Claim. *Let* $m, n,$ *and* $k \geq (n-1)(m+1)$ *be positive integers. Then*

$$\sum_u (-1)^{\sigma(u)} \binom{K}{k - u_0 m \quad k-1-u_1 m \quad \ldots \quad k-(n-1)-u_{n-1}m}$$
$$= 0!\,1!\,\cdots\,(n-1)!\,\frac{K!}{k!\,(k-m)!\,\cdots\,(k-(n-1)m)!}\,m^{\binom{n}{2}}\ ,$$

where $K = nk - (m+1)\binom{n}{2}$ *and* $u = (u_0, \ldots, u_{n-1})$ *runs over all permutations of the set* $\{0, 1, \ldots n-1\}$.

Proof. The left-hand side is $K!\ \det \|1/(k-i-mj)!\|_{0 \leq i,j \leq n-1}$. Dividing the ith row by $i!$ and multiplying the jth column by $(k-mj)!$ (for all $i, j \in [0, n-1]$) we re-write the identity in question as

$$\det \left\| \binom{k-jm}{i} \right\|_{0 \leq i,j \leq n-1} = m^{\binom{n}{2}}.$$

Subtracting from each row a suitable linear combination of the rows above it, we see that the determinant at the left is

$$\det \left\| \frac{1}{i!}\,(k-jm)^i \right\|_{0 \leq i,j \leq n-1} = \prod_{0 \leq i < j \leq n-1} ((k-im)-(k-jm)) \Big/ \prod_{i=0}^{n-1} i! = m^{\binom{n}{2}},$$

as required. ☐