, 225367, 7305788, 23816743, 78023602, 256738751  
SERIES-REDUCED PLANTED TREES. REF CAY 3 246. R11.
- 559 1, 2, 5, 12, 34, 130  
TRIANGULATIONS OF SPHERE. REF MTAC 21 252 67.
- 560 1, 2, 5, 12, 35, 107, 363, 1248, 4460, 16094, 56937, 217117, 805475, 3001211  
POLYMINONES WITHOUT HOLES. REF PA1. JRM 2 182 69. LU2.

- 561 1, 2, 5, 12, 35, 108, 369, 1285, 4655, 17073, 63600, 238591, 901971, 3426576, 13079255, 50107911, 192622052  
POLYMINOES. REF A11 363.
- 562 1, 2, 5, 12, 37, 123, 446, 1689, 6693, 27034, 111630, 467262, 1981353, 8487400, 36695369, 159918120, 701957539, 3101072051, 13779935438, 61557789660  
RESTRICTED HEXAGONAL POLYMINOES. REF EMS 17 11 70. RES.
- 563 1, 2, 5, 12, 53, 171, 566, 737, 4251, 4988, 9239, 41944, 428679, 7329487, 7758166, 115943811, 123701977, 239645788, 731522646953, 731762292741  
CONVERGENTS TO CUBE ROOT OF 5. REF AMP 46 107 1866. LET 67. HPR.
- 564 1, 2, 5, 13, 17, 29, 37, 41, 53, 61, 73, 89, 97, 101, 109, 113, 137, 149, 157, 173, 181, 193, 197, 229, 233, 241, 257, 269, 277, 281, 293, 313, 317, 337, 349, 353, 373, 389  
PRIMES WHICH ARE THE SUM OF 2 SQUARES. REF AMM 56 526 49.
- 565 1, 2, 5, 13, 19, 32, 53, 89, 139, 199, 293, 887, 1129, 1331, 5591, 8467, 9551, 15683, 19609, 31397, 370261, 1357201, 1561919, 2010733, 3826019, 3933599, 4652353  
FROM GAPS BETWEEN PRIME-POWERS. REF DVSS 2 255 1884.
- 566 1, 2, 5, 13, 29, 34, 89, 169, 194, 233, 433  
SOLUTIONS OF A DIOPHANTINE EQUATION. REF LEM 6 19 60.
- 567 1, 2, 5, 13, 33, 80, 184, 402, 840  
EXPANSION OF BRACKET FUNCTION. REF FQ 2 256 64.
- 568 1, 2, 5, 13, 33, 89, 240, 657, 1806, 5026, 13999, 39260, 110381, 311465, 880840, 2497405  
CONNECTED GRAPHS WITH ONE CYCLE. REF R1 150. ST1.
- 569 1, 2, 5, 13, 34, 89, 233, 610, 1597, 4181, 10946, 28657, 75025, 196418, 514229, 1346299, 35224578, 9227465, 24157817, 63245986, 165580141, 433494437  
BISECTION OF FIBONACCI SEQUENCE. REF R1 39. FQ 9 283 71.
- 570 1, 2, 5, 13, 35, 95, 262, 727, 2033, 5714  
PARTIALLY LABELED ROOTED TREES. REF R1 134.
- 571 1, 2, 5, 13, 36, 102, 296, 871, 2599  
NONISENTROPIC BINARY TREES. REF GUS.
- 572 1, 2, 5, 13, 36, 109, 359, 1266, 4731, 18657, 77464, 337681, 1540381, 7330418, 36301105, 186688845, 995293580, 5491595645, 31310124067, 184199228226  
FROM A DIFFERENTIAL EQUATION. REF AMM 67 766 60.
- 573 1, 2, 5, 13, 38, 116, 382, 1310, 4748, 17848, 70076, 284252, 1195240, 5174768, 23103368, 105899656, 498656912, 2404850720, 11879332048, 59976346448  
 $A(N) = A(N-1) + N \cdot A(N-2)$ . REF R1 86 (DIVIDED BY 2).
- 574 1, 2, 5, 13, 44, 191, 1229, 13588, 298597  
DISCONNECTED GRAPHS. REF TAMS 78 459 55. ST1.
- 575 1, 2, 5, 14, 39, 109  
PARAFFINS. REF ZFK 93 437 36.
- 576 1, 2, 5, 14, 39, 120, 358, 1176, 3527, 11622, 36627, 121622, 389560, 1301140, 4215748  
FOLDING A STRIP OF STAMPS. REF JCT 5 151 68.
- 577 1, 2, 5, 14, 42, 132, 429, 1430, 4862, 16796, 58786, 208012, 742900, 2674440, 9694845, 35357670, 129644790, 477638700, 1767263190, 6564120420, 24466267021  
CATALAN NUMBERS OR BINOMIAL COEFFICIENTS  $C(2N, N)/(N+1)$ . REF AMM 72 979 65. GUN1. RCI 101. CO1 1 67. G04.
- 578 1, 2, 5, 14, 44, 152  
PARTITION FUNCTION FOR SQUARE LATTICE. REF AIP 9 279 60.
- 579 1, 2, 5, 14, 46, 166, 652, 2780, 12644, 61136, 312676, 1680592, 9467680, 55704104, 341185496, 2170853456, 14314313872, 97620050080, 687418278544  
THE PARTITION FUNCTION  $G(N, 3)$ . REF CMB 1 87 58.
- 580 1, 2, 5, 14, 50, 233, 1249, 7595  
TRIANGULATIONS OF SPHERE. REF MTAC 21 252 67. GR2 424. JCT 7 157 69.
- 581 1, 2, 5, 14, 51, 267  
NUMBER OF GROUPS OF ORDERS 2, 4, 8, 16, 32, 64. REF HS1.
- 582 1, 2, 5, 15, 32, 99, 210, 650, 1379, 4268, 9055, 28025, 59458, 184021, 390420, 1208340, 2563621, 7934342, 16833545, 52099395  
A TERNARY CONTINUED FRACTION. REF TOH 37 441 33.
- 583 1, 2, 5, 15, 49, 169, 602, 2191  
PERMUTATIONS BY INVERSIONS. REF NET 96.
- 584 1, 2, 5, 15, 51, 196, 827, 3795, 18755, 99146, 556711, 3305017, 20655285, 135399720, 927973061, 6631556521, 49294051497, 380306658250, 3039453750685  
THE PARTITION FUNCTION  $G(N, 4)$ . REF CMB 1 87 58.
- 585 1, 2, 5, 15, 52, 203, 877, 4140, 21147, 115975, 678570, 4213597, 27644437, 190899322, 1382958545, 10480142147, 82864869604, 682076806159, 56327422050E  
BELL NUMBERS. REF MTAC 16 418 62. AMM 71 498 64. PSM 19 172 71. G04.
- 586 1, 2, 5, 16, 52, 208  
INVERSE SEMIGROUPS. REF PL 1. MA4 2 2 67.
- 587 1, 2, 5, 16, 61, 272, 1385, 7936, 50521, 353792, 2702765, 22368256, 19936098, 1903757312, 19391512145, 209865342976, 2404879675441, 29088885112832  
EULER NUMBERS. REF JDM 7 171 1881. JO1 238. NET 110. DKB 262. CO1 2 101.
- 588 1, 2, 5, 16, 63, 318, 2045  
UNLABELED PARTIALLY ORDERED SETS. REF BI1 4. NAMS 17 646 70. WH1. WR1.
- 589 1, 2, 5, 16, 65, 326, 1957, 13700, 109601, 986410, 9864101, 108505112, 1302061345, 16926797486, 236975164805, 3554627472076, 56874039553217  
PERMUTATIONS OF N THINGS. REF R1 16. MST 31 79 63.
- 590 1, 2, 5, 16, 67, 435  
CIRCUITS BY NULLITY. REF AIEE 51 311 32.
- 591 1, 2, 5, 16, 73, 538  
CIRCUITS BY RANK. REF AIEE 51 313 32.
- 592 1, 2, 5, 17, 37, 101, 197, 257, 401, 577, 677, 1297, 1601, 2917, 3137, 4357, 5477, 7057, 8101, 8837, 12101, 13457, 14401, 15377, 15877, 16901, 17957, 21317, 22501  
PRIMES OF FORM  $N^2 + 1$ . REF EUL (1) 3 22 17.