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Samuel Pepys, Isaac Newton, and Probability*

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The lost art of letter writing is certain to deprive future generations of scholars the pleasure and satisfaction of discovering interesting and instructive insights about the subjects they are studying. The role that personal correspondence played in the development of mathematics, science and philosophy, especially prior to 1850, is incompletely realized today.

We are privileged to examine a sequence of letters in which Newton answered a probability question asked by Pepys. I wish to thank Mr. Emil D. Schell of the International Business Machines Corporation for preparing the following discussion.

It is always a little surprising to learn that people famous in quite different fields were acquainted. John Milton's visit to Galileo in 1637 is an instance of this kind. The appearance of the imprimatur of S. Pepys, President of the Royal Society, on the title page of Newton's *Principia* certainly suggests that Newton and Pepys were acquainted. The fact that Pepys consulted Newton by letter on a problem in probability, however, seems not to be generally known, especially to American statisticians. It is our purpose to present the highlights of this correspondence. (1)

Although most readers are familiar with Pepys and Newton, a review of their careers up to the time of the correspondence in 1693 provides a more distinct setting. Pepys advanced from an initial clerical position in 1660 to Secretary of the Admiralty by 1672. The diary, for which he is famous, covered only the first nine years of this period and was discontinued because of failing eyesight. After serving as Secretary for seven years Pepys was accused of participating in a Popish plot during the Titus Oates affair, one frequently compared to similar hysterias of our own time. After a year's imprisonment in the Tower of London, he was freed and exonerated, but deprived of his post. In 1682 he decommissioned the naval base in Tangier. Again in 1684 he was appointed Secretary of the Admiralty and at the same time elected President of the Royal Society. With the accession of William III he lost his Admiralty post for the second time. At the time of the correspondence he had already completed his *Memoirs of the Royal Navy* and was in his fourth year of somewhat active retirement.

Newton was born in 1642 and was Professor at Cambridge from 1669-1701. His work on the binomial theorem had been done in 1666. His *Principia* was published in 1687. Prior to the time of the correspondence he was already making frequent trips to London. Besides his interest in the Royal Society, visits to the city were prompted by his seat in Parliament as the Cambridge University representative. The political upheaval which closed Pepys' Admiralty career for the second time saddled Newton with handling "the delicate question of oaths after the revolution of 1688", a task which he performed in a manner that evoked the admiration of Keynes. (2) At the close of 1692 Newton appears to have had a severe nervous breakdown. He wrote Pepys a bewildering letter at this time, and Pepys was undoubtedly relieved to receive another asking him to disregard the first. It is possible at the time of the present correspondence that Newton had not yet fully recovered. Newton was still three years away from his appointment as Warden of the Mint, which led him to make a permanent move to London. (3)

In the first letter of the correspondence on the probability problem Pepys presents Mr. Smith, a writing master at Christ's Hospital. (4) Thomas Neale, also mentioned in Pepys' letter, was Master of the Mint (a position which Newton held later) and Groom-Porter to the King. He had proposed a public lottery to raise funds immediately needed; expenses of the lottery were to be met by revenues from a new salt tax duty. The proposal was adopted in 1694.

In the opening letter (5), Pepys introduces Mr. Smith and proposes the problem:

Mr. Pepys to Mr. Isaac Newton

Wednesday, November 22, 1693

Sir,—However this comes accompanyed with a little trouble to you, yet I cannot but say that the occasion is welcome to me, in that it gives me an opportunity of telling you that I continue most sensible of my obligations to you, most desirous of rendring you service in whatever you shall think me able, and noe lesse afflicted when I hear of your being in towne without knowing how to wait on you till it be too late for me to doe it.

This sayd, and with great truth and respect, I goe on to tell you that the bearer, Mr. Smith, is one I beare great goodwill to, noe less for what I personally know of his general ingenuity, industry, and virtue, than for the general reputation he has in this towne (inferiour to none, but superiour to most) for his maistery in the two points of his profession, namely, *Faire-Writeing* and *Arithmetick*, soe farr (principally) as is subservient to Accountantship. Now soe it is, that the late project (of which you cannot but have heard) of Mr. Neale the Groom-Porter his lottery, has almost extinguished for some time at all places of publick conversation in this towne, especially among men of numbers, every other talk but what relates to the doctrine of determining between the true proportions of the hazards incident to this or that given chance or lot.

On this occasion it has fallen-out that this gentleman is become concerned (more than in jest) to compass a solution that may be relyed-on beyond what his modesty will suffer him to think his owne alone, or any less than Mr. Newton's to be, to a question which he takes a journey on purpose to attend you with, and prayed my giving him this introduction to you to that purpose, which, not in common friendship only but as due to his soe earnest an application after truth, though in a matter of speculation alone, I cannot deny him, and therefore trust you will forgive me in it, and the trouble I desire you to beare at my instance, of giving him your decision upon it, and the processe of your coming at it. Wherein I shall esteem myselfe on his behalfe greatly owing to you, and remaine, Honoured Sir, Your most humble and most affectionate and faithful servant,

S. P.

* For related discussions on probability in this section, see *The American Statistician* of December 1955, "Instructive Probability Problems" and December 1958, "Probability for the General Reader (and Others)."

The Question

A—has 6 dice in a box, with which he is to fling a 6.
B—has in another box 12 dice, with which he is to fling 2 sixes.
C—has in another box 18 dice, with which he is to fling 3 sixes.
Q—Whether B and C have not as easy a taske as A at even luck?

Nearly any practicing statistician will feel a high degree of kinship with Newton in his stress on formulation in his reply. He is mostly concerned with whether he has understood the question: Are A, B and C to throw independently? Are *exactly* 1, 2 and 3 sixes involved or *at least* 1, 2 and 3 sixes? Is it understood it is expectation that is involved and not the outcome of a particular throw? Apparently, Mr. Smith has not been of much help in answering these questions. Perhaps he was confused by the phrase "even luck" in the original question or by Newton's explanation of why A had the best expectation. The reader may draw his own inference from Newton's reply:

Mr. Isaac Newton to Mr. Pepys

Cambridge, November 26, 1693

Sir,—I was very glad to hear of your good health by Mr. Smith, and to have any opportunity given me of shewing how ready I should be to serve you or your friends upon any occasion, and wish that something of greater moment would give me a new opportunity of doing it so as to become more useful to you than in solving only a mathematical question. In reading the question it seemed to me at first to be ill stated, and in examining Mr. Smith about the meaning of some phrases in it he put the case of the question the same as if A plaid with six dyes tille he threw a six and then B threw as often with 12 and C with 18.—the one for twice as many, the other for thrice as many sixes. To examin who had the advantage, I tooke the case of A throwing with one dye and B with two, the former till he threw a six, the latter as often for two sixes, and found that A had the advantage. But whether A will have the advantage when he throws with 6 and B with 12 dyes I cannot tell, for the number of dyes may alter the proportion of the chances considerably, and I did not compute it in this case, the problem being a very hard one. And indeed, upon reading the question anew, I found that these cases do not come within the question. For here an advantage is given to A by his throwing first till he throws a six; whereas the question requires that they throw upon equal luck, and by consequence that no advantage be given to any one by throwing first. The question is this:—

A has 6 dyes in a box, with which he is to fling a six.
B has in another box 12 dyes, with which he is to fling two sixes.
C has in another box 18 dyes with which he is to fling 3 sixes.
Q Whether B and C have not as easy a task as A at eaven luck?

If this question must be understood according to the plainest sense of words, I think that sense must be this:—

1. Because A, B, and C are to throw upon equal luck, there must be no advantage of luck given to any of them by throwing first or last, or by making any thing depend upon the throw of any one which does not equally depend on the throws of the other two. And therefore to barr all inequality of luck on these accounts, I would understand the question as if A, B, and C were to throw all at the same time.
2. I take the most proper and obvious meaning of the words of the question to be that when A flings more sixes than one he flings a six as well as when he flings but a single six and so gains his expectation, and so when B flings more sixes than two and C more than three they gain their expectations. But if B throw under two sixes and C under three, they miss their expectations, because in the question 'tis exprest that B is to throw 2 and C three sixes.
3. Because each man has his dyes in a box ready to throw, and the question is put upon the chances of that throw without naming any more throws than that, I take the question to be the same as if it had been put thus upon single throws.
   What is the expectation or hope of A to throw every time one six at least with six dyes?
   What is the expectation or hope of B to throw every time two sixes at least with 12 dyes?
   What is the expectation or hope of C to throw every time three sixes or more than three with 18 dyes?
   And whether has not B and C as great an expectation or hope to hit every time what they throw for as A hath to hit his what he throws for?

If the question be thus stated, it appears by an easy computation that the expectation of A is greater than that of B or C; that is, the task of A is the easiest. And the reason is because A has all the chances of sixes on his dyes for his expectation, but B and C have not all the chances on theirs. For when B throws a single six or C but one or two sixes, they miss of their expectations. This Mr. Smith understands, and therefore allows that if the question be understood as I have stated it, then B and C have not so easy a task as A; but he seems of opinion that the question should be so stated that B and C as well as A may have all the chances of sixes on their dyes within their expectations. I do not see that the words of the question as 'tis set down in your letter will admit it, but this being no mathematical question, but a question what is the true mathematical question, it belongs not to me to determin it. I have contented my self therefore to set down how in my opinion the question according to the most obvious and proper meaning of the words is to be understood, and that if this be the true state of the question, then B and C have not so easy a task as A. But whether I have hit the true meaning of the question I must submit to the better judgments of your self and others. If you desire the computation, I will send it you. I am, Sir, Your most humble and most obedient servant,

Is. Newton.

Pepys appears to have had difficulty in meeting Mr. Smith after receiving Newton's reply. He indicates plainly that he understands the question to be as formulated by Newton, but finds it difficult to understand why A has the best chance of success. To make certain they are in agreement, he rewords the question so as to make the stakes high—the life of a condemned man:

Mr. Pepys to Mr. Isaac Newton

York Buildings, December 9, 1693

Sir,—It was my fortune to bee out of towne at Mr. Smith's returne, so as I received the favour of your letter left for mee by him, but have without successe expected every day to see him since my being back, that I might the more particularly render you with my thankes (which I doe most respectfully pay you) my acknowledgments for the satisfaction you are therein pleased to give mee upon the question I troubled you with by him. I am suspitious hee is not well, that I have been soe long without his visit, or that hee is not yet informed of my being returned. I will not however longer respite my observing to you that the construction hee would putt upon the question (and which I would the rather have discoursed with him on, before my offering you any thoughts of mine upon it) seems no more to mee than I find it does to you in any wise warrantable from the terms of it; I carrying about mee just the same notion of its meaning that you doe, viz., How much more or lesse expectation A may (with equal lucke) reasonably have of throwing at one or every throw one sixe at least with six dyes, than B two sixes with twelve, or C three with eighteen dyes?

Now if this wording of the question sorts as well with your conceptions of it as I have endeavoured to make them speak mine, then I discerne your resolution to come clearly up to the question in the terms I understood it in, and that you give it in favour of the expectations of A, and this (as you say) by an easy computation. But yet I must not pretend to soe much conversation with numbers as presently to comprehend as I ought to doe all the force of that which you are pleased to assigne for the reason of it, relating to their having or not having the benefit of all their chances; and therefore, were it not for the trouble it must have cost you, I could have wished for a sight of the very computation. But I have abundant reason to sitt downe (as I doe) without hesitancy under your determination, rather than keep-up an enquiry that I have already given you more interruption by than I can reasonably expect your excuse for.

I must confesse, were I now (after soe much chawing of the question) to begin my pursuit afresh after a solution to it, I think I should avoid some of the ambiguitys that commonly hang about our discoursings of it, by changing the characters of the dice from numbers to letters, and supposing them instead of 1, 2, 3, etc., to bee branded with the 6 initiall letters of the alphabet A, B, C, D, E, F. And the case should then bee this:

Peter a criminal convict being doomed to dye Paul his friend prevails for his having the benefitt of one throw only for his life, upon dice soe prepared; with the choice of any one of these three chances for it, viz., One F at least upon six such dice. Two F's at least upon twelve such dice, or Three F's at least upon eighteen such dice. Question:—Which one of these chances should Peter in this case choose?

I have the rather pitched upon this method of stating it, for the rendring it receptive of as simple and succinct an answer as (for the answerer's ease) I could. And therefore though I can't absolve my selfe of impertinence in the offering it, yet if you shall please, to what you have already indulged mee in it, to throw-in one act of kindnesse more, and tell mee your thought in the matter as thus drest, without creating more worke to your selfe in your reply than by giving it mee in either of these 2 words, the First—The Second—or the Third; I shall yet think I have asked too much, and rest ever, Your true honorer, and most faithfull humble servant,

S. Pepys.

Those who see philosophical difficulties in applying probability to the outcome of a single event will be interested to note that Newton sees none. He describes the method of his computation to Pepys and works out the stakes that each player should pay to enter the game:

Mr. Isaac Newton to Mr. Pepys

Cambridge, December 16, 1693

Sir,—In stating the case of the wager you seem to have exactly the same notion of it with me; and to the question which of the three chances should Peter chuse were he to have but one throw for his life, I answer, that if I were Peter, I would chuse the first. To give you the computation upon which this answer is grounded, I would state the question thus:—

A hath six dice in a box, with which he is to fling at least one six for a wager laid with R.

B hath twelve dice in another box, with which he is to fling at least two sixes for a wager laid with S.

C hath eighteen dice in another box, with which he is to fling at least three sixes for a wager laid with T.

The stakes of R, S, and T are equal; what ought A, B, and C to stake, that the parties may play upon equal advantage?

To compute this I set down the following progressions of numbers:

Progr. 1	1	2	3	4	5	6	the number of the dice
Progr. 2	0	1	3	6	10	15	
Progr. 3	6	36	216	1296	7776	46656	the number of all the chances upon them
Progr. 4	5	25	125	625	3125	15625	the number of chances without sixes
Progr. 5	1	5	25	125	625	3125	
Progr. 6	1	10	75	500	3125	18750	chances for one six and no more
Progr. 7		1	5	25	125	625	
Progr. 8		1	15	150	1250	9375	chances for two sixes and no more

The progressions in this table are thus found: the first progression, which expresses the number of the dice, is an arithmetical one, viz., 1, 2, 3, 4, 5, etc.; the second is found by adding to every term the term of the progression above it, viz., $0+1=1$, $1+2=3$, $3+3=6$, $6+4=10$, $10+5=15$, etc.; the third progression, which expresses the number of all the chances upon the dice, is found by multiplying the number 6 into itself continually; and the fourth, fifth, and seventh are found by multiplying the number 5 into itself continually; the sixth is found by multiplying the terms of the first and fifth, viz., $1\times 1=1$; $2\times 5=10$, $3\times 25=75$, $4\times 125=500$, etc.; and the eighth is found by multiplying the terms of the second and seventh, viz., $1\times 1=1$, $3\times 5=15$, $6\times 25=150$, $10\times 125=1250$, etc.; and by these rules the progressions may be continued on to as many dice as you please.

Now since A plays with six dice, to know what he and R ought to stake I consult the numbers in the column under six, and there from 46656, the number of all the chances upon those dice expressed in the third progression, I subduct 15625, the number of all the chances without a six expressed in the fourth; and the remainder, 31031, is the number of all the chances with one six or above. Therefore the stake of A must be to the stake of R, upon equal advantage, as 31031 to 15625, or $\frac{31031}{15625}$ to 1 for their stakes must be as their expectations, that is, as the number of chances which make for them. In like manner, if you would know what B and S ought to stake upon 12 dice, produce the progressions to the column of twelve dice, and the sum of the numbers in the fourth and sixth progressions, viz., $244140625 + 585937500 = 830078125$, will be the number of chances for S; and this number subducted from the number of all the chances in the third progression, viz., 2176782336, will leave 1346704211, the number of chances for B. Therefore the stake of B would be to the stake of S as 1346704211 to 830078125, or $\frac{1346704211}{830078125}$ to 1. And so by producing the progressions to the number of eighteen dice, and taking the sum of the numbers in the fourth, sixth, and eighth progressions for the number of the chances for T, and the difference between this number and that in the third column for the number of the chances for C, you will have the proportion of their stakes upon equal advantage. And thence it will appear that when the stakes of R, S, and T are units (suppose one pound or one guinea) and by consequence equal, the stake of A must be greater than that of B and that of B greater than that of C, and therefore A has the greatest expectation. The question might have been thus stated, and answered in fewer words: if Peter is to have but one throw for a stake of £1000 and has his choice of throwing either one six at least upon six dice, or two at least upon twelve, or three at least upon eighteen, which throw ought he to chuse, and of what value is his chance or expectation upon every throw, were he to sell it? Answer: Upon six dice there are 46656 chances, whereof 31031 are for him; upon 12, there are 2176782336 chances, whereof 1346704211 are for him; therefore his chance or expectation is worth the $\frac{31031}{46656}$ th. part of £1000 in the first case, and the $\frac{1346704211}{2176782336}$th part of £1000 in the second; that is , £665 0s 2d in the first case, and £618 13s 4d in the second. In the third case the value will be found still less. This I think, Sir, is what you desired me to give you an account of, and if there be any thing further you may command, Your most humble and most obedient servant,

Is. Newton.

Pepys frankly replies that he does not understand the computations of Newton. Furthermore, he finds it hard to believe the answer. Since B is throwing 12 dice, why can't he be regarded as two A's, and thus have at least as good an opportunity for success as A:

Mr. Pepys to Mr. Isaac Newton

December 21, 1693

Sir,—If to what you have done, and which I can in no wise sufficiently acknowledge your favour in, it could bee excusable to come once more to you upon the same errand, it should bee to ask you whether B's disadvantage (in his contest with A) bee any thing different under his obligation to fling 2 sixes at one throw with twelve dyes, from what it would bee were hee to doe it at twice with 6 dyes at a time out of one box, or at once out of 2 boxes with that number in each; I being yet (I must owne) unable to satisfie my selfe touching the difference, i.e., how it ariseth, though at the same time you have putt mee beyond all doubt of A's having the advantage in the main of B. Nor must I conceal my being at the same loss how to comprehend, even when flinging 12 dyes at one throw out of a single box (the said dyes being tinged, halfe green, half blew) my being less provided for turning up a six with either of these different coloured parcels while flung together out of the same box, than were the six blew to bee thrown out of one box and the 6 green from another; in which latter case, I presume each of them severally would bee equally entituled to the producing of a six with A's six white ones, and by consequence of 2 when flung together. I am conscious enough that this is but fumbling, and that it ariseth only from my not knowing how to make the full use of your Table of Progressions; but pray bee favourable to my unreadiness in keeping pace with you therein, and give mee one line of further helpe. I am most thankfully, Dear Sir, Your obliged and most humble and faithful servant,

S. Pepys.

At this stage in Pepys published correspondence the computations of a friend of Pepys, Mr. George Tollett are given. Tollett considers B as throwing two separate sets of six dice and finds, mistakingly, that B has the same chance of success as A. Following Tollett's manuscript there is a summary of the computations of Newton and Tollett by J. J. of M. C. This is presumed to be John Jackson of Magdalene College, Pepys' nephew. At the close of his summary the nephew indicates that the discrepancy might be resolved most simply by raising the question of B throwing six dice of one color and six of another and asking "what reason can be alledged why hee should not have the same expectation upon each of those setts as A upon his single sett?" This is the same form in which Pepys had put the question in the preceeding letter. I do not present these computations, but their remarkable outcome is that Pepys' nephew finds the flaw in Tollett's computation and decides Newton's result is correct. This is before Newton's reply to Pepys' question is written. Newton begins by discussing Peter and James and almost as an afterthought mentions that Peter is A and James is B:

Mr. Isaac Newton to Mr. Pepys

Cambridge, December 23, 1693

Sir,—I take it to be the same case whether a man, to throw two sixes, have one throw with twelve dyes or two throws with six, but I reccon it an easier task to throw with six dyes one six at one throw than two sixes at two throws. Were James to have twice as many throws as Peter, and as often as he throws a six to win half as much as Peter doth by the like throws, and by consequence were James to win as (much) at every two such throws as Peter doth at every one such throw and half as much at every such single throw, their cases would be equal. But this is not the case of the wager. As the wager is stated, Peter must win as often as he throws a six, but James may often throw a six and yet win nothing because he can never win upon one six alone. If Peter flings a six (for instance) four times in eight throws, he must certainly win four times, but James upon equal luck may throw a six eight times in sixteen throws and yet win nothing. For as the question in the wager is stated, he wins not upon every single throw with a six as Peter doth, but only upon every two throws wherein he throws at least two sixes. And therefore if he flings but one six in the two first throws, and one in the two next, and but one in the two next, and so on to sixteen throws, he wins nothing at all, though he throws a six twice as often as Peter doth, and by consequence have equal luck with Peter upon the dyes. Mr. Smith, being sensible to this disadvantage, would put such a sense upon the question that James may in some cases have some advantage of a single six, but this I was not satisfied in because it seemed to me contrary to the words of the question. He represents that it was their meaning, when they laid the wager, that James could do twice as much with 12 dyes as Peter with six, which is true if all the chances of sixes be considered, but in the wager all the chances are not considered. It requires that B (here called James) throw two sixes with twelve dyes at once, or (which is all one) with six dyes at twice. One six is not considered. 'Tis a losing cast, and this gives A (here called Peter) the advantage. In what proportion A has the advantage I computed in my last. If there be anything else, pray command, Your most humble and most obedient servant,

Is. Newton.

A further letter appears by Mr. Tollett to Pepys dated February 8, 1694. He now agrees with Newton and gives the detailed computations for A, B and C. A few days later Pepys replies, thanking him for the reasonable arrival of his letter for "being upon the very brink of a wager (£10 deep) upon my former belief. But apostacy (we all know) is now no novelty, and therefore like others I shall endeavor to make the best of mine, and face my antagonist downe that I always meant Thus. But then I must begg your ayde, that I may not be outbraved (as I have sometimes seen it done at Garroway's (a coffee house) by a cross-offer, and for want of knowing well why, not know which to stick to."

Pepys goes on to propose the time when he plans to welch on his original position in placing the wager. Whether he was able to find takers and whether he was successful, I have not found recorded. Certainly from the amount of effort he devoted to trying to understand the problem and the caliber of the consultants he enlisted, we can hope he profited handsomely.

NOTES

(1) I have not found any reference to this correspondence in any books on probability. Some twenty years ago I came across the reference in Volume II of Chrystal's *Algebra.* Only recently, I noticed that Todhunter's *History of Probability* does not contain the reference, which may explain its ommission from other books.

(2) J. M. Keynes' essay is reprinted in Newman's *World of Mathematics.*

(3) For data on Pepys and Newton, I used the Columbia Encyclopedia.

(4) I was tempted to refer to Mr. Smith as a gambler, having in mind the historical parallel of the chevalier de Mere and the problem he brought to Pascal. Oystein Ore in the American Mathematical Monthly for May 1960 points out that contrary to the widespread accounts, de Mere was not a professional gambler and "would turn in his grave at such a characterization of his main occupation in life."

(5) Newton's correspondence is just now being published by Cambridge University Press. Up to now a complete collection has not been attempted. The volumes dealing with the correspondence in this article will not appear before next year. The source for all letters given in this article has been "Private Correspondence and Miscellaneous Papers of Samuel Pepys," Edited by J. R. Tanner, G. Bell and Son, 1926.