$$| Y | \geq \sum_{k=0}^{[n/2]} \binom{n}{n - m + k} = \sum_{k=0}^{m} \binom{n}{k}$$

with equality only if $\alpha_j = \delta_{j(n-m)}$.

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# A Short Proof of Sperner's Lemma

Let $S$ denote a set of $N$ objects. By a *Sperner collection on S* we mean a collection of subsets of $S$ such that no one contain another. In [1], Sperner showed that no such collection could have more than ${}_NC_{[N/2]}$ members. This follows immediately from the somewhat stronger

THEOREM. *Let $\Gamma$ be a Sperner collection on S. Then*

$$\Sigma_{A \in \Gamma} {}_NC_{|A|}^{-1} \leq 1,$$

*where $| A |$ denotes the cardinality of A.*

PROOF. For each $A \subset S$, exactly $| A |!(N - | A |)!$ maximal chains of S (as a lattice under set inclusion) contain $A$. Since none of the $N!$ maximal chains of $S$ meet $\Gamma$ more than once, we have

$$\Sigma_{A \in \Gamma} | A |!(N - | A |)! \leq N!,$$

proving the theorem.

## REFERENCE

1. E. SPERNER, Ein Satz über Untermenger einer endlichen Menge, *Math. Z.* **27** (1928), 544–548.

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