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PI OF THE CIRCLE

(A Canto On-line Edition)

π

By

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it is a common property of all, like air, water, light etc – author

DEDICATION

I bow my head and dedicate this work

to

THE CREATOR OF THIS COSMOS

PREFACE

3.141.....is an approximation to π and is really the length of the perimeter of the inscribed polygon with 192 sides and above. The circle is the limit for the inscribed polygon. And this method is called Classical method or Archimedean method.

3.146..... is the new value to π . This is the value of the circumference of the inscribed circle. Square is the superscribed entity. The value obtained for π from the circle square entity is $(14 - \sqrt{2})/4 = 3.1464466.....$

The difference between the Archimedean method and the proposed method is simple. Archimedes had adopted **polygon – circle system**. The proposed method involves a **circle – square system**. In the former method attribution of the value of polygon to the circle is clear and in the proposed method square helps but the value belongs to the circle.

π equal to 3.141..... has failed to solve (arithmetically) the Hippocretean crescents of 500 B.C. and no hope of solving in the future too, with that π value. Can any body say with the knowledge of Euclid's **Elements** that Hippocretean crescents are wrong? However, the proposed π value has solved them. If the reader believes in the above facts, there is something new and interesting work detailed in the coming pages. For the reader who believes 3.141..... is final and transcendental, turning the pages of the book is waste of his precious time.

Coming to the details of the book :

1. New value of π has been obtained for the first time from the Gayatri method (pages 6 and 244).
2. and has been believed from the Siva method (pages 13, 53, 171, 173, 175, 176, 177, 181, 197, 209, 220, 228, 249, 260, 289, and 293).
3. Third method having much more convincing study of the interrelationship between the circle and the square in the calculation of the length of the circumference of the inscribed circle is explained in page no.218.
4. Many geometrical methods in the computation of π are given from the beginning to page 81. These are simple methods. Nothing is great in them. They help us in understanding that $(14 - \sqrt{2})/4$ is not wrong.
5. Archimedes' first theoretical proposition from his "**Measurement of a Circle**" is given a practical touch in page no.87. A length equal to π is given in the construction.
6. Hippocrates of Chios, Greece (500 B.C.) has given construction for squaring lunes, and **squaring lune and circle put together**, in one construction. Theoretical diagrams are given by him. He did not know the value of π . 3.1415926.... could not make practical the concepts of Hippocrates. From page 89 to 100 arithmetical values have been given to the theoretical propositions of Hippocrates.
7. Constantine Karapappoulos of University of Patra, Greece has "Squared the circle" with his modified diagram, page 102.

8. Gernot Hoffmann of Germany has “Circled a Square” with a refined approach and a proof of his own, page 202.
9. Trisection of 90^0 angle has been done is page no.110. The author has no prior knowledge of it though it was done and known to every body earlier. “Duplication of the cube” is done approximately in page 116.
10. Inscribed polygon and superscribed circle can not merge, as one entity. In page 137, it is explained they are two separate entities and are linked by a certain number.
11. Segmental areas of some constructions are calculated by different workers with the formulas deduced in terms of π constant only. Concerned line segments of these constructions have no role to play in the arrival of values to their areas. **When line segments are involved and areas are calculated these constructions have given the new value to π .** The constructions of the following workers are analysed / interpreted in terms of their line segments. Authors and their constructions are given (page nos in brackets).
 - Prof. Trevor Easingwood (pages 135 and 145)
 - Prof. C.R. Fletcher (140)
 - Prof. J.V. Narlikar (142)
 - Prof. K. Capell (158)
 - Prof. Louis Loynes (160 and 164)
 - Prof. Crockett Johnson (162)
 - Prof. Ing. Helmut Sander (165)
 - Bhaskar I of India (191)
 - Archimedes of Greece (269)
 - Prof. Alfred S. Posamentier (276)

12. **Squaring of a circle** is explained in many ways (Pages. 102, 129, 155, 159, 175, 178 and 255, semicircle into a triangle).
13. **Circling a square** is detailed in the pages 155, 179, 180 and 202.
14. A circle is drawn with a known value of radius. The length of the circumference and area of the circle are calculated. Similarly, with a known value of side a square can be drawn. The length of perimeter and diagonal and the area of square are calculated when a circle is inscribed in square or four equidistant tangents are drawn on the circle, a square can be created. In either of the case, the diameter of the inscribed circle and the side of the superscribed square **become equal**. Values of perimeter, diagonal and area of a square can be calculated very easily with the known side length.

In the present work the circle – square composite system has helped to calculate the length of the circumference and area of the inscribed circle.

The same procedure is adopted and with the **known** values of radius, circumference and area of the inscribed circle, the lengths of the side, diagonal of its superscribed square can also equally be calculated **exactly** (Pages 183, 188, 212, 233, 234, 241 and 295).

This way it is clear that the values of the line segments of **both** the circle and square can be calculated exactly *vice versa*, and is a **proof** of being the correctness of the values of the circumference and area of the circle.

15. Some interesting articles:

- a. Experimental method to arrive at the π value (168)
- b. Circle of different dimensions (182)
- c. Pythagorean way of calculating arcual length (185)
- d. Belly – Button of Human body (195)
- e. A circle of $\sqrt{2}$ area (240)
- f. A circle of $\sqrt{2}$ length of circumference (242)
- g. $\sqrt{5}$ and the area of a circle (243)
- h. Circle turning into a top (255)
- i. Circle and polygon have a clear interrelationship (259)
- j. Wrong choice of 3.1415926..... to π (266)

Mr. A. Narayanaswamy Naidu has read the manuscript and his suggestions have helped me to finish and write this work. Friends, colleagues and well-wishers have also helped directly and / or indirectly in the preparation of the difference^t articles of the book. I thank them all cordially.

Author

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1. SOME EARLIER METHODS TO COMPUTE π

The first mathematician who has explained the relationship between the area and the radius of the circle is **Euclid**. He has proved with his theorem that if A is the area of a circle of radius r and A' is the area of a circle of radius r' the $A/r^2 = A'/r'^2$. That is, the area of a circle divided by the square of the radius gives the same numerical result no matter how large or small a circle is. This number A/r^2 is called π ; hence we have the familiar formula $A = \pi r^2$ for the area of any circle. However, he has not computed the value of π .

From the later period of Euclid to the present day many people have attempted to estimate π . All the methods can broadly be divided into three types. They are

1. The method of exhaustion or the classical method of Archimedes
2. From the length of the chords of circle subtended by central angles of each degree and half degree proposed by Claudius Ptolemy and
3. From the application of infinite series of James Gregory.

Let us have a glance of these methods:

- I. Archimedes (240BC) has adopted a scientific method for the first time which is called the method of exhaustion where the area of the circle is computed to its nearest approximation from the inscribed polygon and circumscribed polygon about it. π value is arrived at from the so called area of the circle.

It can be done in the following way. In a circle of radius one, π is exactly area of the circle. A circle is taken and a square is inscribed with it and also another square is circumscribed about it. Area of the circle is in between these two. Then the number of sides are doubled. As we increase the number of sides, the area of the inscribed or circumscribed regular polygon approaches the area of the circle as a limit. Archimedes has carried this kind of process far enough to ascertain $3 \frac{10}{71} < \pi < 3 \frac{1}{7}$ using a polygon of 96 sides. π is approximated to two decimals, 3.14.....

π value has also been arrived at from the perimeters of the inscribed and circumscribed polygons of innumerable sides (2^{62}). This method is also called as Classical method of computing π .

Using polygons

Francois Viète (1579AD)	found π correct to	9	decimals
Adriaen Van Roomen (1593)	"	15	"
Ludolph Van Ceulen (1610)	"	35	"
Grienberger (1630) using Snell's refinement	"	39	"

- II. Claudius Ptolemy (150AD) in his book **Syntaxis Mathematica** has given π as equal to 3.1416... He has derived this value from the table of chords which appears in the treatise. The table gives the length of the chords of circle subtended by central angles of each degree and half degree. If the length of the chord of the 1° central angle is multiplied by 360, and the result divided by the length of the diameter of the circle, the above value for π is obtained.

Besides these mathematicians, some more people earlier to 1630AD, Tsu Chung - Chih(480AD), Aryabhata(530), Bhaskara(1150), Adriaen Anthoniszoon (1585), W.Snell (1621) and others have given similar value to π , but with differing decimals.

John Wallis (1650) has proposed a curious expression to π

$$\frac{\pi}{2} = \frac{2.2.4.4.6.6.8...}{1.3.3.5.5.7.7...}$$

Lord Brouncker, has converted Wallis' result into the continued fraction

$$\frac{4}{\pi} = 1 + \frac{1^2}{2 + \frac{3^2}{2 + \frac{5^2}{2 + \dots}}}$$

Neither of these expressions, however, has served an extensive calculation of π .

- III. The Scotch mathematician James Gregory (1671) has obtained the infinite series

$$\text{Arctan } x = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \dots$$

$$(-1 \leq x \leq 1)$$

Not noted by Gregory is the fact that for $x = 1$ the series becomes

$$\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \dots$$

This very slowly converging series was known to Leibniz in 1674.

Using Gregory's series or a modified form of it.

Abraham Sharp (1699 AD)	found	71	correct decimals to π
John Machin (1706)	"	100	"
De Lagny (1719)	"	112	"
William Rutherford (1841)	"	208	"
(1853)	"	400	"
William Shanks (1873)	"	707	"
D.F. Ferguson (1948)	"	710	"
J.W. Wrench Jr. (1948)	"	808	"

From Ptolemy (150AD) to Wrench(1948) all people have been saying that π is 3.141.... adopting Classical or exhaustion method / infinite series etc. The Classical method is based on a polygon inscribed / circumscribed with the circle. We derive the value for polygon's area and or perimeter and **attributing** this to the circle. Therefore the Classical method involving an element of presumption ("that the inscribed regular n-gon with n large enough we can approximate π as closely as we please / growing inscribed regular polygon gradually exhausts the area of the circle") and computing π value, instils in every mind a need for new approach.

As the result 3.1415926... for π obtained from the infinite series is identical to that of Classical method, the element of presumption of Classical method must have crept also into the infinite series unknowingly.

When two methods, say, A and B are there for one concept and both of which result in a common value and further if A is based on some assumption, the method B which supports the value of A, will also acquire the status of A.

In the circumstances explained above the author has come forward with the grace of God and proposed some methods to compute π . The area method has a mathematical proof in which the area of the inscribed circle is estimated and π derived out of it. Therefore, it is a direct method and naturally π computed from this method is an accurate approximation, whether it agrees or not with the Classical method or infinite series but definitely is in obedience with the Euclid's theorem of A/r^2 .

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2. HOW A CIRCLE AND A SQUARE ARE RELATED BY SQUARE ROOT TWO.

Four tangent lines on the circle equally apart from each other intersect at four points. As a result one more mathematical entity is created. It is called a square. With the creation of this square the original circle becomes an inscribed

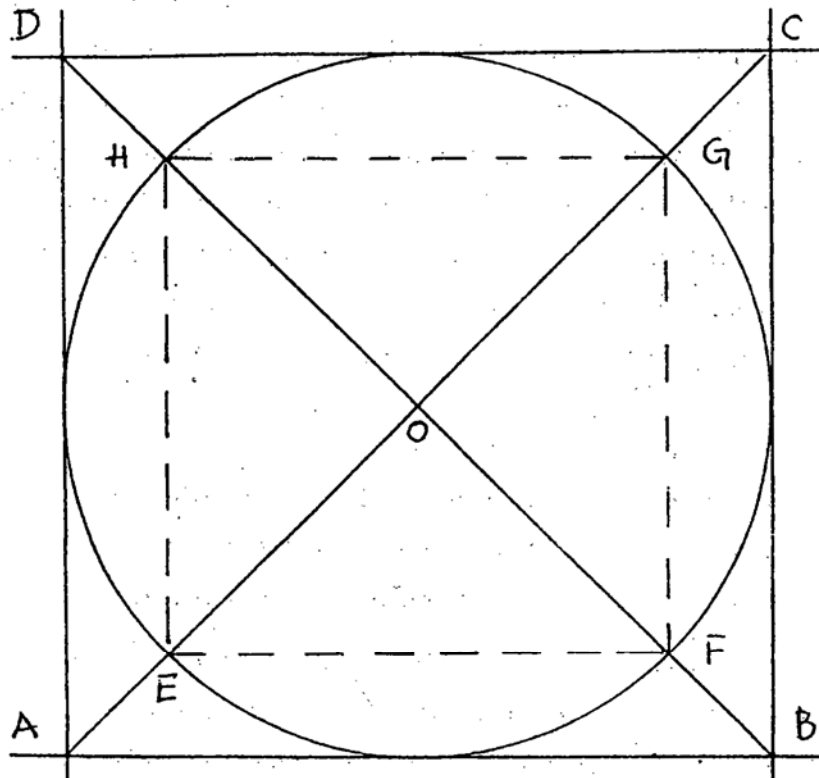


Fig -1

entity and the square a superscribed entity. The diameter of the circle and the side of the square are thus equal. If the diameter / side is of 1 unit length the diagonals of the square are equal to $\sqrt{2}$ each. From the foregoing lines it is clear that a circle, square and diagonals of $\sqrt{2}$ each, are interrelated one another significantly.

The diagonals AC and BD intersect the inscribed circle at four points E, F, G and H. When these points are joined another square EFGH is created anew. It is called the second or inscribed square. The side of it is equal to $\frac{\sqrt{2}}{2}$. Further, the second square can be divided into four equal right angled triangles with laterals each is equal to the radius of the circle i.e., 0.5 unit. Now, the side becomes the hypotenuse of the newly created triangle. When the Pythagorean theorem is applied the hypotenuse can be calculated to verify the side / hypotenuse of the inscribed square / right angled triangle.

The perimeter of the second square is equal to $4 \times \frac{\sqrt{2}}{2}$ when the perimeter of the superscribing square is 4 ($= 4 \times 1$). Thus the perimeter of the first square is $\sqrt{2}$

times of the perimeter of the second square. Therefore, we can say that the square and the circle are interrelated by $\sqrt{2}$.

1. Four tangent lines on the circle form a square.
 2. The points of intersection of the diagonals and the circle will form another smaller square.
 3. The first and second squares are interrelated in perimeter with $\sqrt{2}$.
 4. The relationship between the two squares with $\sqrt{2}$ is due to the presence of a circle in between them. Hence the square and the inscribed circle are related.
-

3. AN ACCURATE APPROXIMATION TO MATHEMATICAL π **

ABSTRACT.

It is established that a diagonal of a square has a significant relationship with the inscribed circle. The π value has thus been arrived at from this relationship and is 3.1464466... Two new formulae $3d + \left(\frac{d - \frac{\sqrt{2}d}{2}}{2} \right)$ and $4a - \left(\frac{\sqrt{2}}{4} a + \frac{1}{2} a \right)$ are proposed for the circumference of a circle. On simplification both lead to $(14-\sqrt{2})/4$ d or a.

INTRODUCTION :

The constant π is used in the calculation of the dimensions of a circle and a sphere. It is introduced by Archimedes who has shown that the exact value of π lies between $\frac{220}{70}$ and $\frac{223}{71}$. It is the ratio of the circumference of a circle to its diameter and is always constant. Its true value is considered to be 3.1415926...

The length of the circumference of any circle has about $3 \frac{1}{7}$ number of diameters in it. As it is accepted that this number is not an exact one, everybody, who is interested in elementary mathematics has been trying to find an accurate value for π since then.

In this direction the author has been trying to approximate accurately the value of π . Several methods have been tried earlier and different values for π have been obtained. All those values thus obtained are in agreement with the standard π value upto 2nd decimal only (i.e., 3.14...) but differ from 3rd decimal onwards. The following is the latest one.

METHOD - 1:

With a given side of a square, a circle is inscribed first and later second square is inscribed with the circle. Thus, we have two large squares and one circle in between these two. In addition, a third square very small in nature (and four in number of similar dimensions) is drawn at one of the corners between the first square and the inscribed circle.

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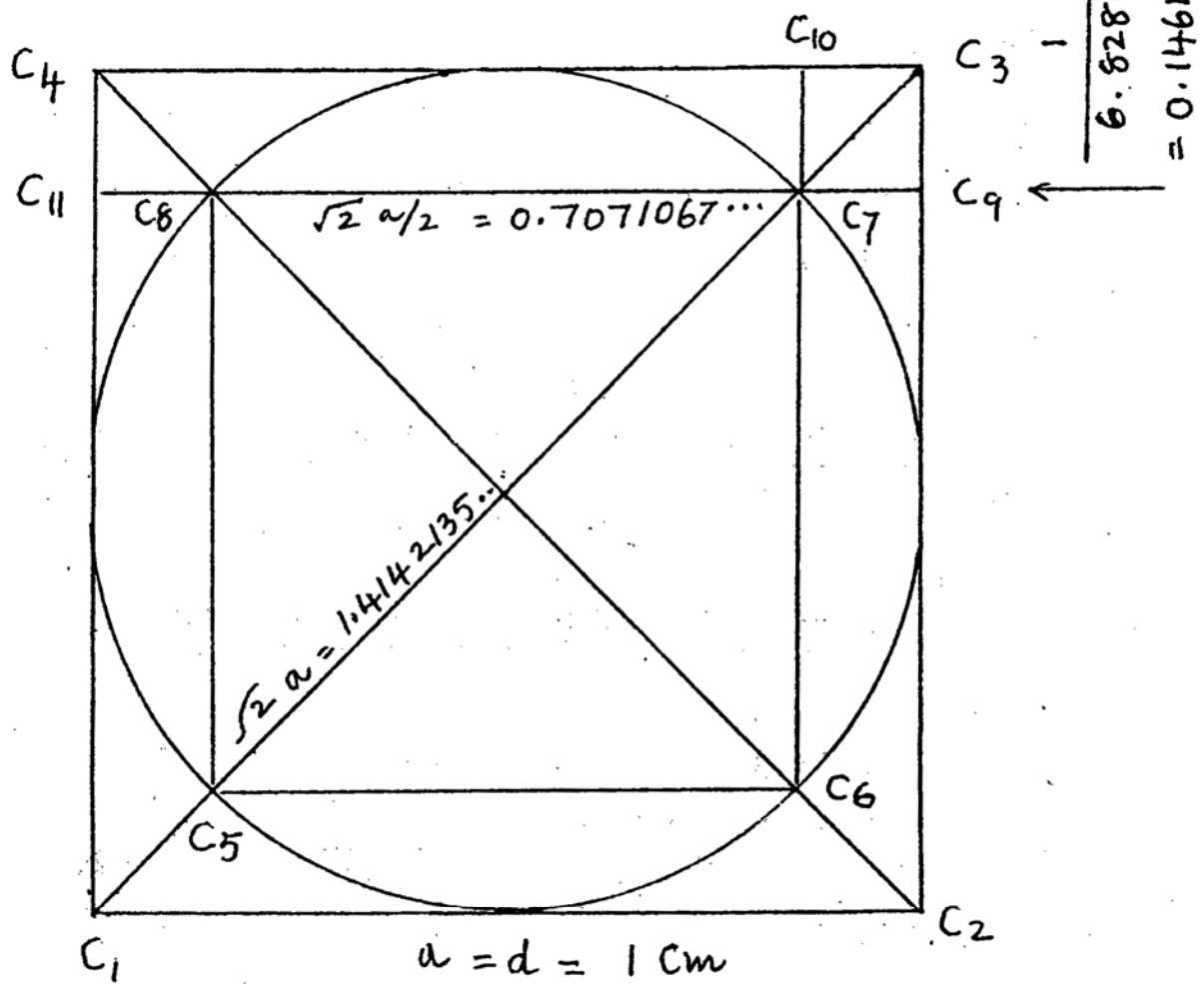


Fig - 2

In the diagram $C_1 C_2 C_3 C_4$ and $C_5 C_6 C_7 C_8$ are two squares. $C_1 C_3$ and $C_2 C_4$ are the two diagonals of the first square. $C_5 C_7$ and $C_6 C_8$ are the other two diagonals of the inscribed square. $C_7 C_9 C_3 C_{10}$ is the third square.

$C_5 C_6 C_7$ and C_8 are also the cross points of $C_1 C_2$ and $C_3 C_4$ diagonals of the first square. The side of the inscribed square, for example, $C_7 C_8$ when extended on either side, it crosses $C_2 C_3$ side and $C_1 C_4$ side of the first square at C_9 and C_{11} respectively. Similarly, the $C_6 C_7$ side of the inscribed square when extended crosses $C_3 C_4$ side of the first square at C_{10} , thus making a third (small) square of $C_7 C_9 C_3 C_{10}$. Similar squares of these dimensions can also be made at the remaining three corners of the first square.

From the diagram one can draw the following conclusions:

1. Each side of the inscribed square is equal to the half of the diagonal of the first square.

If $a = d = 1$ cm.

$$\text{the diagonal} = \sqrt{2}a = \sqrt{2} \times 1 = 1.4142135...$$

$$\text{The side of the inscribed square} = \frac{\sqrt{2}a}{2} = 0.7071067...$$

$$\text{i.e., } C_7 \text{ } C_8 = 0.7071067...$$

$$\text{the side of the third square} =$$

$$\frac{\text{the side of the first square} - \text{the side of the inscribed square}}{2}$$

$$2$$

$$\frac{d - \frac{\sqrt{2}d}{2}}{2} = \frac{1 - 0.7071067}{2} = 0.1464466...$$

therefore the length of $C_3 \text{ } C_9$ is 0.1464466... Cm. This value when represented in fraction it would be equal to $\frac{1}{6.8284275} = \frac{1}{4+2\sqrt{2}}$

We know πd is the formula for the circumference of a circle. We also know that about $3 \frac{1}{7}$ number of diameters constitute the length of the circumference of a circle. Regarding the value 3 there is no ambiguity. But doubt persists with regards to the vague fraction $\frac{1}{7}$. It is clear from the diagram and deductions based on this the fraction one is forced to accept is $\frac{1}{6.8284275}$ as the correct one.

When the diameter (d) is 1 cm. the circumference of a circle is equal to the π value.

$$\begin{aligned} \text{Therefore} &= 3 \frac{1}{6.8284275} = 3 + 0.1464466 \\ &= 3.1464466..... \end{aligned}$$

Further, the length of the sides of the first square between the points C_2 and C_9 via C_1 , C_{11} , C_4 , C_{10} , C_3 is equal to the circumference of the inscribed circle. This can be viewed as a visual proof of the correctness of the value of the circumference of a circle.

2. The sides of the inscribed square are decided by the crosspoints C_5 , C_6 , C_7 , and C_8 of the two diagonals of the first square on the inscribed circle.
3. The sides of the inscribed square and the sides of the third square are directly proportional – if the side of the inscribed square increases the side of the third square also increases.
4. $\frac{22}{7}$ or $3\frac{1}{7}$ is a simple fraction to π value. But the accurate approximation deduced from the diagram is $3\frac{1}{6.8284275}$.

As the diagonal value itself is an irrational number the resulting value obtained from such a value will also be an irrational number.

5. The standard π value is 3.1415926... and this value if represented (for comparison with the proposed fraction) it would be $3\frac{1}{7.0625159}$

FORMULA DERIVATION:

The following formula is deduced to calculate the circumference of the circle

$$3d + \left(\frac{d - \frac{\sqrt{2}d}{2}}{2} \right)$$

Therefore the length of the circumference of a circle is equal to the sum of the lengths of $C_2 C_1$ (d), $C_1 C_4$ (d), $C_4 C_3$ (d) and $C_3 C_9$ where,

$$\begin{aligned} C_3 C_9 &= \left(\frac{d - \frac{\sqrt{2}d}{2}}{2} \right) = \left(\frac{C_4 C_3 - \left(\frac{\sqrt{2} C_4 C_3}{2} \right)}{2} \right) \\ &= \frac{C_4 C_3 - C_7 C_8}{2} \end{aligned}$$

To be exact $3 \frac{1}{6.8284275}$ diameters make the circumference of any circle.

METHODS - 2:

The circumference of a circle can also be known from the value of perimeter of a square. It is possible because that the side of a square and the diameter of the inscribed circle are same. Further, as the circumference of an inscribed circle is

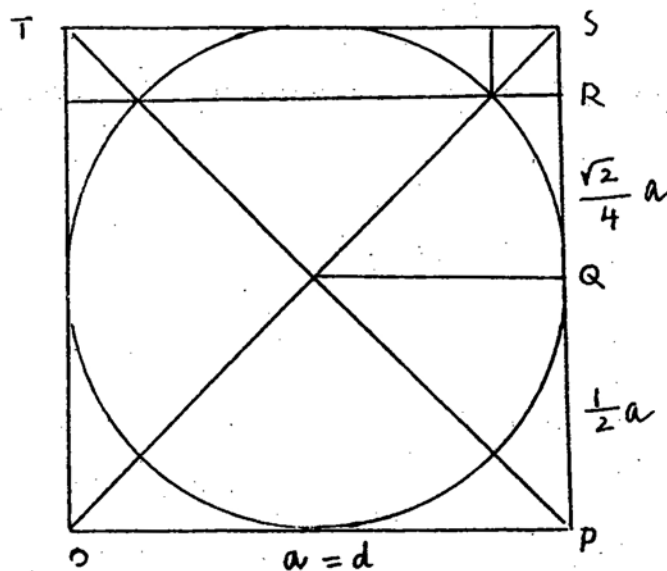


Fig -3

smaller than the perimeter of superscribed square one can deduce the following formula for the circumference of a circle.

$$4a - \left(\frac{\sqrt{2}}{4} a + \frac{1}{2} a \right)$$

where a is the side of a square ($=d$)

d is the diameter of a circle

$$\frac{\sqrt{2}}{4} a = RQ$$

$$\frac{1}{2} a = \text{radius} = PQ$$

hence

$$\text{perimeter of a square} - (RQ + PQ) = \text{Circumference of a circle.}$$

METHOD - 3

In the first article of the author entitled "A non-conventional formula to calculate the area of a circle" a ratio between the dimensions of square and its inscribed circle has been established. 'A new constant K has been derived.

$$\frac{\text{Area / perimeter of a square}}{\text{Area / circumference of a circle}} = K$$

$$\text{Where } a = d = 2r$$

$$\frac{4a}{2\pi r} = \frac{8r}{2\pi r} = \frac{4}{\pi} = K = \frac{4}{\frac{14 - \sqrt{2}}{4}} = \frac{16}{14 - \sqrt{2}}$$

$$\frac{a^2}{\pi r^2} = \frac{4r^2}{\pi r^2} = \frac{4}{\pi} = K =$$

From the present CBTS method also we can derive K value.

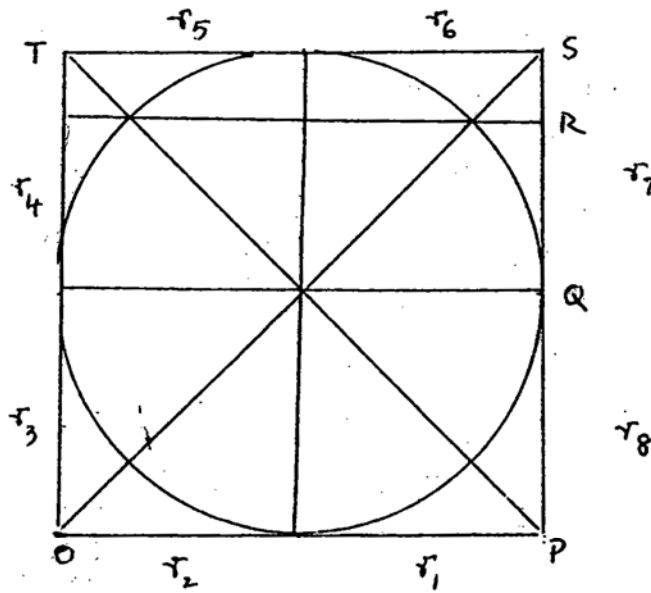


Fig - 4

$$a = d = 1 \text{ cm.}$$

$$r = 0.5 \text{ cm.}$$

$$SR = 0.1464466... \text{ cm.}$$

$$RQ = 0.3535533... \text{ cm.}$$

$$\text{If } r = \sqrt{2} = 1.4142135$$

$$SR = 0.4142134$$

$$RQ = 1$$

$$POTSQ = 7\sqrt{2}$$

$$POTSQ - RQ = 7\sqrt{2} - 1$$

We know that the perimeter of a square comprises of 8 radii (r) and if r is taken as $\sqrt{2}$ then the perimeter of a square will be $8\sqrt{2}$. This is because $\sqrt{2}$ has played a significant role in the derivation of the formula for the circumference of a circle i.e. $3d + \left(\frac{d - \frac{\sqrt{2}d}{2}}{2} \right)$. Similarly, the circumference of a circle lies between $6\sqrt{2}$ and $7\sqrt{2}$ (i.e., between PS and PQ via OT). To get an accurate figure from this range the following derivation is made.

If QS (1/2 of the side or radius of the inscribed circle) is taken as $\sqrt{2}$ then SR will be 0.4142134... and RQ will be 1. Therefore the circumference of a circle will be $7\sqrt{2} - 1$ ($POTSQ = 7\sqrt{2}$; $RQ = 1$; $POTSQ - RQ = 7\sqrt{2} - 1$) which is equal to the length between P and R via OTS. Therefore the value of K is equal to

$$\frac{\text{Perimeter of a square}}{\text{Circumference of a circle}} = \frac{8\sqrt{2}}{7\sqrt{2}-1}$$

This is in total agreement with the following derivation from the formulae of the perimeters of square and circle.

$$\frac{4d}{3d + \left(\frac{d - \frac{\sqrt{2}d}{2}}{2} \right)} = \frac{8\sqrt{2}}{7\sqrt{2}-1} = K = 1.2712753$$

where a, is the side of a square = d

d, the diameter of the inscribed circle.

From K we can calculate π value.

$$\frac{4}{K} = \pi = \frac{1}{1.2712753} = 3.1464467$$

$$\frac{\frac{4}{8\sqrt{2}}}{7\sqrt{2}-1} = \frac{7\sqrt{2}-1}{2\sqrt{2}} = 3.1464465$$

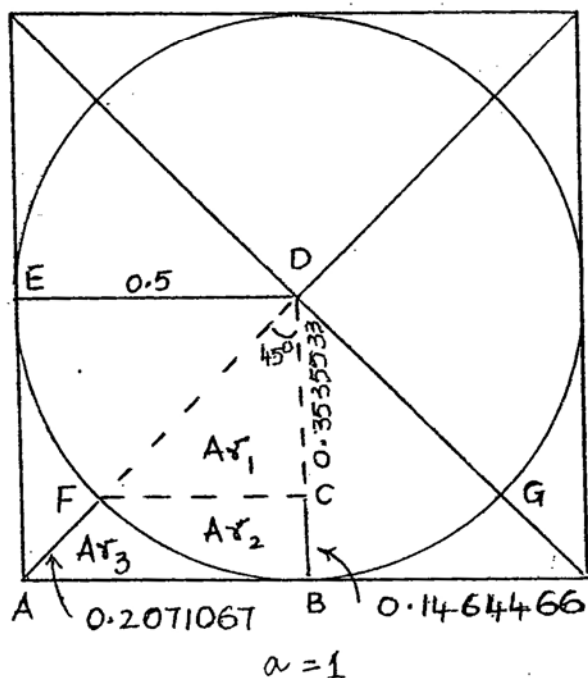
$$\text{therefore } \pi = \frac{7\sqrt{2}-1}{2\sqrt{2}} = 3.1464465$$

AN ACCURATE APPROXIMATION TO MATHEMATICAL PI

AREA METHOD

In the previous articles with the above same title the value of Pi has been derived from the value of the **circumference** of a circle (πd) with 1 unit as its diameter. All the eight methods proposed in these articles have resulted in the Pi value as 3.1464466... From these methods ten different formulae have been derived to yield the same Pi value.

For the first time an **area** method is proposed to arrive at the value of Pi. In this method, a square of 1 unit as its side is drawn and a circle is inscribed with it. Naturally, the diameter of the circle is same as the side of the superscribing square. Diagonals are drawn in the square. They intersect the inscribed circle at four points. Two intersecting points are F and G. The square can be divided into four equal parts. Each part is again a smaller square of 0.5 unit as its side. One smaller square out of the four is ABDE.



The two intersecting points F & G when joined become equal to $0.7071067... (\sqrt{2}/2)$. Half of this length FC is equal to $0.3535533... (\sqrt{2}/4)$.

In the smaller square of ABDE (which is one fourth of the superscribing square) an arc of the circumference FEB is also equal to $1/4^{\text{th}}$ of the circumference of the circle. Due to a part of the one of the diagonals i.e., AD the ABDE square is divided into two equal right angled triangles, ABD and AED. Therefore, the arc EFB is also divided into two equal halves i.e., EF and FB.

The values of the following lengths are

1. $AB = BD = DE = AE = \text{Side} = 0.5 \text{ Unit} \left(\frac{a}{2} \right)$
2. $AD = FG = 0.7071067... \left(\frac{\sqrt{2}}{2} \right)$
3. $FD = \text{Radius} = 0.5 \left(\frac{d}{2} \right)$
4. $AD - FD (0.7071067 - 0.5) = 0.2071067... \left(\frac{\sqrt{2}-1}{2} \right)$
5. $FC = CD = 0.3535533... \left(\frac{\sqrt{2}}{4} \right)$
6. $BD - CD (0.5 - 0.3535533) = 0.1464466... \left(\frac{\sqrt{2}-1}{2\sqrt{2}} \right)$

Thus, finally, we get a right angled triangle of ABD with an inscribed sector of DFB and a smaller triangle of FCD. Because of FCD triangle and FB arc of the circle the bigger triangle ABD is divided into three areas of different dimensions. They are area Ar_1 , area Ar_2 and area Ar_3 .

As ABD and FCD are right angled triangles their areas can be obtained using the well known formula $\frac{1}{2} ab$.

$$\text{Area of ABD} = \frac{1}{2} \times 0.7071067 \times 0.3535533 = 0.125 \text{ Sq. Units.}$$

$$\text{Area of FCD (Ar}_1\text{)} = \frac{1}{2} \times 0.5 \times 0.25 = 0.0625 \text{ Sq. Units.}$$

$$\begin{aligned} \text{Areas of Ar}_2 \text{ and Ar}_3 &= \text{ABD} - \text{FCD} \\ &= 0.125 - 0.0625 = 0.0625 \text{ Sq. Units.} \end{aligned}$$

Ar₂ and Ar₃ are bounded each by three sides. Out of the three sides one side is an arc FB, which is common to both. Therefore the areas Ar₂ and Ar₃ can not be evaluated **directly** with the conventional methods (due to the nature of curvature of FB).

To arrive at the areas Ar₂ and Ar₃, a selection method is adopted. This approach involves adopting two known method Arctan method and CBTS method. These methods, however, have given two different Pi values. Using these Pi values, the area of the inscribed sector FBD is calculated.

	Arctan Pi Value 3.1415926...	CBTS Pi Value 3.1464466...
1. Area of ABD = 0.125		
2. Area of FBD Sector	0.0981747	0.0983264
3. Area of Ar ₁	0.0625	0.0625
4. Area of Ar ₂	0.0356747	0.0358264
5. Area of Ar ₃	0.0268253	0.0266736

The areas of Ar₂ and Ar₃ can be calculated devising new formulae. Further, CBTS Pi value alone helps in this direction utilizing one of the sides in both the cases. In Ar₂, CB (0.1464466...) and in Ar₃, AF (0.2071067) are the **critical sides** which help in the calculation of these areas.

$$\begin{aligned}
 \text{Area of ABD} &= \text{areas of Ar}_1 + \text{Ar}_2 + \text{Ar}_3 &= & 0.125 \\
 \text{Area of FCD (Ar}_1) &= & 0.0625 \\
 &+ & \\
 \text{Areas of Ar}_2 + \text{Ar}_3 &= & 0.0625 &= & 0.125
 \end{aligned}$$

I. The area of Ar₂ can be calculated in the following way:

$$\frac{0.0625}{2} \times 1.1464466... = 0.0358264... \quad \text{---(1)}$$

where CB = 0.1464466...

With the **guidance** of above equation (1) the following formula is devised

$$\frac{r^2}{8} \left[1 + \frac{\sqrt{2}-1}{2\sqrt{2}} \right] \quad -(2)$$

where r is the radius of the circle of 0.5 unit.

II. The area of Ar₁ can be calculated from the following

$$\frac{a^2}{16} = \frac{r^2}{4} = 0.0625 \quad -(3)$$

The Sum of areas of Ar₁ and Ar₂ gives the area of the sector of 45° angle. 8 times of the sector gives the area of full circle.

$$(Ar_1 + Ar_2) 8 = \pi r^2 \quad -(4)$$

A general formula for the area of a circle can be devised with the help of equations (2), (3) and (4).

$$8 \frac{r^2}{4} + 8 \left[\frac{r^2}{8} \left(1 + \frac{\sqrt{2}-1}{2\sqrt{2}} \right) \right] \quad -(5)$$

on simplification we get

$$2r^2 + \left[r^2 \left(1 + \frac{\sqrt{2}-1}{2\sqrt{2}} \right) \right] = \pi r^2 \quad -(6)$$

$$\pi = \frac{2r^2 + \left[r^2 \left(1 + \frac{\sqrt{2}-1}{2\sqrt{2}} \right) \right]}{r^2} = 3.1464464..... \quad -(7)$$

III. The area of Ar₃ can be calculated in the following way

$$\frac{0.0625..}{8 \times \sqrt{2} \times 0.2071067..} = 0.0266735... \quad -(8)$$

where AF = 0.2071067

With the above equation (8) the following formula is devised

$$\frac{\frac{a^2}{16}}{8 \times \frac{\sqrt{2}-1}{\sqrt{2}}} = \frac{r^2}{32 \times \frac{\sqrt{2}-1}{\sqrt{2}}} \quad -(9)$$

where a is the side of the square with 1 unit r is the radius of 0.5 unit
(a = diameter = 2r)

$\frac{\sqrt{2}-1}{\sqrt{2}}$ is equal to $\sqrt{2} \times 0.2071067...$ and

$$\frac{a^2}{16} = \frac{1 \times 1}{16} = 0.0625$$

As the area of a square minus the area of the inscribed circle is equal to 8 times the area of Ar_3 , the area of any circle can be calculated using the following general formula.

$$\text{Area of the square} - (8 \times Ar_3) = \text{area of the circle} \quad - (10)$$

$$a^2 - 8 \left[\frac{r^2}{32 \times \frac{\sqrt{2}-1}{\sqrt{2}}} \right] \quad - (11)$$

where **a** is equal to **2r**

$$4r^2 - \left[\frac{r^2}{4 \times \frac{\sqrt{2}-1}{\sqrt{2}}} \right] = \pi r^2 \quad - (12)$$

Finally,

$$\pi = \frac{4r^2 - \left[\frac{r^2}{4 \times \frac{\sqrt{2}-1}{\sqrt{2}}} \right]}{r^2} = 3.1464464... \quad - (13)$$

From the equations (7) and (13) it is clear that the CBTS method is the right method in the arrival of Pi value of a circle for the calculations of its various dimensions.

Further, it is for the first time in the annals of mathematics that the area of Ar_3 (i.e., the area between square and its inscribed circle) is calculated by a new and independent formula and consequently the area of every circle.

CONFIRMATION METHOD

Previously the Pi value obtained with the circumference of a circle was confirmed by several methods. And Pi value derived with the Arctan Series (3.1415926....) and with the CBTS method (3.1464466....) were compared in a letter-cum article dated 10-11-1998 addressed to the Mathematical Societies and individual professors and was brought to their perusal that Pi value of Arctan Series is unequivocally defective.

The present confirmation method is an area method (fig.2). All the five circles are of equal dimensions. Here the entire sector, DEBC, of 90° angle is divided into six parts. Out of six, four are identical of one dimension and the other two are again identical of another dimension.

Further, all the eight areas of Ar_3 and Ar_2 (4+4) are the individual constituents of the square of 0.5 unit side. The sum of these areas is exactly equal to the area of the square.

1. Square ABCD

$$\begin{array}{rclcl}
 Ar_2 = 4 & Ar_2 = 0.0358264 \times 4 & = & 0.1433056 \\
 Ar_3 = 4 & Ar_3 = 0.0266736 \times 4 & = & 0.1066944 \\
 & \text{Area} & = & \underline{0.2500000}
 \end{array}$$

2. Sector DEBC of 90°

$$\begin{array}{rclcl}
 Ar_2 = 4 & = 0.0358264 \times 4 & = & 0.1433056 \\
 Ar_3 = 2 & = 0.266736 \times 2 & = & 0.0533472 \\
 & \text{Area} & = & \underline{0.1966528}
 \end{array}$$

3. Full Circle

$$\begin{array}{rclcl}
 \text{Sectors} = 4 & = 0.1966528 \times 4 & = & 0.7866112 \\
 \pi r^2 & & = & 0.7866112 \\
 \pi & = \frac{0.7866112}{0.25} & = & \underline{3.146448....}
 \end{array}$$

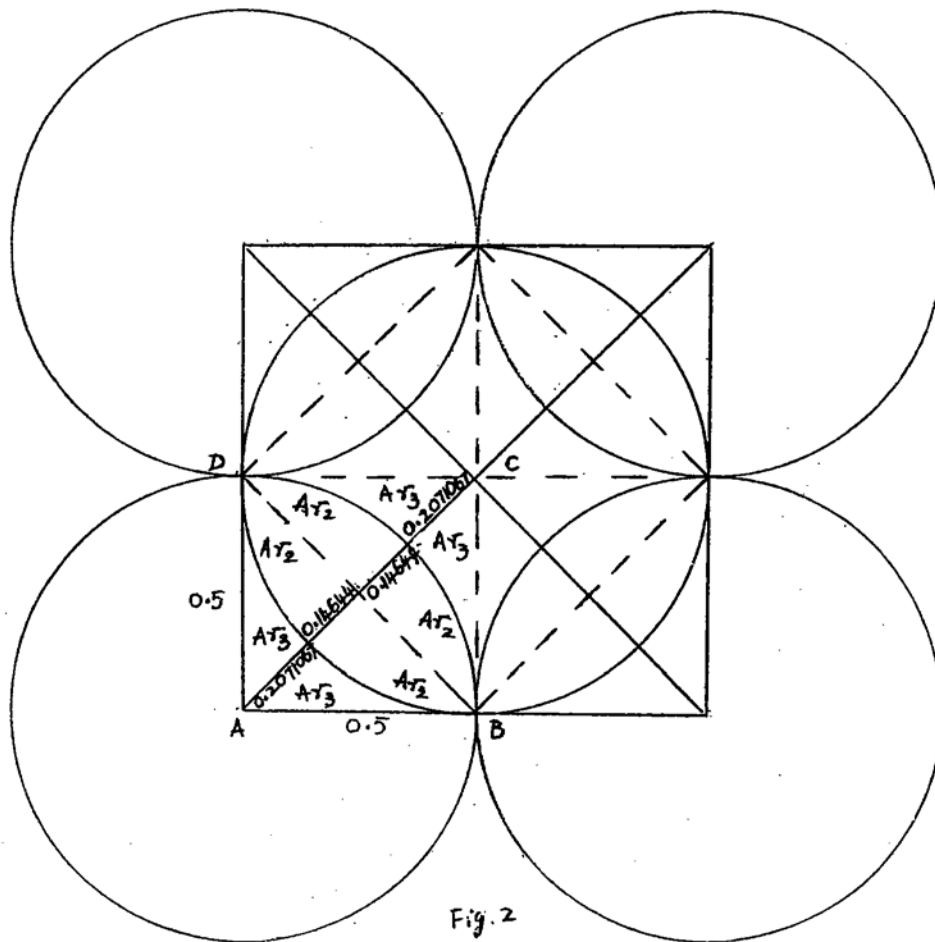
Finally the inscribed circle of 1 unit as its diameter has 16 Ar₂s and 8 Ar₃s by area and the superscribing square naturally of 1 unit side (= diameter) has 16 Ar₂s and equal number of Ar₃s.

$$\begin{array}{rclcl}
 \text{Ar}_2 & = & 0.0358264 \times 16 & = & 0.5732224 \\
 \text{Ar}_3 & = & 0.0266736 \times 16 & = & 0.4267776 \\
 & & & & \hline
 & & & & 1.0000000
 \end{array}$$

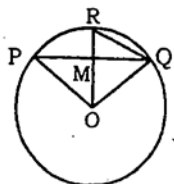
$$16 \left[\frac{4r^2}{2} \left(1 + \frac{\sqrt{2}-1}{2\sqrt{2}} \right) \right] + 16 \left[\frac{4r^2}{8 \times \frac{\sqrt{2}-1}{\sqrt{2}}} \right] = a^2$$

where $a = 2r$

As we are able to get the area of the square exactly from the constituent areas of Ar₂ and Ar₃ of sector, and Ar₃ outside the sector and within the square, we can take this as a confirmation method. Thus, the Pi value 3.1464466.... obtained by the CBTS method is an accurate approximation.



Classical method of computing π



Classical method involves the inscription of polygon in the circle. Starting with the known perimeter of a regular polygon (here we start with a regular hexagon) of n sides inscribed in a circle, the perimeter of the inscribed regular polygon of $2n$ sides can be calculated by the application of Pythagorean theorem. Let C be a circle with centre O and radius r , and let $PQ = s$ be a side of a regular inscribed polygon of n -sides having a known perimeter. Then the apothem, $OM = u$ is given by $\sqrt{r^2 - (s/2)^2}$ hence the sagitta, $MR = v = r - u$ is known. Then the side $RQ = w$ of the inscribed polygon of $2n$ sides is found from $w = \sqrt{v^2 + (s/2)^2}$ hence the perimeter of this polygon is known.

S.No.	n	$PQ=s$	ns Perimeter = circumference = π	$s/2 = t$	$\sqrt{r^2 - t^2} = u$	$r - u = v$	$\sqrt{v^2 + t^2} = w$
1.	6	0.5	3.0	0.25	0.433012701	0.066987299	0.258819045
2.	12	0.258819045	3.1058828544	0.129409522	0.482962913	0.17037087	0.13052619
3.	24	0.13052619	3.132628565	0.065263095	0.49572243	0.00427757	0.065403128
4.	48	0.065403125	3.139350157	0.032701564	0.498929461	0.001070539	0.032719082
5.	96	0.032419082	3.141031907	0.016359541	0.499732293	0.000267707	0.016361731
6.	192	0.016361731	3.141452431	0.00818065705	0.499933069	0.000066931	0.008181139495
7.	384	0.008181139495	3.141557566	0.004090569748	0.499983267	0.000016733	0.004090603972
8.	768	0.004090603972	3.14158385	0.002045301986	0.499995816	0.000004184	0.002045306262
9.	1536	0.002045306262	3.141590424	0.001022653133	0.499998654	0.000001046	0.001022653368
10.	3072	0.001022653668	3.141592067	-	-	-	-

CC APPROACH TO APPROXIMATE π VALUE

INTRODUCTION

In the π series, by the grace of God the author so far has submitted 21 methods and equal number of formulae to approximate π value equal to $(14-\sqrt{2})/4$. The proposed method is called **Curvature Constant method**.

It has been established by the author in his 1st article (Ref.1) that a square and its inscribed circle are related by a constant called k .

$$k = \frac{a^2}{\pi r^2} = \frac{4a}{2\pi r} = \frac{16}{14 - \sqrt{2}} = 1.2712752.....$$

Where $a = 2r$ and $\pi = (14-\sqrt{2})/4..$

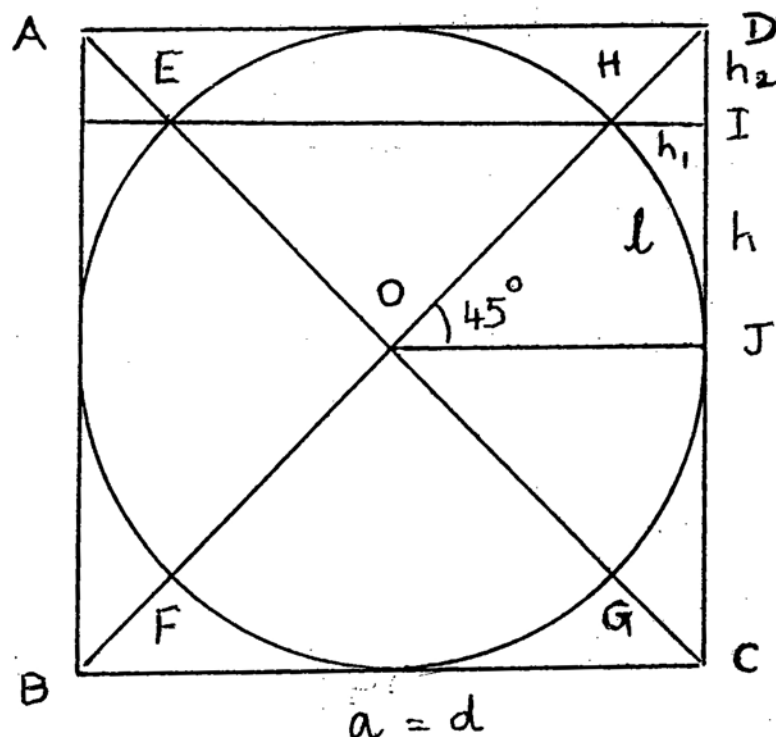
From k we can also calculate the dimensions of any circle and sphere (for formulae Ref.1)

PROPOSED METHOD

A square ABCD of a given side with two diagonals AC & BD is drawn. AC & BD intersect at O. Taking O as the centre a circle is inscribed with the square. The circle intersects the two diagonals at E, F, G and H.

The nature of the curvature to a circle is due to the uniformity of the radius throughout 360° . An arc for example JH is away and deviated to the left from the straight line JI at right angle to OJ. As the arc JH which is away from JI due to the constancy of radius, a new parameter $h = HI$ has originated and this in turn led to the **increased length of the arc** l vis-à-vis the length h of the straight line.

The square ABCD can also be an outcome of 4 equidistant tangent lines of the circle and further the nexus between the circle and consequential square is related by a surd $\sqrt{2}$ (Ref.2) and the arcual length l can be obtained from the known value of h .



$$h = JI = \frac{\sqrt{2}a}{4}$$

$$h_1 = h_2 = IH = ID = \frac{\sqrt{2}a - 1a}{2\sqrt{2}}$$

$$h + h_2 = OJ = \text{radius} = a/2$$

$$l = JH = \frac{\pi d}{8}$$

$$\text{Angle HOJ} = 45^\circ$$

The ratio of the surd $\sqrt{2}$ and k is equal to a new constant called **Curvature Constant, cc**.

$$cc = \sqrt{2}/k = \frac{\sqrt{2}(14 - \sqrt{2})}{16}$$

The arcual length $l = h \times cc$

$$= \frac{\sqrt{2}a}{4} \times \frac{\sqrt{2}(14 - \sqrt{2})}{16} = \frac{\pi d}{8} = \frac{\pi a}{8}$$

Finally, π can be derived from the above formula and is equal to $(14 - \sqrt{2})/4$...

REFERENCES:

1. A non – conventional formula to calculate the area of a circle (accepted for publication.)
2. How a circle and square are related by square root two.

PYTHAGOREAN APPROACH TO COMPUTE π

INTRODUCTION

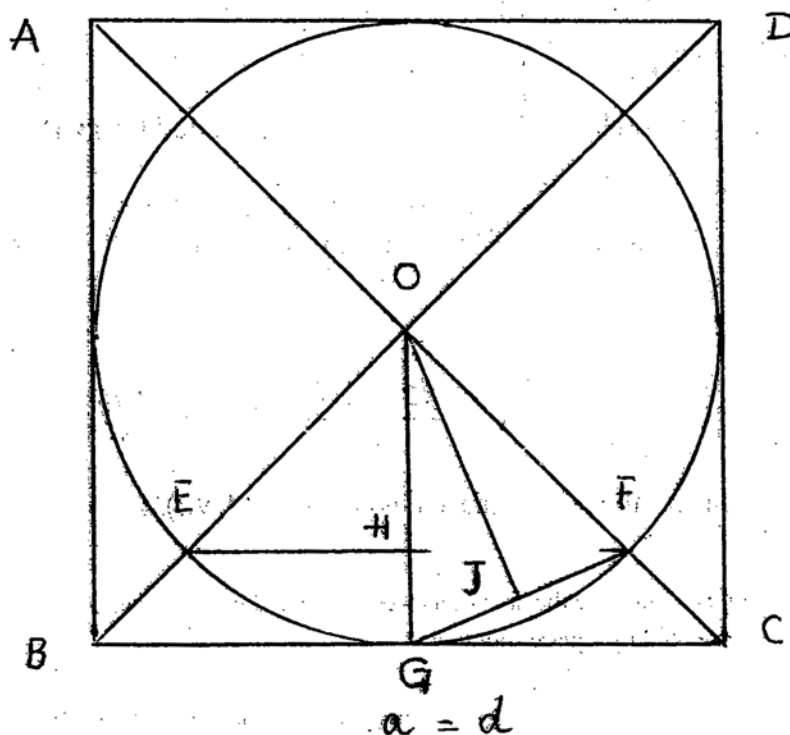
By the grace of God 22 methods have already been submitted to the Learned Societies of Mathematics all over the world, explaining that $\pi = (14-\sqrt{2})/4$. The present method is the latest in the series and submitted humbly for approval.

PROPOSED METHOD

A square ABCD of a given side with two diagonals AC & BD is drawn. The diagonals intersect at O. Taking O as the center a circle is inscribed with the square. The circle intersects the diagonals at E & F. G is the mid point of BC. GF is the side of the octagon inscribed with the circle. J is the mid point of GF. From GH, the length of GF can be obtained. The length of OJ can be calculated applying Pythagorean theorem.

A square and its inscribed circle are related by a constant called k.

$$k = \frac{\text{Square}}{\text{Circle}} = \frac{a^2}{\pi r^2} = \frac{4a}{2\pi r} = \frac{16}{14-\sqrt{2}}$$



where $a = 2r$ and $\pi = (14 - \sqrt{2})/4$. Further, the square and the inscribed circle are related by a surd $\sqrt{2}$.

The arc GF is $1/8^{\text{th}}$ of the circumference of the circle. From the values of 1. the side GF of the octagon and 2. OJ, the altitude of the triangle OGF, we can obtain the arcual length GF which is equal to $(\pi d)/8$.

1.	AB=BC=CD=DA= diameter	=	a
2.	EF	=	$\frac{\sqrt{2}a}{2}$
3.	GH	=	$\frac{\sqrt{2}a - 1a}{2\sqrt{2}}$
4.	GF	=	$\left(\sqrt{\frac{\sqrt{2} - 1}{2\sqrt{2}}} \right) a$
5.	GJ	=	$\frac{GF}{2}$
6.	OJ	=	$\sqrt{(OG)^2 - (GJ)^2}$

The arcual length GF can be calculated by the following equation

$$2 \left(\frac{\sqrt{2}}{ka} \right) \times OJ \times GF = \frac{\pi d}{8} = \frac{\pi a}{8}$$

When the values are substituted in the above equation we get a formula as shown below

$$2 \left\{ \frac{\sqrt{2}(14 - \sqrt{2})}{16 \times a} \right\} \times \sqrt{\left(\frac{a}{2} \right)^2 - \left\{ \frac{\left(\sqrt{\frac{\sqrt{2} - 1}{2\sqrt{2}}} \right) a}{2} \right\}^2} \times \left\{ \sqrt{\frac{\sqrt{2} - 1}{2\sqrt{2}}} \right\} a = \frac{\pi d}{8} = \frac{\pi a}{8}$$

Finally, π value can be derived from the above formula and is equal to $(14 - \sqrt{2})/4$

7. AN ACCURATE APPROXIMATION TO MATHEMATICAL π

INTRODUCTION:

Earlier, a few articles proposing different methods have been submitted to the mathematical world in support of π value 3.1464466 ... obtained from CBTS method. This paper is the latest in the series.

In yet another article of the author entitled **How a circle and square are related by square root two**, it has been explained that the circle and square are related by a surd $\sqrt{2}$.

METHOD:

A circle can give rise to a superscribing square by its 4 tangent lines on the circumference when they are equally apart from each other. When the diameter/side is equal to 1 unit, diagonal of a square is equal to a surd $\sqrt{2}$. The two diagonals intersect the circle at 4 points dividing the circumference of the circle into 4 equal arcs.

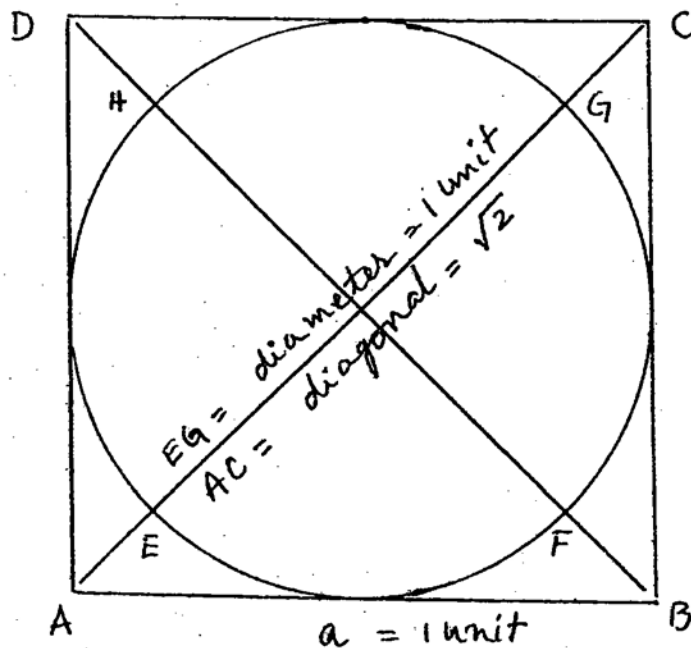


Fig -10

It has rightly been presumed that when a square can be constructed from a circle, it is equally possible, in converse, to get the unknown dimensions of the circle from the known values of square and its diagonals with the help of $\sqrt{2}$.

The square and circle composite system has 4 sides, 4 corner lengths and 4 arcs. Sides and corner lengths are straight lines.

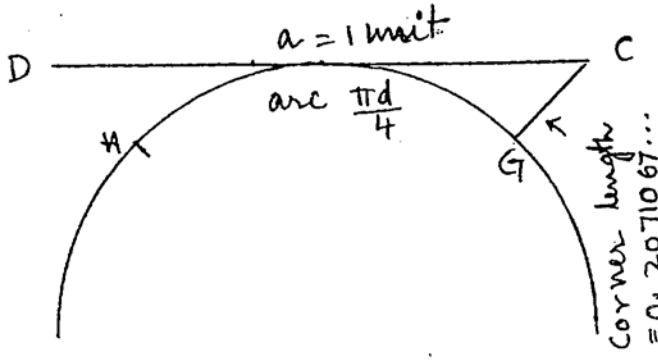


Fig -11

A corner length say GC is equal to $(\sqrt{2} - 1) / 2$ i.e.,

$$\frac{\text{diagonal} - \text{diameter}}{2} =$$

0.2070167 .. if the side/diameter is 1 unit.

The unknown length of the arc which is 1/4th of the circumference can be calculated by the following formula.

$$\frac{\text{Side} + \text{Corner length}}{4\sqrt{2}} = S_1 \quad - (1)$$

the equation (1) can be represented as

$$\frac{DC + CG}{4\sqrt{2}} = \frac{a + \left(\frac{a\sqrt{2} - 1a}{2} \right)}{4\sqrt{2}} = S_1 \quad - (2)$$

where S is a factor called **SIVA** **

$$\begin{aligned} \text{Arc} = \frac{\pi d}{4} &= \text{Side} - S_1 \\ &= a - S_1 \end{aligned} \quad - (3)$$

** The birth of 1st article took place on a Mahasivaratri festival day last year (24-02-1998) and ended with 16th article again, Mahasivaratri day this year, 14-02-1999. Siva (or shiva) in Sanskrit connotes auspicious, dear, benevolent. Numerous are His names. He is one of the major gods, perhaps a later development of the Vedic god Rudra, who forms a triad, called Trimurti, with Brahma, the Creator and Vishnu, the Preserver. He is worshipped in many aspects; as fierce destroyer, naked ascetic, Lord of the cosmic dance.

The circumference of a circle = $\pi d =$

$$4(a - S_1) = 4 \left\{ a - \left[\frac{a + \left(\frac{a\sqrt{2} - 1a}{2} \right)}{4\sqrt{2}} \right] \right\} \quad -(4)$$

$$= 3.1464466.....$$

when diameter (d) or side (a) is equal to 1 unit, π value is same as the circumference of the circle.

Or, straight away the circumference of a circle can be calculated using the following formula.

$$4a - S_2 \quad -(5)$$

where 4 a is the perimeter of the square

$$S_2 = \frac{DC + CG}{\sqrt{2}} = \frac{\text{Side} + \text{Corner length}}{\sqrt{2}} \quad (6)$$

$$4a - \left[\frac{a + \left(\frac{a\sqrt{2} - 1a}{2} \right)}{\sqrt{2}} \right] = \pi d = 3.1464466.... \quad -(7)$$

From the equations (4) and (7) it is clear that the π value is 3.1464466 ... which is same as CBTS π value.

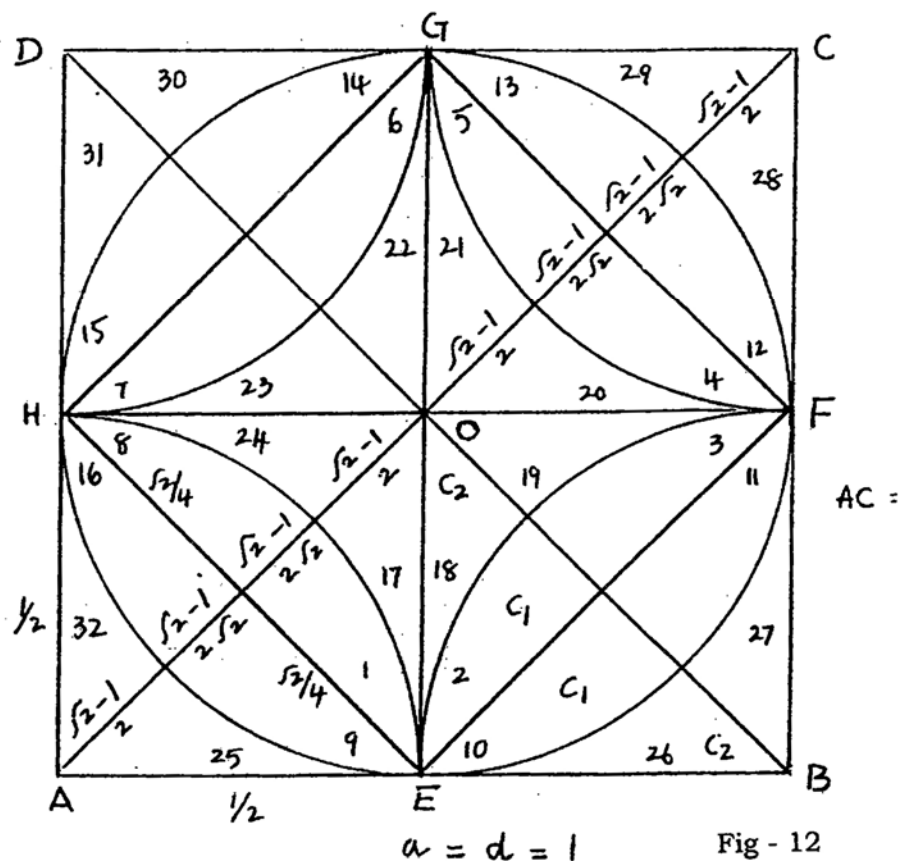
8. AREA OF A CIRCLE AND MATHEMATICAL π

The conventional formula to calculate the area of a circle is πr^2 where π is a constant representing a ratio of the circumference of a circle to its diameter, and r is the radius of the circle. In the proposed formula the area of a circle can be calculated **without** the help of π .

METHOD

Draw a square ABCD with a given side. E, F, G and H are the midpoints of four sides. Join AC and BD to get two diagonals. Similarly, join EG and FH which are equal to the side of the ABCD square. AC, BD, EG and FH intersect at O. Take O as the centre and half the side of the square ABCD as radius inscribe a circle with the square. The side of the square and the diameter of the inscribed circle are thus the same.

Join E, F, G and H to get another square EFGH which is smaller than ABCD square. Take A, B, C and D as the centres and with the same radius draw four arcs as shown in the figure.



At the end we find that the ABCD square is divided exactly into 32 parts and are numbered 1 to 32. 1 to 16 are of one dimension called C_1 entities. 17 to 32 are of another dimension called C_2 entities.

Two formulae for C_1 and C_2 are devised with the help of (1) their approximate quantum of area in the total area of the ABCD square; (2) one or two parameters of either C_1 or C_2 ; and (3) $\sqrt{2}$ relationship between the superscribed square and its inscribed circle.

The formulae are :

$$C_1 = \frac{a^2}{32} \left(1 + \frac{\sqrt{2}-1}{2\sqrt{2}} \right) \quad \text{or} = \frac{a^2}{32} \left[1 + \left(\frac{\sqrt{2}}{4} \times \frac{\sqrt{2}-1}{2\sqrt{2}} \times 2\sqrt{2} \right) \right]$$

$$\text{or} = \frac{a^2}{32} (\pi - 2)$$

$$C_2 = \frac{\frac{a^2}{32}}{4 \left(\frac{\sqrt{2}-1}{\sqrt{2}} \right)} \quad \text{or} = \frac{a^2}{32} \left[\frac{1}{2} + \left\{ \frac{\sqrt{2}-1}{2} \left(1 + \frac{\sqrt{2}}{2} \right) \right\} \right]$$

$$\text{or} = \frac{a^2}{32} (4 - \pi)$$

On close observation we can see ABCD square having triangles, smaller square, rectangles and finally an important entity, a circle.

The inscribed circle has $16C_1$ and $8C_2$ entities. Therefore the area of the circle is equal to $16C_1 + 8C_2 = \pi r^2$

From the above equation
we can get π which is equal to

$$\frac{16C_1 + 8C_2}{r^2} = \frac{16C_1 + 8C_2}{\left(\frac{a}{2}\right)^2}$$

$$\pi =$$

$$\frac{16 \left\{ \frac{a^2}{32} \left[1 + \left(\frac{\sqrt{2}}{4} \times \frac{\sqrt{2}-1}{2\sqrt{2}} \times 2\sqrt{2} \right) \right] \right\} + 8 \left\{ \frac{a^2}{32} \left[\frac{1}{2} + \left\{ \frac{\sqrt{2}-1}{2} \left(1 + \frac{\sqrt{2}}{2} \right) \right\} \right] \right\}}{\left(\frac{a}{2}\right)^2}$$

$$= \frac{14 - \sqrt{2}}{4} = 3.1464466.....$$

11. COMPUTATION OF π FROM DIVERSIFIED CONSTITUENTS OF A CIRCLE (CELLULAR METHOD)

With a geometrical archaic method of inscription - of - a polygon - with - the - circle, π value has been computed to 3.1415926.... Unfortunately, this value has been taken as final and concluded wrongly that π number is a transcendental number.

By the grace of God the author has proposed 33 methods, some with proofs and computed $\pi = (14-\sqrt{2})/4$. In the earlier two methods a square - circle nexus has been divided into 32 parts and 40 parts and derived π from the area of the inscribed circle. Now the square - circle nexus is divided into 20 parts of varied entities of earlier methods and re-affirmed that $\pi = (14-\sqrt{2})/4$.

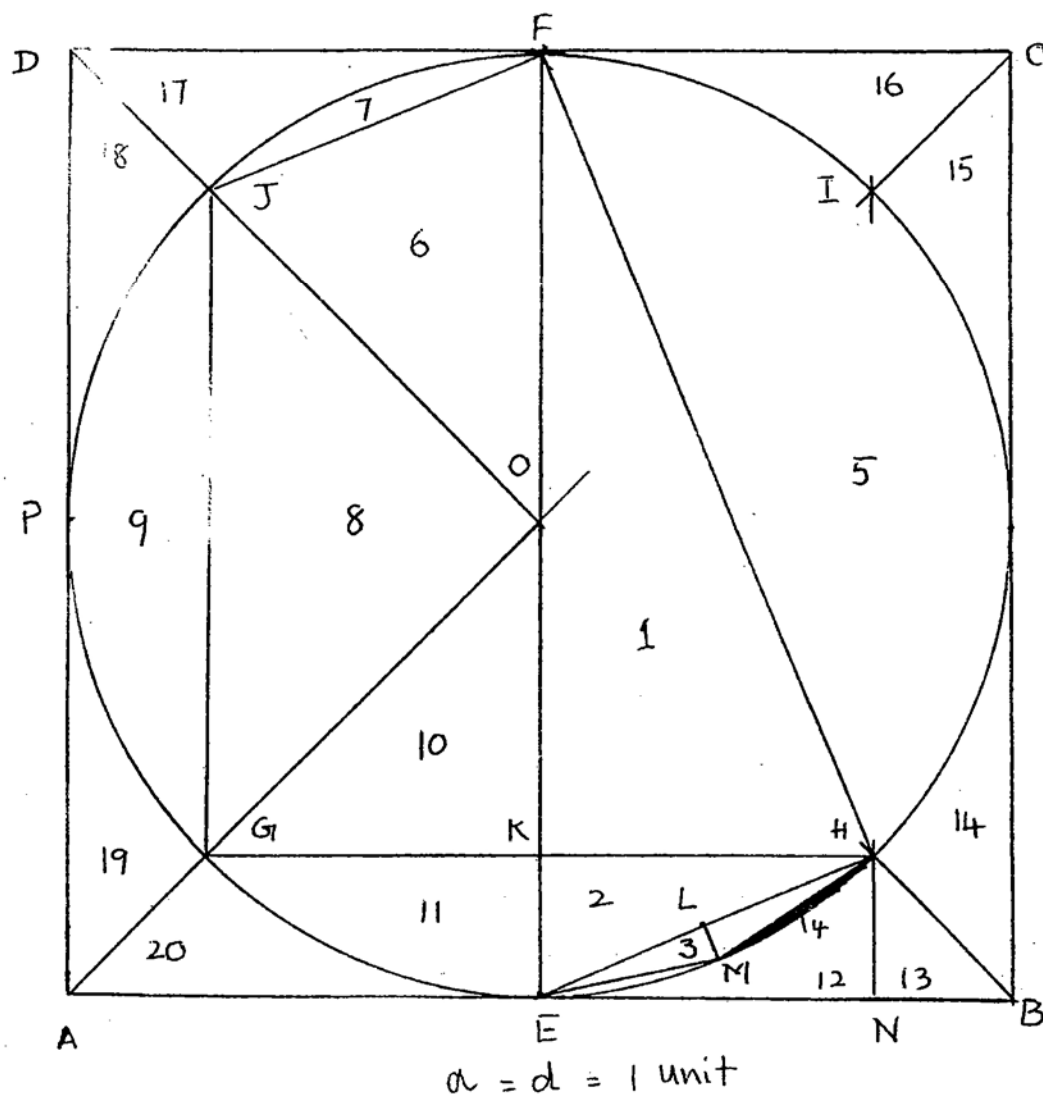


Fig -15

$$\left\{ \frac{1}{2} \times \frac{\sqrt{2}a}{4} \times \left(\frac{a}{2} + \frac{\sqrt{2}a}{4} \right) \right\} + \left\{ \frac{\frac{\sqrt{2}a}{4} \times \frac{2a - \sqrt{2}a}{4}}{2} \right\} \quad (1)$$

(2)

$$\left[a \sqrt{\frac{2 - \sqrt{2}}{4}} \times \frac{2}{2} + 2 \left\{ \frac{\frac{a}{2} - \sqrt{\left(\frac{a}{2}\right)^2 - \left(a \sqrt{\frac{2 - \sqrt{2}}{4}}\right)^2}}{2} \right\} \right] + \quad (3)$$

$$+ 2 \left\{ \frac{14 - (\sqrt{2} + 16 \sqrt{2 - \sqrt{2}})}{256} a^2 \right\} + \left\{ \left(\frac{42 - 11\sqrt{2}}{128} \right) a^2 \right\} + \left\{ a \sqrt{\frac{2 - \sqrt{2}}{4}} \times \frac{2}{2} \sqrt{\left(\frac{a}{2}\right)^2 - \left(a \sqrt{\frac{2 - \sqrt{2}}{4}}\right)^2} \right\} + \left\{ \left(\frac{14 - 9\sqrt{2}}{128} \right) a^2 \right\} + \quad (4)$$

(5)

(6)

(7)

$$+ \left\{ \frac{\frac{a}{2} \times \frac{a}{2}}{2} \right\} + 2 \left\{ \frac{\frac{a^2}{32} \left(1 + \frac{\sqrt{2} - 1}{2\sqrt{2}} \right) \right\} + \left\{ \frac{\frac{\sqrt{2}a}{4} \times \frac{\sqrt{2}a}{4}}{2} \right\} + \left\{ \frac{\frac{a^2}{32} \left(1 + \frac{\sqrt{2} - 1}{2\sqrt{2}} \right) \right\} + \left\{ \frac{\frac{a^2}{16}}{3 + 2\sqrt{2}} \left(\sqrt{2} + \frac{\sqrt{2} - 1}{4\sqrt{2}} \right) \right\} + \quad (8)$$

(9)

(10)

(11)

(12)

$$+ \left\{ \frac{1}{2} \left(\frac{\sqrt{2}a - 1a}{2\sqrt{2}} \right)^2 \right\} + 7 \left\{ \frac{\frac{a^2}{16}}{8 \left(\frac{\sqrt{2} - 1}{\sqrt{2}} \right)} \right\} = a^2 \quad (13)$$

(14)

Formulae of the Sum of the areas of from 1 to 11 is equal to the area of the inscribed circle and from 1 to 14 is equal to the area of the square.

CONSTITUENT ENTITIES OF THE SQUARE/CIRCLE
SIDE/DIAMETER = a / d

Sl.No:	ENTITY	AREA	
		a = 1 Unit	a = 9 Units
1.	Triangle FKH	0.1508883 Sq.units	12.221954 Sq.Units
2.	Triangle KHE	0.0258883	2.0969562
3.	Triangle LEM & LMH	0.0072824	0.5898830
4.	Curvature entity HM & EM	0.0026556	0.2151036
5.	Curvature entity HIF	0.2065910	16.733871
6.	Triangle OJF	0.0883883	7.1594555
7.	Curvature entity FJ	0.0099381	0.8049861
8.	Triangle JGO	0.1250000	10.125000
9.	Curvature entity GJP	0.0716528	5.8038854
10.	Triangle GKO	0.0625000	5.0624985
11.	Curvature entity GKE	0.0358264	2.9019427
Area of the circle (1 to 11)		0.7866112	63.715534
12.	Curvature entity EMHN	0.0159502	1.2919690
13.	Triangle NBH	0.0107232	0.8685871
14.	Curvature entities (14 to 20)	0.1867148	15.123904
Area of the square (1 to 14)		0.9999994	80.999994

$$\pi = \frac{\text{Area}}{r^2} = \frac{0.7866112}{0.25} = 3.1464448$$

$$\frac{63.715534}{20.25} = 3.1464461$$

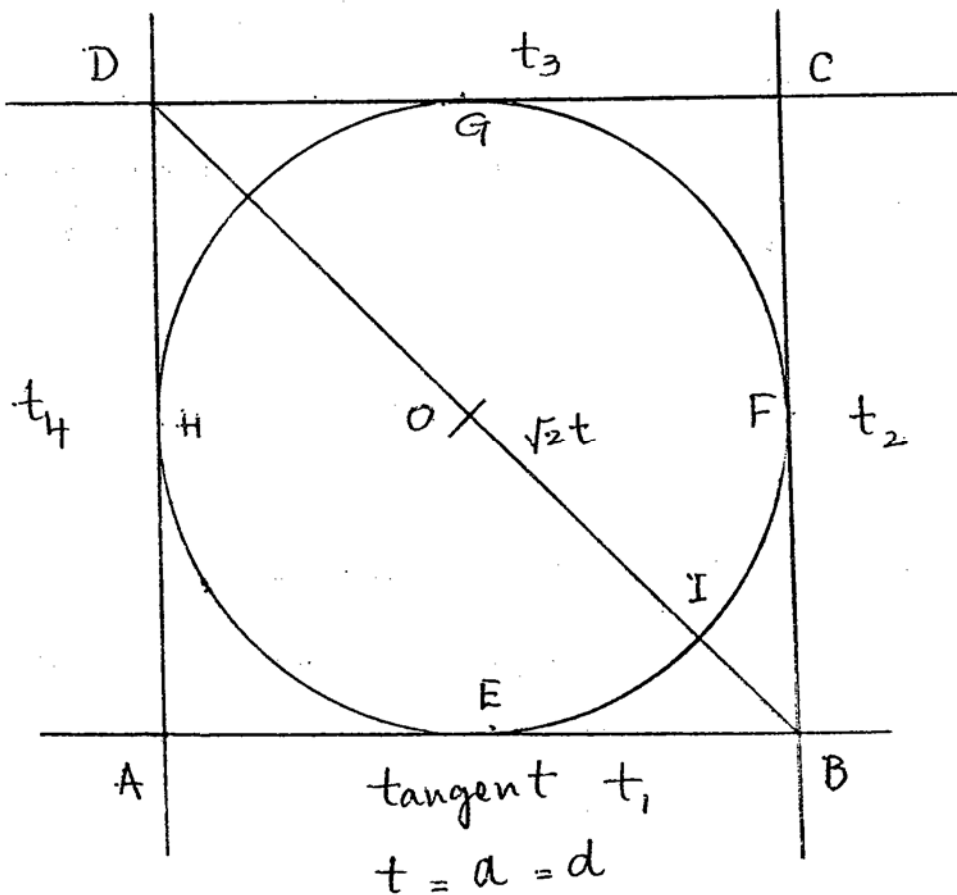
To be exact $\pi = (14-\sqrt{2})/4.$

COMPUTATION OF π THROUGH TANGENTS

So far by the grace of God 26 methods have been proposed to compute π value which is exactly equal to $(14-\sqrt{2})/4$. Two more formulae for the circumference of a circle are proposed now.

PROPOSED METHODS

A circle with unit radius is drawn. Four tangents at equidistant points of E, F, G & H on the circumference are drawn, which all the four tangents, t_1 , t_2 , t_3 & t_4 intersect among each other at A, B, C & D. (Figure 1)



Thus a square ABCD is created from the 4 tangents of equal lengths. The length of the each tangent or side of the square is equal to the diameter of the circle. The center of the circle is O, and a line BD passing through O is diagonal of the newly created square from the tangents. The midpoints of the tangents are E, F, G & H.

The circle intersects BD at I. OI ($=t/2$) is the radius, OB ($=\sqrt{2}t / 2$) is half of the diagonal and IB is equal to $((\sqrt{2}t - t)/2)$

Due to the presence of IB at four corners between the square and the circle, the circumference of the circle is lesser than the perimeter of the square or the sum of the four tangents.

As four tangents and IB at four corners, are straight lines and their creation is due to the circular entity i.e., circle yet another attempt is made here to arrive at the circumference of the circle from the tangents, diagonal and IB interrelating among each other.

Two formulae are devised for the circumference of the circle and are given below.

Formula No. 27

$$\left(\frac{t_1 + t_2 + t_3 + t_4}{= 4t} \right) - \left(\frac{2t + \sqrt{2}t}{4} \right) = \pi t = \pi a = \pi d$$

Formula No. 28

$$\left(\frac{t_1 + t_2 + t_3 + t_4}{= 4t} \right) - \left\{ \left(1 + \frac{\sqrt{2}t - t}{2t} \right) \left(\frac{\sqrt{2}t}{2} \right) \right\} = \pi t = \pi a = \pi d$$

Where t, the length of the tangent or a, side of the square or d, the diameter of the circle. Ultimately π value can be derived from the above two formulae and is exactly equal to $(14-\sqrt{2})/4$.

All the 28 formulae so far proposed clearly show that π value is $(14-\sqrt{2})/4$. i.e., 3.1464466..... but not 3.1415926...

Reference :

1. **Square - Circle nexus by Square root 2 and Mathematical Pi**
R. Sarva Jagannadha Reddy
1st Edition 1999, 71 pages
2. **Solutions to three famous unsolved problems**
R. Sarva Jagannadha Reddy
1st Edition 1999, 62 pages

THREE MORE NEW FORMULAE FOR THE AREA OF A CIRCLE & Pi

INTRODUCTION

Mathematicians have been believing that π is a transcendental number. With the earlier 28 methods the author has proved that π is **not** a transcendental number and its exact value is equal to $(14-\sqrt{2})/4$.

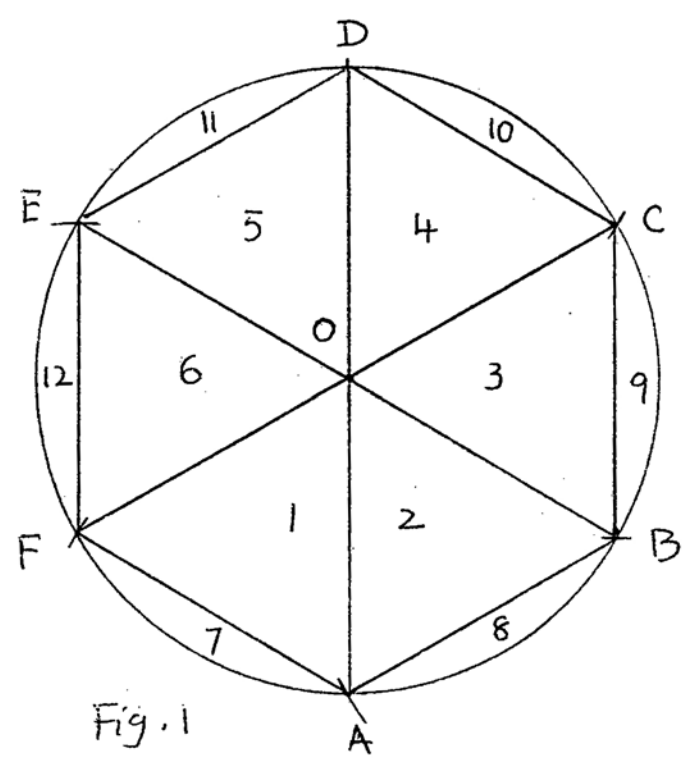
With this formula that it has become possible to calculate area of the curvature entities in a circle when they are associated with the triangles. In this article three more methods are proposed for the calculation of the area of a circle from the known formulae of the other curvature entities and conventional formula for the triangle.

PROPOSED METHOD – 1

With the center O and with a unit radius a circle is drawn. With the same radius the circumference is divided into six equal parts and at the points A,B,C,D,E and F. When these six points are joined we get a hexagon. Further all these points when joined to the center O we get six equilateral triangles numbered 1 to 6. The area between each triangle and the circumference is a curvature entity. As there are six triangles we get six curvature entities and are numbered 7 to 12.

Formula No. 29.

The following formula has been devised to get the area of a circle. π value can be derived from this formula and is equal to $(14-\sqrt{2})/4$. With the present approximation of π i.e., 3.1415926... it is impossible to devise a formula to calculate the area of any curvature entity.



$$6 \left\{ \frac{1}{2} \left(\frac{a}{2} \times \sqrt{\left(\frac{a}{2}\right)^2 - \left(\frac{a}{4}\right)^2} \right) + a^2 \left(\frac{14 - (\sqrt{2} + 6\sqrt{3})}{96} \right) \right\} = \frac{\pi d^2}{4} = \frac{\pi a^2}{4}$$

Area of the triangle (1 to 6)

area of the curvature entity

(7 to 12)

Finally π can be derived from the above equation and is exactly equal to $(14 - \sqrt{2})/4$.

PROPOSED MEHTOD – 2

With the center O and with a unit radius a circle is drawn. With the same radius the circumference is divided into six parts and at the points A,B,C,D,E & F. A & D are joined. Similarly B,D and F are joined to get an equilateral triangle(1). AB and AF are also joined to get two more smaller triangles (2 & 3), they are triangles AGB & AGF. The curvature entities AGB and AGF are divided into smaller triangles AGB & AGF, and AB & AF curvature entities(6 & 7). The area between BD and the circumference of the circle is a bigger curvature entity (4) and another area between DF and the circumference of the circle is the second bigger curvature entity(5). (Fig2)

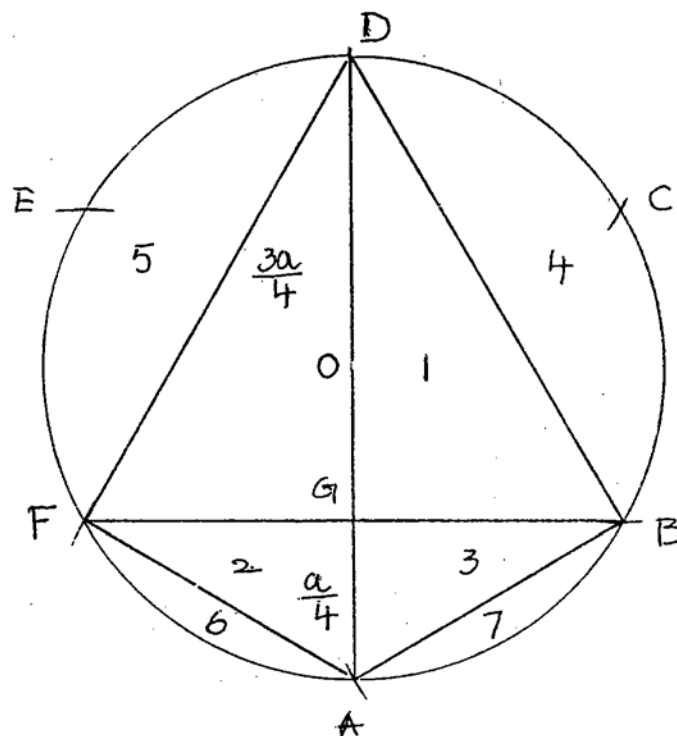


Fig. 2

FORMULA No. 30

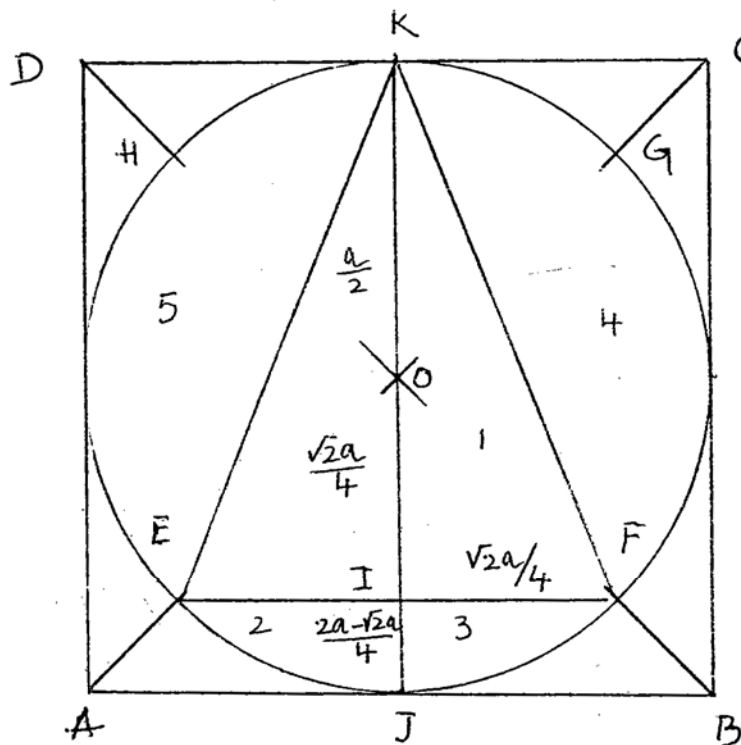
Area of the Circle = $\frac{\pi d^2}{4} = \frac{\pi a^2}{4}$

$= \frac{1}{2} \left(\sqrt{a^2 - \left(\frac{a}{2}\right)^2} \right) \left(\frac{3a}{4} \right) + 2 \left\{ \frac{1}{2} \left(\frac{a}{4} \times \left(\frac{\sqrt{a^2 - \left(\frac{a}{2}\right)^2}}{2} \right) + a^2 \left(\frac{14 - (\sqrt{2} + 3\sqrt{3})}{48} \right) + a^2 \left(\frac{14 - (\sqrt{2} + 6\sqrt{3})}{96} \right) \right\}$	Area of the triangle(1)	Area of the triangle(2 & 3)	Area of theCurvature entity (4 & 5)	Area of theCurvature entity (6 & 7)
--	-------------------------	-----------------------------	-------------------------------------	-------------------------------------

Finally π can be derived from the above equation and is exactly equal to $(14 - \sqrt{2})/4$.

PROPOSED METHOD – 3

A square ABCD with two diagonals AC & BD is drawn. Taking O as the centre with the half of the side of the square as radius a circle is inscribed with the square. The two diagonals intersect the circle at E, F, G & H. E, F & K are joined to get a triangle. The mid points of AB and CD are J & K respectively. J & K are joined. Finally, we have one bigger triangle EFK (1), two smaller curvature entities IJE & IJF (2 & 3) and two bigger curvature entities FGK & EHK (4 & 5). (Fig.3)



FORMULA No. 3!

Fig. 3

$$\text{Area of the circle} = \frac{\pi d^2}{4} = \frac{\pi a^2}{4}$$

$$= \left(\frac{1}{2} \times \frac{\sqrt{2}a}{2} \left(\frac{a}{2} + \frac{\sqrt{2}a}{4} \right) \right) + 2 \frac{a^2}{32} \left(1 + \frac{2-\sqrt{2}}{4} \right) + 2 \left(\frac{42 - 11\sqrt{2}}{128} \right) a^2$$

Area of the triangle(1)

Area of the Curvature
entity (2 & 3)

Area of the Curvature
entity (4 & 5)

Finally π can be derived from the above equation and is exactly equal to $(14-\sqrt{2})/4$.

we are not opposing the formula $\frac{\pi d^2}{4}$ for the area of the circle but we are questioning the approximation of π value 3.1415926...

NEO - CLASSICAL METHOD OF ARCHIMEDES TO COMPUTE π

48

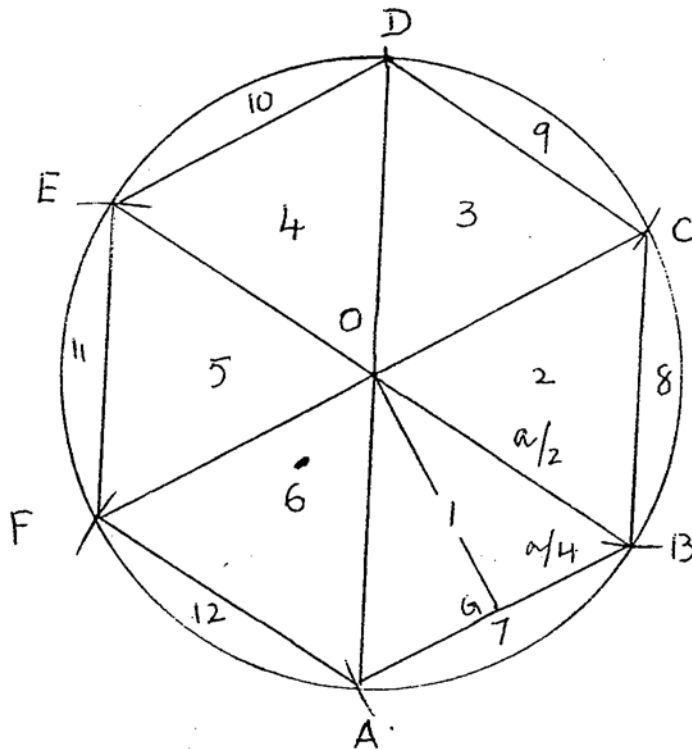
INTRODUCTION:

Euclid's theorem says that the area of a circle divided by the square of the radius gives the value of π . But he did not estimate the value of π . Archimedes, a brilliant mathematician and physicist who lived in the century following Euclid had developed a scientific method for the first time to compute π . He inscribed a polygon with a minimum number of sides, say 6. As we increase the number of sides the area of the inscribed regular polygon approaches the area of the circle as a limit. In fact the area of the circle may be defined as the limit of the area of the inscribed regular n-gon as n increases indefinitely. We can approximate π as closely as we please by increasing the number of sides of the polygon. Archimedes carried this kind of process far enough to ascertain $3 \frac{10}{71} < \pi < 3 \frac{1}{7}$ using a polygon of 96 sides. Archimedes' method is called the Method of Exhaustion or Classical Method – because the growing inscribed regular polygon gradually exhausts the area of the circle.

Slightly deviating from the Archimedes' method the author has simultaneously calculated the area of the polygon (1 to 6) using conventional formula and the area of the curvature entities (7 to 12) devising new formula. The sum of these two gives the area of the circle, from which π derived and is exactly equal to $(14-\sqrt{2})/4$.

The sides of the polygon is increased from 6 to 3072. At every stage the sum of the areas of both the polygon and the curvature entities remain same and is ≈ 0.7866116 sq.unit when $a = d = 1$ unit. Progressively the area of the polygon increases with the increasing number of sides and the area of the curvature entities decreases, Table (2). Here for 10 stages formulae have been devised. In the last stage the polygon has 3072 sides. It can be increased further and further. But the value for π remains at $(14-\sqrt{2})/4$ only

With the center O and with a unit radius a circle is drawn. With the same radius the circumference is divided into six equal parts and at the points A,B,C,D,E and F. When these six points are joined we get a hexagon. Further all these points when joined to the center O we get six equilateral triangles numbered 1 to 6. The area between each triangle and the circumference is a curvature entity. As there are six triangles we get six curvature entities and are numbered 7 to 12.



$$OB = \frac{a}{2} \text{ (Hypotenuse)}$$

$$AB = \frac{a}{2} \text{ (Base)}$$

$$OG = \sqrt{\left(\frac{a}{2}\right)^2 - \left(\frac{a}{4}\right)^2}$$

(Altitude)

$$\frac{1}{2} ab$$

$$OG = \sqrt{(OB)^2 - (GB)^2}$$

$$6 \left\{ \frac{1}{2} \left(\frac{a}{2} \times \sqrt{\left(\frac{a}{2}\right)^2 - \left(\frac{a}{4}\right)^2} \right) + a^2 \left(\frac{14 - (\sqrt{2} + 6\sqrt{3})}{96} \right) \right\} = \frac{\pi d^2}{4} = \frac{\pi a^2}{4}$$

Area of the triangle (1 to 6)

area of the curvature entity

(7 to 12)

Finally π can be derived from the above equation and is exactly equal to $(14 - \sqrt{2})/4$.

Table 1

S.No.	Number of sides of the Polygon	No. of Triangles	Formula for the area of each triangle	Area of the Polygon	No. of Curvature entities	Formula for the area of each curvature entity	Total area of all the curvature entities	Total area of the Circle
1	2	3	4	5	6	7	8	9
1	6	6	$\frac{1}{2} \left(\frac{a}{2} \times \sqrt{\left(\frac{a}{2} \right)^2 - \left(\frac{a}{4} \right)^2} \right)$	3 x 4	6	$\left\{ a^2 \left(\frac{14 - (\sqrt{2} + 6\sqrt{3})}{96} \right) \right\}$	6 x 7	5 + 8 $= \frac{\pi d^2}{4}$ $= \frac{\pi a^2}{4}$ “
2	12	12	$\frac{1}{2} \left(\frac{a}{4} \times \sqrt{\left(\frac{a}{2} \right)^2 - \left(\frac{a}{8} \right)^2} \right)$	“	12	$\left\{ a^2 \left(\frac{14 - (\sqrt{2} + 3\sqrt{15})}{192} \right) \right\}$	“	“
3	24	24	$\frac{1}{2} \left(\frac{a}{8} \times \sqrt{\left(\frac{a}{2} \right)^2 - \left(\frac{a}{16} \right)^2} \right)$	“	24	$\left\{ a^2 \left(\frac{28 - (2\sqrt{2} + 3\sqrt{63})}{768} \right) \right\}$	“	“
4	48	48	$\frac{1}{2} \left(\frac{a}{16} \times \sqrt{\left(\frac{a}{2} \right)^2 - \left(\frac{a}{32} \right)^2} \right)$	“	48	$\left\{ a^2 \left(\frac{56 - (4\sqrt{2} + 3\sqrt{255})}{3072} \right) \right\}$	“	“
5	96	96	$\frac{1}{2} \left(\frac{a}{32} \times \sqrt{\left(\frac{a}{2} \right)^2 - \left(\frac{a}{64} \right)^2} \right)$	“	96	$\left\{ a^2 \left(\frac{112 - (8\sqrt{2} + 3\sqrt{1023})}{12288} \right) \right\}$	“	“

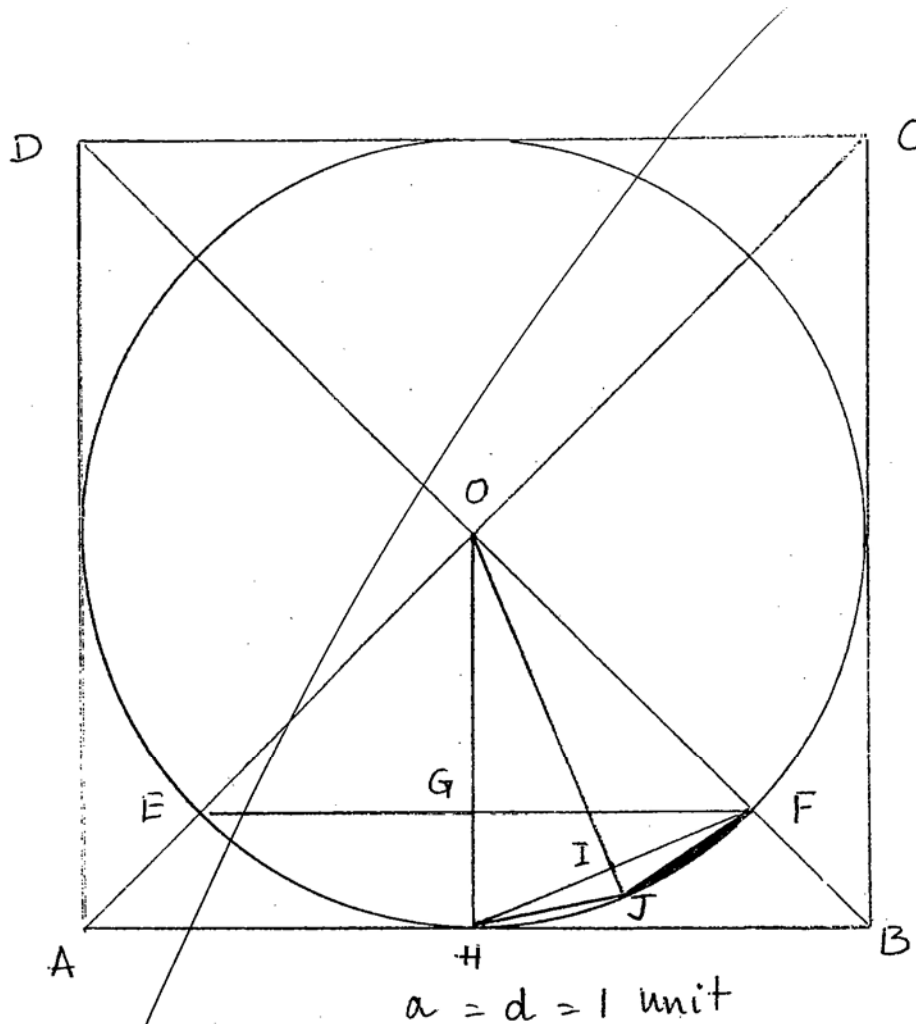
S.No.	Number of sides of the Polygon	No. of Triangles	Formula for the area of each triangle	Area of the Polygon	No. of Curvature entities	Formula for the area of each curvature entity	Total area of all the curvature entities	Total area of the Circle
1	2	3	4	5	6	7	8	9
6	192	192	$\frac{1}{2} \left(\frac{a}{64} \times \sqrt{\left(\frac{a}{2} \right)^2 - \left(\frac{a}{128} \right)^2} \right)$	3 x 4	192	$\left\{ a^2 \frac{224 - (16\sqrt{2} + 3\sqrt{4095})}{49152} \right\}$	6 x 7	5 + 8 = $\frac{\pi d^2}{4}$ = $\frac{\pi a^2}{4}$
7	384	384	$\frac{1}{2} \left(\frac{a}{128} \times \sqrt{\left(\frac{a}{2} \right)^2 - \left(\frac{a}{256} \right)^2} \right)$	"	384	$\left\{ a^2 \frac{448 - (32\sqrt{2} + 3\sqrt{16383})}{196608} \right\}$	"	"
8	768	768	$\frac{1}{2} \left(\frac{a}{256} \times \sqrt{\left(\frac{a}{2} \right)^2 - \left(\frac{a}{512} \right)^2} \right)$	"	768	$\left\{ a^2 \frac{896 - (64\sqrt{2} + 3\sqrt{65535})}{786432} \right\}$	"	"
9	1536	1536	$\frac{1}{2} \left(\frac{a}{512} \times \sqrt{\left(\frac{a}{2} \right)^2 - \left(\frac{a}{1024} \right)^2} \right)$	"	1536	$\left\{ a^2 \frac{3584 - (256\sqrt{2} + 6\sqrt{262143})}{6291456} \right\}$	"	"
10	3072	3072	$\frac{1}{2} \left(\frac{a}{1024} \times \sqrt{\left(\frac{a}{2} \right)^2 - \left(\frac{a}{2048} \right)^2} \right)$	"	3072	$\left\{ a^2 \frac{7168 - (512\sqrt{2} + 6\sqrt{1048575})}{25165824} \right\}$	"	"

TABLE 2

S. No.	No.of Sides	Area of the Polygon		Area of all the curvature entities		Area of the circle
1	6	0.6495186	+	0.1370922	=	0.7866108
2	12	0.7261836	+	0.0604272	=	0.7866108
3	24	0.7441176	+	0.0424920	=	0.7866096
4	48	0.7485312	+	0.0380784	=	0.7866096
5	96	0.7496256	+	0.0369792	=	0.7866048
6	192	0.7498944	+	0.0367104	=	0.7866048
7	384	0.7499520	+	0.0366336	=	0.7865856
8	768	0.7499520	+	0.0366336	=	0.7865856
9	1536	0.7498752	+	0.0367104	=	0.7865856
10	3072	0.7498752	+	0.0365568	=	0.7864320

PROPOSED METHOD

A square ABCD with unit side is drawn. Its diagonals are AC & BD. Taking O as the centre and radius equal to half of the side, a circle is inscribed with the square. The circle intersects the two diagonals at E & F. E & F are joined.



H is the mid point of AB. OH is the radius which bisects EF at G. H & F are joined and is the hypotenuse of the triangle GFH. I is the mid point of HF. OJ is the radius which bisects HF at I. A third triangle IJF is created. Finally we get a sector OJF. The bigger sector OHF consists of three triangles of different dimensions and two curvature entities. The triangles are 1. OGF 2. GHF and 3. IJF & IJH. The areas of these triangles can be arrived at with the known formula $1/2 ab$. The formula for the area of the curvature entity is devised and the sum of these areas is equal to $1/8$ th area of the circle. π value can be derived and is equal to $(14-\sqrt{2})/4$

14. TRIANGLE – FROM – CIRCLE METHOD TO COMPUTE π^{54}

INTRODUCTION

From the earlier 38 methods it had been established that π is equal to $[14-\sqrt{2}]/4$. The present method too is in complete agreement with the earlier methods.

Archimedes' Classical method in the computation of π involves inscription – of-a-polygon-with-the-circle; James Gregory's arctan series is infact the **extension** of Classical method beyond 39th decimal place to infinity for π . David Bailey, Peter Borwein and Simon Plouffe have recently computed the tenth billionth digit in the hexadecimal expansion of π . They utilized an astonishing formula

$$\pi = \sum_{n=0}^{\infty} \left(\frac{4}{8n+1} - \frac{2}{8n+4} - \frac{1}{8n+5} - \frac{1}{8n+6} \right) \left(\frac{1}{16} \right)^n$$

Dr. Kanada, University of Tokyo calculated 206,158,430,000 decimal digits of π [September 1999]. The later approximations to π , powers of π , summation of π series have nothing to do with the parameters of a circle. Still the infinite series is considered to be superior method. No doubt, the π value equal to 3.1415926 satisfied mathematicians since 240BC [i.e. for the last 2240 years] has now been under question from the proposed methods. **The reason is, that the conventional methods to compute π have no proofs**, like Pythagorean theorem. However, it may not be irrelevant to quote here Brouwer, D.Hilbert, F. Bacon, Pierre Duhem etc.

“.....At that time L.E.J. Brouwer [1881-1966] began to develop an “intuitionistic” philosophy of mathematics, admitting only constructive proof – methods. Meanwhile, D.Hilbert [1862-1943] advocated “formalism”, where mathematical systems are studied for properties such as consistency and completeness”

“F. Bacon [1561-1626] thought one could enumerate all possible paths [theories] available. At first, all would seem reasonable, but an appropriate crucial instance would show which one fitted with the facts.”

“Pierre Duhem [1861-1916] has said that “ - - - any hypothesis is embedded within a complex of assumptions and auxiliary hypotheses. Duhem contended, one can never disprove any hypothesis and can not, there fore, prove that another is true”.

According to I.Lakatos [1922-74] while no crucial experiment exists to offer “instant rationality”, i.e. definitive results, nonetheless so called minor crucial experiments can determine which of specific competing hypotheses within a more general research programme accord better with the data. Major crucial experiments can test the predictions of two general research programmes; the one with better results is thus rationally demonstrated to be preferable”.

METHOD

A circle with a given radius is drawn. Three equidistant tangents at the points D,E,&F when drawn on the circumference, the tangents intersect among each other at the points A,B,C creating an equilateral triangle ABC. Each side of the triangle is equal to $\sqrt{3} a$ where a , is the diameter of the circle.

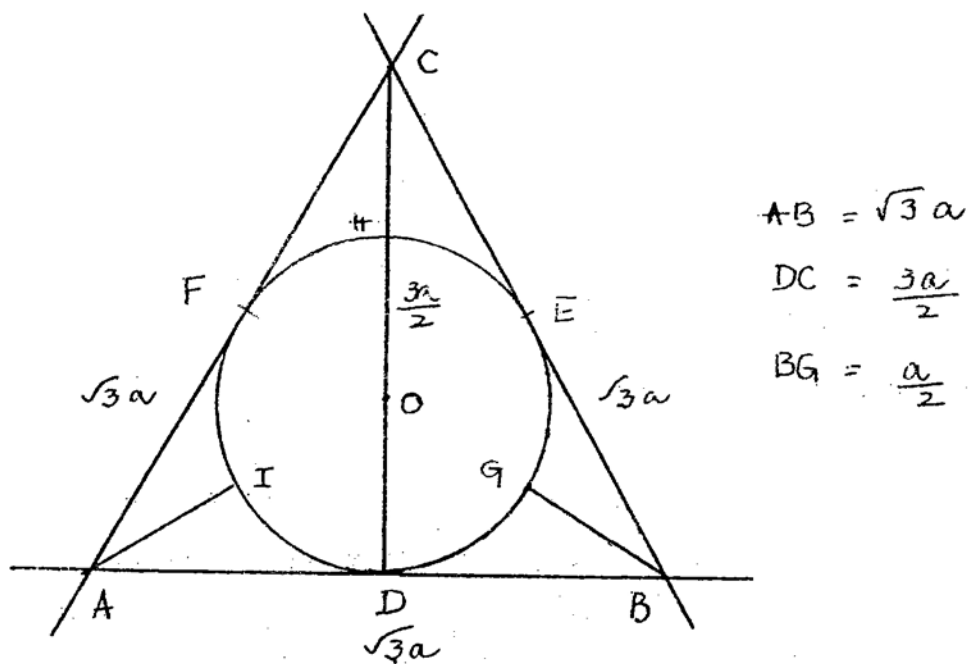


Fig - 18

With the guidance of proposed π value equal to $[14-\sqrt{2}]/4$, the 3 areas outside the circle but within the triangle have been calculated using the following formula.

FORMULA :

Area of the triangle — area of the 3 corner areas = area of the inscribed circle

$$\left[\frac{1}{2} \times \sqrt{3}a \times \frac{3a}{2} \right] - 3 \left[\left(\frac{\sqrt{3}a}{2} \times \frac{a}{2} \right) - \left(\frac{7a}{12} \times \frac{a}{2} \right) + \left(\frac{\sqrt{2}a}{24} \times \frac{a}{2} \right) \right] = \frac{\pi a^2}{4}$$

With the slight modifications of the above formula the circumference of the inscribed circle can also be calculated.

FORMULA :

Perimeter of the triangle – length of a segment of the triangle = circumference of the circle

$$3\sqrt{3}a - 3 \left(\sqrt{3}a - \frac{7a}{6} + \frac{\sqrt{2}a}{12} \right) = \pi d$$

From the above two formulae π value can be derived and is exactly equal to $[14-\sqrt{2}]/4$.

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16. IS π REALLY A TRANSCENDENTAL NUMBER ?

C.Hermite showed in 1873 that e is a transcendental number. Following a similar line of thought as that taken by Hermite and basing on **Euler's formula** $e^{ix} + 1 = 0$, **C.L.F. Lindemann** showed that π is also transcendental (1882).

However, if we see critically, we are compelled to question the validity of transcendency of π .

- 1a. *"The theory of transcendental numbers is, however, far from complete. There is no general criterion that can be utilized to characterize transcendental numbers" (Encyclopaedic Dictionary of Mathematics page No.1310, Japan Mathematical Society).*

- b. *"One of the men who hit on this latter relationship ($e^{ix} = -1$) between the four most important numbers in all mathematics said that he did not understand it and barely believed it- but that he knew that it must be true since it was backed by unchallengeable logic.*

For what does this statement mean when it is - very crudely - interpreted in words? We start with a number which can never be accurately written out. We take another number of the same type - first met with as something to do with circles - and subject it to a process which can at best be visualized as a kind of twisting in space. We then take as many of the first number as are represented by the second one (after twisting) and multiply them up. And as a result of all this, we are told, we reach minus one -1 an expression which was itself once regarded as all but meaningless" (Men and Discoveries in Mathematics, Bryan Morgan (1972) page No. 140.

- c. *"There is a famous formula - perhaps the most compact and famous of all formulas - developed by Euler from a discovery of de Moivre : $e^{ix} + 1 = 0$ It appeals equally to the mystic, the scientist, the philosopher the mathematician. For each it has its own meaning. Though known for over a century, de Moivre's formula came to Benjamin Peirce (1809 -1880) as some thing of a revelation. Having discovered it one day, he turned to his students... "Gentlemen" he said, "that is surely true, it is absolutely paradoxical ; we cannot understand it, and we don't know what it means, but we have proved it, and therefore we know it must be the truth" (E. Kasner and J. Newman).*

2. The Euler's formula, $e^{ix} + 1 = 0$ holds good only when $\pi = 180^\circ$

$$\pi = \pi \text{ radians} = 180^\circ$$

$$\pi \text{ constant} \neq \pi \text{ radians}$$

3. We know that π besides having 180° as its value, it also stands for a value equal to 3.1415926... Does the above formula hold good for 3.1415926..... also ?
4. Are we justified in calling π constant 3.1415926... as transcendental number without incorporating it in the Euler's formula ? **π constant is divine.** π radians, a human creation.
5. The proposed π value $(14-\sqrt{2})/4$ can be expressed as an algebraic equation with integral coefficients i.e. $16x^2 - 112x + 194 = 0$ from which we can say that π is **not** a transcendental number.

17. GEOMETRICAL CONSTRUCTION TO THE ARCHIMEDES' 1st PROPOSITION

Archimedes, in his treatise **Measurement of a Circle** has put forward 3 Propositions. The first one is that the problem of squaring the circle and of rectifying its circumference are equivalent. The second one is that the area of a circle is to the square on its diameter as 11 to 14. And the third one is that circumference of a circle is less than $3 \frac{1}{7}$ times and greater than $3 \frac{10}{71}$ times the diameter.

In his Proposition 1 he has explained further that the area of any circle is equal to a right angled triangle in which one of the sides about the right angle is equal to the **radius**, and the other to the **circumference** of the circle.

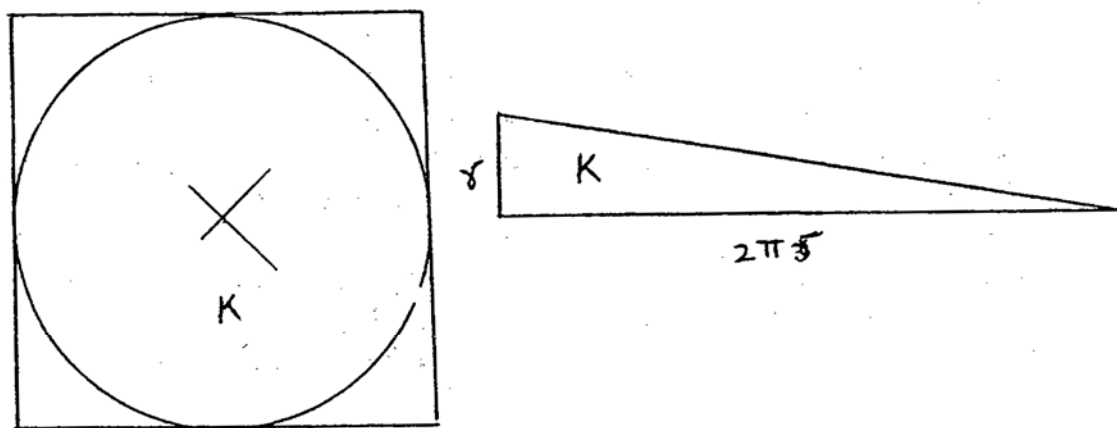


Fig - 20

The author in his earlier paper has calculated the circumference of an inscribed circle from a CBTS (a circle in between two squares) method in which a certain segment of the perimeter of the superscribing square is marked off as exactly equal to the circumference of the circle.

A square ACDF is drawn and a circle is inscribed with it. AD and CF are the two diagonals and they intersect the circle at G, H, I, J. The length between D to E via C, B, A, F is equal to the circumference of the inscribed circle. $AC = CD = DF = AF = 1$ unit. EF, then is equal to $0.1464466\dots$ unit which can be obtained from the formula $\frac{\sqrt{2d} - d}{2\sqrt{2}}$ where d, the diameter or, a the side is equal to 1 unit.

$$BE = BG + GJ + JE = \frac{\sqrt{2d} - d}{2\sqrt{2}} + \frac{\sqrt{2d}}{2} + \frac{\sqrt{2d} - d}{2\sqrt{2}} = d = a = 1 \text{ unit.}$$

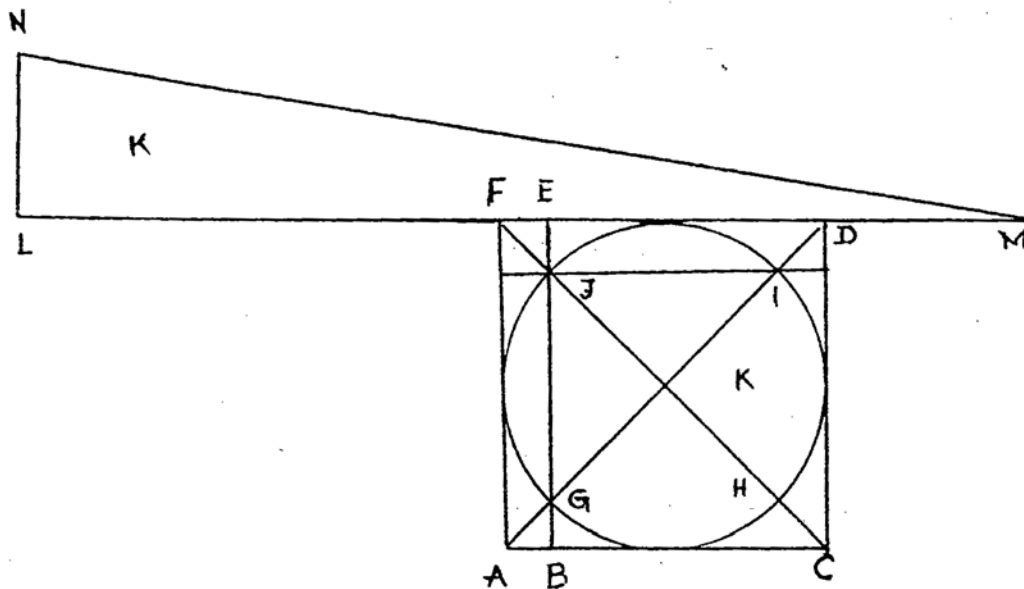


Fig - 21

A straight line equivalent to the circumference of a circle can be obtained in the following way (the difficulty is to get a segment equivalent to, over and above 3 i.e. 0.1464466.. when the diameter is 1 unit).

A line from EF (0.1464466..) is extended both the ways to get FL and EM equal to $1 \frac{1}{2}$ times of d each; so that, the sum of the lengths LF, FE and EM is equal to the circumference of the circle ($1.5 + 0.1464466... + 1.5 = 3.1464466...$).

A right angled triangle LMN is constructed with its sides LN - the radius, r (0.5 unit) and LM = the circumference $2\pi r$ ($3.1464466...$).

$$\begin{array}{lcl} \text{Area of the circle} & = & \text{area of the rightangled triangle} \\ \text{(k of Archimedes)} & & \text{(k)} \end{array}$$

$$\pi r^2 = \pi(d^2/4) = \frac{r \times 2\pi r}{2}$$

or

$$\frac{14 - \sqrt{2}}{4} \times \frac{d \times d}{4} = \frac{(3d + \frac{\sqrt{2}d - d}{2\sqrt{2}}) \times \frac{d}{2}}{2}$$

$$\text{Where } \pi = \frac{14 - \sqrt{2}}{4}$$

18. HIPPOCRATES' METHOD OF SQUARRING LUNES AND COMPUTATION OF π

"Archimedes's procedure for finding approximate numerical values of π (without, of course, referring to π as a number), by establishing narrower and narrower limits between which the value must lie, turned out to be the only practicable way of squaring the circle. But the Greeks also tried to square the circle exactly, that is they tried to find a method, employing only straight edge and compasses, by which one might construct a square equivalent to the given circle. All such attempts failed, though Hippocrates of Chios did succeed in squaring lunes.

Hippocrates begins by noting that the areas of similar segments of circles are proportional to the squares of the chords which subtend them

1. Consider a semi-circle ACB with diameter AB (Fig 7.7). Let us inscribe in this semi-circle an isosceles triangle ACB, and then draw the circular arc AMB which touches the lines CA and CB at A and B respectively. The segments ANC, CPB and AMB are similar. Their areas are therefore proportional to the squares of AC, CB and AB respectively, and from Pythagoras's theorem the greater segment is equivalent to the sum of the other two. Therefore the lune ACBMA is equivalent to the triangle ACB. It can therefore be squared.

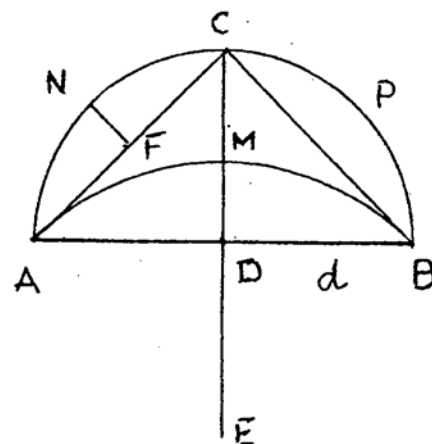


Fig - 22

The Circular arc AMB which touches the lines CA and CB at A and B respectively can be drawn by taking E as the centre and radius equal to EA or EB.

$$\begin{aligned} AB &= \text{diameter, } d. \\ DE &= DC = \text{radius, } d/2 \\ F &= \text{mid point of AC} \\ N &= \text{mid point of arc AC} \\ NF &= \frac{\sqrt{2}d-d}{2\sqrt{2}} & DM &= \frac{\sqrt{2}d-d}{2} \\ MC &= \frac{\sqrt{2}d-d}{\sqrt{2}} \end{aligned}$$

With the guidance of the formulae of earlier methods of the author where a Circle is inscribed with the Square, the formulae for the areas of ANC, CPB, ACM and BCM are devised.

$$1. \quad \text{Area of ANC} = \text{area of CPB} = 2 \left\{ \frac{d^2}{32} \left(1 + \frac{\sqrt{2}-1}{2\sqrt{2}} \right) \right\}$$

$$2. \quad \text{Area of AMB} = \text{areas of ANC} + \text{CPB (Hippocrates)}$$

$$3. \quad \text{Area of ACM} = \text{area of BCM} = \frac{\frac{(\sqrt{2}d)^2}{16}}{8 \left(\frac{\sqrt{2}-1}{\sqrt{2}} \right)}$$

4. Area of ACB triangle = $\frac{1}{2} \times \frac{d}{2} \times d$

5. According to Hippocrates the area of the lune ACBMA is **equivalent** to the area of the triangle ACB

$$\begin{array}{ccc} \text{Lune ACBMA} & = & \text{triangle ACB} \\ (\text{ANC} + \text{ACM} + \text{BCM} + \text{CPB}) & & \end{array}$$

$$\text{i.e. } 4 \left\{ \frac{d^2}{32} \left(1 + \frac{\sqrt{2}-1}{2\sqrt{2}} \right) \right\} + 2 \left\{ \frac{\frac{(\sqrt{2}d)^2}{16}}{8 \left(\frac{\sqrt{2}-1}{\sqrt{2}} \right)} \right\} = \frac{1}{2} \times \frac{d}{2} \times d$$

ANC + CPB ACM + BCM ACB

From the above equation it is clear that the devised formulae for the areas of different segments is **exactly** correct.

6. Area of AMB = areas of ANC + CPB

7. Area of the semicircle = $\frac{\pi d^2}{8}$ = Areas of ANC + CPB + ACM + BCM + AMB

8. $\pi = \frac{8 \times \text{Area of the semicircle}}{d^2}$

$$= \frac{8}{d^2} \left[4 \left\{ \frac{d^2}{32} \left(1 + \frac{\sqrt{2}-1}{2\sqrt{2}} \right) \right\} + 2 \left\{ \frac{\frac{(\sqrt{2}d)^2}{16}}{8 \left(\frac{\sqrt{2}-1}{\sqrt{2}} \right)} \right\} + 4 \left\{ \frac{d^2}{32} \left(1 + \frac{\sqrt{2}-1}{2\sqrt{2}} \right) \right\} \right]$$

$$= \frac{14 - \sqrt{2}}{4}$$

Reference:

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19. SUBMISSION OF VALUES FOR SQUARING OF A HIPPOCRATEAN CRESCENT

INTRODUCTION

By the grace of God the author had already submitted values for squaring one Hippocratean crescent. The present method too supports the proposed π value of the author i.e. $(14 - \sqrt{2})/4$. Hippocrates had succeeded in squaring lunes. Though the constant π was not in vogue in the Hippocratean period (460 BC) **he foresaw the algebraic nature of π number.** ★

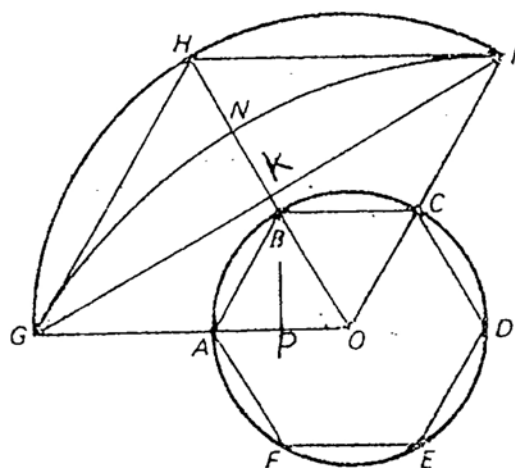


Fig - 23

“ Consider two concentric circles with common centre O and radii such that the square of the radius of the larger circle is six times the square of the radius of the smaller one (Fig 7.11). Let us inscribe in the smaller circle the regular hexagon ABCDEF. Let OA cut the larger circle in G, the line OB in H and the line OC in I. On the line GI we construct a circular segment GNI similar to the segment GH. Hippocrates shows that the lune GHIN plus the smaller circle is equivalent to the triangle GHI plus the hexagon.”

MATHEMATICS AND MATHEMATICIANS
P. Dedron Jitard

$$\begin{aligned}
 1. \quad OA &= \text{radius of the smaller circle} = a/2, & (\text{diameter} = a) \\
 OH &= \text{radius of the larger circle} = \sqrt{6 \left(\frac{a}{2}\right)^2} = GH = HI.
 \end{aligned}$$

$$OK = OH/2 = HK, \quad GK = \sqrt{(GH)^2 - (HK)^2}, \quad GI = 2GK$$

$$\text{Area of the GHI triangle} = 1/2 \times GI \times HK$$

★The author feels that had the later mathematicians seriously thought about the truth in the Hippocratean Propositions, the π value of Archimedean range 3.1408.... to 3.1428.. would have been corrected in the **3rd decimal place** long back. Next to Archimedes (240 BC) Claudius Ptolemy (150 AD) was the first mathematician who gave π value as 3.1416. In the 17th century curious expressions in the form of infinite series had been proposed. The infinite series is of two categories : arctan series and non-arctan series. As the arctan series is the outcome of right angled triangles inscribed with the circle, the π value derived out of this did support in toto the value of π obtained with the polygon inscribed with the circle of Archimedes. Whereas the non-arctan series is totally unrelated with the parameters of the circle. The number 3.1415926.... (supposed to be equal to π) was the only guiding factor for the expression, ofcourse, for the purpose of arrival of infinite decimal places.

$$2. \quad OA = a/2 = AB, \quad AP = a/4, \quad PB = \sqrt{(AB)^2 - (AP)^2}$$

$$\text{Area of the triangle } AOB = 1/2 \times AO \times PB$$

$$\text{Area of the hexagon} = \text{area the triangle } AOB \times 6.$$

$$3. \quad \text{Area of the smaller Circle} = \frac{\pi d^2}{4} = \frac{\pi a^2}{4} = \left(\frac{14 - \sqrt{2}}{16} \right) a^2$$

$$4. \quad \text{Area of the lune GHIN} = \text{Segments of GH} + HI + \text{GHN} + \text{INH}$$

According to Hippocrates

$$\text{Area of the lune GHIN} + \text{Area of the smaller circle} = \text{area of the triangle GHI} + \text{area of the hexagon ABCDEF}$$

$$= \text{segments of GH} + HI + \text{GHN} + \text{INH} + \frac{\pi a^2}{4} = (1/2 \times GI \times HK) + 6 (1/2 \times AO \times PB)$$

92

$$2 \left\{ 6a^2 \left(\frac{14 - (\sqrt{2} + 6\sqrt{3})}{96} \right) \right\} + 2 \left\{ 6a^2 \left(\frac{8\sqrt{3} + \sqrt{2} - 14}{64} \right) \right\} + a^2 \left(\frac{14 - \sqrt{2}}{16} \right)$$

$$GH + HI$$

$$+$$

$$GHN + INH$$

$$= \left\{ \frac{1}{2} \times 2 \left(\sqrt{6 \left(\frac{a}{2} \right)^2} - 6 \left(\frac{a}{4} \right)^2 \right) \times \sqrt{6 \left(\frac{a}{4} \right)^2} \right\} + 6 \left\{ \frac{1}{2} \times \frac{a}{2} \times \sqrt{\left(\frac{a}{2} \right)^2 - \left(\frac{a}{4} \right)^2} \right\} \quad 63$$

triangle GHI

+ hexagon ABCDEF

20. HIPPOCRATEAN CRESCENTS AND THEIR VALUES FROM THE PROPOSED π VALUE

Values for two Hippocratean lunes or crescents have been given with the proposed π value equal to $(14-\sqrt{2})/4$. Now for two more crescents are submitted again using the same approximation to π number.

HIPPOCRATEAN CRESCENT No.1

"Let ABC be an isosceles right angled triangle inscribed in the semicircle ABOC, whose centre is O. On AB and AC as diameters described semicircles as in the figure. Then, since by Euc. I, 47,

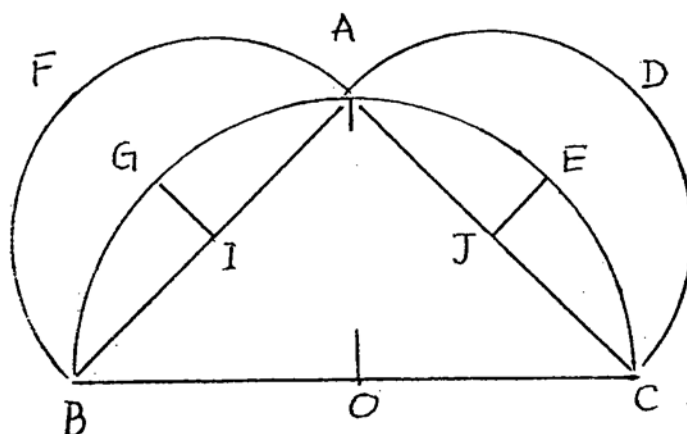


Fig - 24

Sq. on BC = sq. on AC + sq. on AB.
therefore, by Euc. XII, 2,

area semicircle on BC = area semicircle on AC + area semicircle on AB.

Take away the common parts

$\therefore$ area triangle ABC = sum of areas of lunes AECD and AFBG.

Hence the area of the lune AECD is equal to half that of the triangle ABC".

1. BC = diameter = $d = a$

$$OB = OC = \text{radius} = a/2, \quad BI = IA = AJ = JC = \frac{\sqrt{2}a}{4}$$

$$AB = AC = \frac{\sqrt{2}a}{2} \quad (\text{diameter of the semicircle } ABF = ACD)$$

$$GI = EJ = \frac{\sqrt{2}a - a}{2\sqrt{2}}$$

$$\text{Sq. on BC} = \text{sq. on AC} + \text{sq. on AB}$$

$$\text{Area of the larger semicircle} = \frac{\pi d^2}{8} = \frac{\pi a^2}{8}$$

2. Area of the smaller semicircle (ABF or ACD)

$$\text{Diameter}(a) = \frac{\sqrt{2}a}{2}$$

$$\text{Area} = \frac{\pi a^2}{8} = \frac{\pi \left(\frac{\sqrt{2}a}{2}\right)^2}{8}$$

$$\text{Areas of two semicircles} = 2 \left\{ \frac{\pi \left(\frac{\sqrt{2}a}{2}\right)^2}{8} \right\} = 2 \left\{ \frac{\left(\frac{14 - \sqrt{2}}{4}\right) \left(\frac{\sqrt{2}a}{2}\right)^2}{8} \right\}$$

3. Area of the triangle ABC = $\frac{1}{2}$ x base x altitude.

$$\text{Base} = BC = a$$

$$\text{Altitude} = OA = a/2$$

$$\text{Area} = \frac{1}{2} \times a \times a/2 = a^2/4.$$

4. Segment AGB = segment AEC

$$\text{Area of AGB or AEC segment} = 2 \left\{ \frac{a^2}{32} \left(1 + \frac{\sqrt{2} - 1}{2\sqrt{2}} \right) \right\}$$

5. Lune AGBF = lune AECD

$$\text{Area of the lune (AECD or AFBG)} = \text{Semicircle (ABF or ACD)} - \text{Segment (AGB or AEC)}$$

6. Area of triangle ABC = sum of the areas of lunes AECD and AFBG.

$$\frac{a^2}{4} = \left[2 \left\{ \frac{\left(\frac{14 - \sqrt{2}}{4}\right) \left(\frac{\sqrt{2}a}{2}\right)^2}{8} \right\} - 4 \left\{ \frac{a^2}{32} \left(1 + \frac{\sqrt{2} - 1}{2\sqrt{2}} \right) \right\} \right]$$

HIPPOCRATEAN CRESENT No.2

"Hippocrates next inscribed half a regular hexagon ABCD in a semicircle whose centre was O, and on OA, AB, BC, and CD as diameters described semicircles of which those on OA and AB are drawn in the figure. The AD is double any of the lines OA, AB, BC and CD,

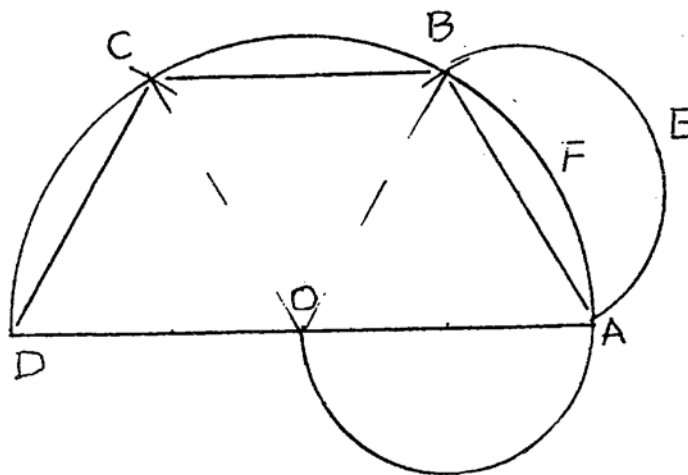


Fig - 25

∴ Sq. on AD = sum of sqs. on OA, AB, BC and CD,

∴ area semicircle ABCD = sum of areas of semicircles on OA, AB, BC and CD.

Take away the common parts.

∴ area trapezium ABCD = 3 lune AEFB + semicircle on OA".

ABCD = $\frac{1}{2}$ hexagon.

OA = AB = BC = CD = $a/2$ (diameters for OA, AB, BC, and CD)

1. Area of semicircle on AD = $\frac{\pi a^2}{8}$
2. Area of semicircles on OA, AB, BC and CD = $\frac{\pi \left(\frac{a}{2}\right)^2}{8} = \frac{\left(\frac{14 - \sqrt{2}}{4}\right) \left(\frac{a}{2}\right)^2}{8}$
3. Area of semicircle on AD = sum of sqs. on OA, AB, BC and CD.

∴ area semicircle ABCD = sum of areas of semicircles on OA, AB, BC, and CD.

Take away the common parts.

4. ∴ area trapezium ABCD = 3 lune AEBF + semicircle on OA

5. Lune AEBF = semicircle on AB - AFB segment.

$$= \left\{ \frac{\left(\frac{14 - \sqrt{2}}{4}\right) \left(\frac{a}{2}\right)^2}{8} \right\} - \left\{ a^2 \left(\frac{14 - (\sqrt{2} + 6\sqrt{3})}{96} \right) \right\} = X$$

So X = area of the lune.

6. Area of the trapezium ABCD = $3X + \text{semicircle on OA}$

$$= 3X + \left\{ \frac{\left(\frac{14 - \sqrt{2}}{4}\right) \left(\frac{a}{2}\right)^2}{8} \right\}$$

by conventional method

Area of the trapezium = areas of 3 triangles

Area of each triangle (ODC or COB or OBA) = $\frac{1}{2} \times \text{base} \times \text{altitude}$

$$= \frac{1}{2} \times \frac{a}{2} \times \sqrt{\left(\frac{a}{2}\right)^2 - \left(\frac{a}{4}\right)^2}$$

$$\text{area of 3 triangles} = 3 \times \frac{\sqrt{3}a^2}{16} \text{ or } \frac{1}{2} h (a + b) = \frac{1}{2} \times \frac{\sqrt{3}a}{4} \left(a + \frac{a}{2}\right)$$

$$\left(\frac{3\sqrt{3}}{16}\right)a^2 = 3a^2 \left\{ \frac{24\sqrt{3} + \sqrt{2} - 14}{384} \right\} + \left\{ \frac{\left(\frac{14 - \sqrt{2}}{4}\right) \left(\frac{a}{2}\right)^2}{8} \right\}$$

3x

The geometrical constructions of Hippocrates could be possible only when value of π is $(14 - \sqrt{2})/4$.

Reference:

W.W.Rouse Ball (1960), **A Short Account of the History of Mathematics**, Dover Publications, New York.

21. THE LUNULAE OF HIPPOCRATES

The following is the extract from the book entitled **The VNR Concise Encyclopedia of Mathematics** edited by W.G.H. Kustner & M.H.H. Kastner, Van Nostrand Reinhold Company, 1975, page.no.175.

"There are also some famous moonshaped figures. The best known of these are the *crescents(or lunulae) of Hippocrates*. By the theorem of Thales the triangle ABC in the first figure is right angled: Thus $p^2 = m^2 + n^2$. The semicircle on AB = p has the area

$A_{AB} = \pi p^2 / 8$; the sum of the areas of the semicircles on AC and BC is $A_{AC} + A_{BC} = \pi (n^2 + m^2) / 8$ and is thus equal to A_{AB} . From this it follows that:

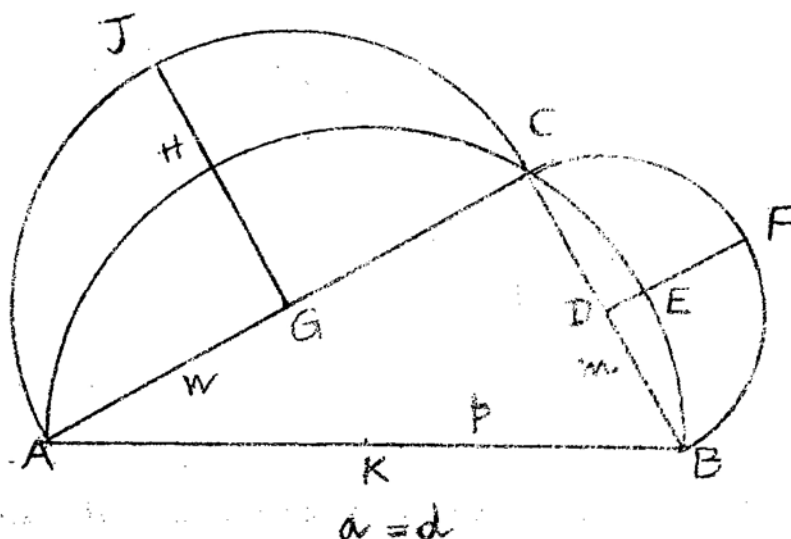


Fig. 25

The sum of the areas of the two crescents is the area of the triangle".

1. $AB = a$
2. $BC = a/2$
3. $AC = \frac{\sqrt{3}a}{2}$
4. $DF = a/4$

$$DE = \left(\frac{2a - \sqrt{3}a}{4} \right)$$

$$6. \quad EF = \left(\frac{\sqrt{3}a - a}{4} \right)$$

$$7. \quad GH = a/4$$

$$8. \quad GJ = \frac{\sqrt{3}a}{4}$$

$$9. \quad HJ = GJ - GH = \frac{\sqrt{3}a}{4} - \frac{a}{4} = \frac{\sqrt{3}a - a}{4}$$

$$10. \text{ Area of the semicircle BDCF} = \left(\frac{14 - \sqrt{2}}{4} \right) \left\{ \frac{\left(\frac{a}{2} \right)^2}{8} \right\}$$

$$11. \text{ Area of the semicircle AGCJ} = \left(\frac{14 - \sqrt{2}}{4} \right) \left\{ \frac{\left(\frac{\sqrt{3}a}{2} \right)^2}{8} \right\}$$

$$12. \text{ Area of the triangle ABC} = \frac{a \times \sqrt{3}a}{8}$$

$$13. \text{ Area of the curvature entity BDCE} = \left\{ \frac{14 - (\sqrt{2} + 6\sqrt{3})}{96} \right\} a^2$$

$$14. \text{ Area of the curvature entity AGCH} = \left\{ \frac{14 - (\sqrt{2} + 3\sqrt{3})}{48} \right\} a^2$$

$$15. \text{ Area of the crescent BECF} = \left\{ \frac{24\sqrt{3} + \sqrt{2} - 14}{384} \right\} a^2$$

$$16. \text{ Area of the crescent AHCJ} = \left\{ \frac{24\sqrt{3} + 14 - \sqrt{2}}{384} \right\} a^2$$

17. The sum of the areas of the two crescents = the area of the triangle

$$\left\{ \frac{24\sqrt{3} + \sqrt{2} - 14}{384} \right\} a^2 + \left\{ \frac{24\sqrt{3} + 14 - \sqrt{2}}{384} \right\} a^2 = \frac{a \times \sqrt{3}a}{8}$$

Crescent: 2

"Similarly, the sum of the areas of the crescents in the second figure is the area of the square'.

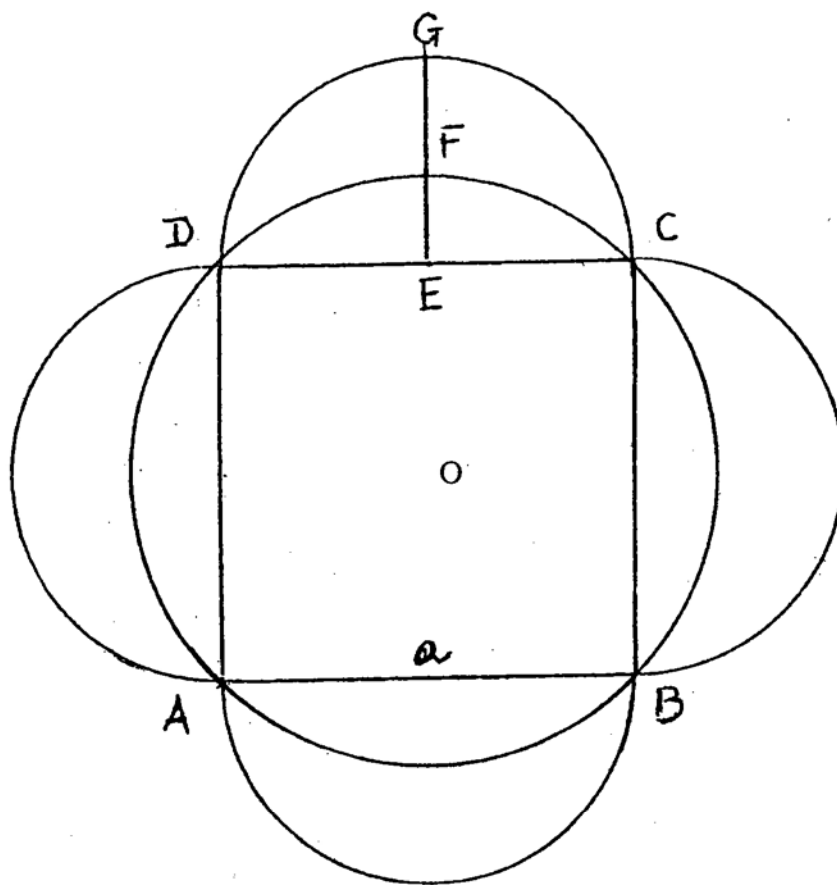


Fig. 26

$$1. \quad AB = CD = AD = BC = a$$

$$2. \quad DE = EC = a/2$$

$$3. \quad AO = OC = \frac{\sqrt{2}a}{2}$$

$$4. \quad EF = \frac{\sqrt{2}a - a}{2}$$

$$5. \quad FG = \frac{\sqrt{2}a - a}{\sqrt{2}}$$

$$6. \quad \text{Diameter of the circle } ABCFD = \sqrt{2}a$$

$$7. \text{ Area of the circle} = \frac{\pi d^2}{4} = \frac{\pi a^2}{4} = \left(\frac{14 - \sqrt{2}}{4}\right) \left\{ \frac{(\sqrt{2}a)^2}{4} \right\}$$

$$8. \text{ Area of the semicircle DECG} = \left(\frac{14 - \sqrt{2}}{4}\right) \left\{ \frac{a^2}{8} \right\}$$

$$9. \text{ Area of the curvature entity DECF} = \left(\frac{6 - \sqrt{2}}{32}\right) a^2$$

$$10. \text{ Area of the crescent DFCG} = \left\{ \frac{a}{2} + \frac{\sqrt{2}a}{4} \right\} \left\{ \frac{\sqrt{2}a - a}{\sqrt{2}} \right\}$$

11. **The sum of the areas of the**

$$\mathbf{4 \text{ crescents} = \text{the area of the square}}$$

$$4 \left\{ \frac{a}{2} + \frac{\sqrt{2}a}{4} \right\} \left\{ \frac{\sqrt{2}a - a}{\sqrt{2}} \right\} = a^2$$

"Misled by the remarkable fact that π does not occur in these formulae many mathematicians of ancient times (including Hippocrates himself) continued the hopeless search for a method of squaring the circle".

(It is unfortunate that Hippocrates and mathematicians of ancient times were misunderstood and commented upon thus.)

22. DURGA METHOD OF SQUARING A CIRCLE

Squaring a circle is a problem in which we have to construct a square whose area is exactly equal to that of a given circle. "Let the radius of the circle be 1. Then the area of the circle is π . But if x is the side of the square to be constructed we must have $x^2 = \pi$. If we could construct a length equal to π we could also construct $x = \sqrt{\pi}$ and conversely".

For the area, there is an oldest surviving document which refers to the problem of squaring the circle is the **Rhind papyrus** which was written in about 1800 B.C. by a scribe named Ahmes who was copying from a document which was then already one or two centuries old.

Ahmes finds the area of circle with diameter d . He calculates

$(1 - 1/9)d \times d - 1/9(1 - 1/9)d \times d$ which is $(8/9)d^2$. This makes the circle equivalent to a square whose side is $8/9$ th (from subepimoric ratios) diameter of the circle. Lets us consider the accuracy of this value by an example. If the side of the square is a and the diameter of the circle is d then we have

$$a^2 = \pi (d^2/4)$$

$$\text{therefore } a/d = 1/2 \sqrt{\pi}$$

The value of $\sqrt{\pi}$ is 1.7738225... ($\pi = 3.1464466...$)

$$\text{so } a/d = 0.8869112..$$

$8/9$ is the value chosen by the Egyptians.

METHOD

Draw a square ABCE of unit side, $a (=d)$ with two diagonals AC and BE. The diagonals intersect at O. Taking O as the centre and radius equal to half the side inscribe a circle with the square. The diagonals AC & BE intersect the circle at L & M. Join L&M, extending beyond such that LM meets CE side of square at D.

The goddess mother Durga is a consort of Lord Siva, the destroyer of the universe. THEY are easily satisfied when the devotees pray to THEM and shower THEIR blessings very generously. The author when he was a mischievous boy, was blessed by the mother Durga and got education upto M.Sc., and got the entire work on mathematics in solving the unsolved problems and approximation to π recently. The author's residence, at present, is nearer to the temple of mother Durga and got the mother's blessings when ever he feels that he is in a helpless situation

1. Side = AB = BC = CE = AE = a = d = diameter.

$$2. CD = \frac{\sqrt{2}a - a}{2\sqrt{2}}$$

Extend DE up to F such that DF is equal to $(3a)/2$. Similarly, extend EC up to J such that CJ is also equal to $(3a)/2$. Finally, FJ is equal to the length of circumference of the inscribed circle.

$$3. FJ = FD + DC + CJ = \pi d = \pi a$$

$$= \frac{3a}{2} + \frac{\sqrt{2}a - a}{2\sqrt{2}} + \frac{3a}{2} = 3a + \frac{\sqrt{2}a - a}{2\sqrt{2}} = \pi a$$

$$4. GJ = \text{diameter of the semicircle} = a = d$$

$$5. GI = IJ = \text{radius} = a/2 = d/2$$

Bisect FJ twice to get HJ which is equal to $1/4^{\text{th}}$ of FJ = $\pi d/4$. HZ is the perpendicular line to GJ at H.

$$6. HJ = \pi d/4 = \left(\frac{14 - \sqrt{2}}{16} \right) a$$

Taking I as the centre and radius equal to IJ or IG draw a semicircle over GJ. The semicircle intersects HZ at K. Join GK and JK. Finally, we get a right angled triangle GKJ.

$$7. GJ = \text{diameter} = AB = a = d$$

$$8. IJ = IG = \text{radius} = a/2 = d/2$$

$$9. HJ = \frac{\pi d}{4} = \frac{\pi a}{4} = \left\{ \left(\frac{14 - \sqrt{2}}{16} \right) a \right\}$$

$$10. GH = \text{diameter} - \frac{\pi a}{4} = a - \left\{ \left(\frac{14 - \sqrt{2}}{16} \right) a \right\}$$

$$= \left(\frac{2 + \sqrt{2}}{16} \right) a$$

$$11. HI = GI - GH = \text{radius} - GH = \frac{a}{2} - \left(\frac{2 + \sqrt{2}}{16} \right) a$$

$$= \left(\frac{6 - \sqrt{2}}{16} \right) a$$

$$12. KI = \text{radius} = a/2$$

$$\begin{aligned}
 13. HK &= \sqrt{(KI)^2 - (HI)^2} \\
 &= \sqrt{\left(\frac{a}{2}\right)^2 - \left\{\left(\frac{6 - \sqrt{2}}{16}\right)a\right\}^2} \\
 &= \left(\frac{\sqrt{26 + 12\sqrt{2}}}{16}\right)a
 \end{aligned}$$

HK can also be obtained applying **altitude theorem**

$$\begin{aligned}
 14. GK &= \sqrt{(GH)^2 + (HK)^2} \\
 &= \left(\frac{\sqrt{2 + \sqrt{2}}}{4}\right)a
 \end{aligned}$$

$$\begin{aligned}
 15. JK &= \sqrt{(GJ)^2 - (GK)^2} \\
 &= \sqrt{a^2 - \left\{\left(\frac{\sqrt{2 + \sqrt{2}}}{4}\right)a\right\}^2} \\
 &= \left(\frac{\sqrt{14 - \sqrt{2}}}{4}\right)a
 \end{aligned}$$

JK is the **required side of the square** whose area is exactly equal to the area of the inscribed circle.

23. TRISECTION OF ARBTIRARY ANGLES

(Approximate geometrical constructions)

The three famous problems are, **the duplication of the Cube, the trisection of an Angle and the quadrature of the Circle**. The importance of these problems lies in the fact that they cannot be solved, except by approximation with ruler and compass, although these tools successfully serve for the solution of so many other construction problems.

Trisection of an angle, is a problem, in which a given angle is divided into three equal parts. Notable people in this field are **Archimedes, Hippias of Elis** and his invented curve **Quadratrix** and **Nicomedes** and his invented curve known as **Conchoid** or the shell shaped curve. An equation was developed by Nicomedes, $r = a \sec \theta \pm d$. It is easy with its aid to trisect a given angle.

With simplest geometric instruments that is, a straightedge and compasses a solution proved possible only in few special cases – for example for angles of 90° and $90^\circ/2n$ where n is a natural number. Hence, in the general case, an angle cannot be trisected by means of only a straightedge and compasses.

1. Trisection of an Angle of 30°

An angle ABC of 30° is drawn with equal sides AB & BC (fig – 29). D and E are the mid points of AB and BC respectively. A perpendicular BF to BC is drawn at B . C and D are joined and extended to meet BF at G . G and E are joined. GE intersects AB at H . H & C and A & E are joined. They intersect each other at I . A line BK is drawn passing through I which results in an angle KBC of $1/3^{\text{rd}}$ angle ABC . ABK is bisected by conventional method. Finally, we get three equal angles of 10° each, ABJ , JBK and KBC .

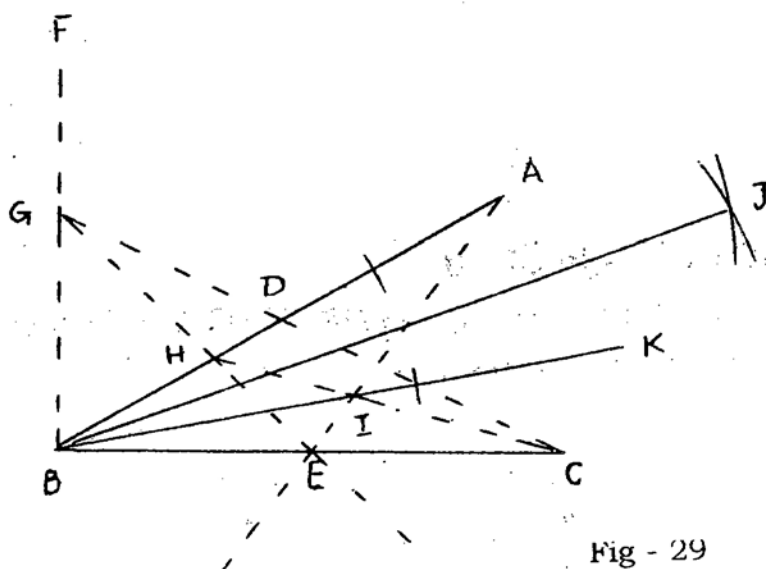


Fig - 29

2. Trisection of an Angle of 40°

An angle ABC of 40° is drawn with equal sides AB and BC (Fig. 30). D and E are the midpoints of AB and BC respectively. A and E are joined. A right angle at D to AB when drawn it intersects AE at G . A line BI is drawn passing through G . We get an angle IBC which is $1/3^{\text{rd}}$ of angle ABC . The angle ABI is bisected by conventional method. Thereby we get 3 equal angles of ABJ , $JB I$ and IBC .

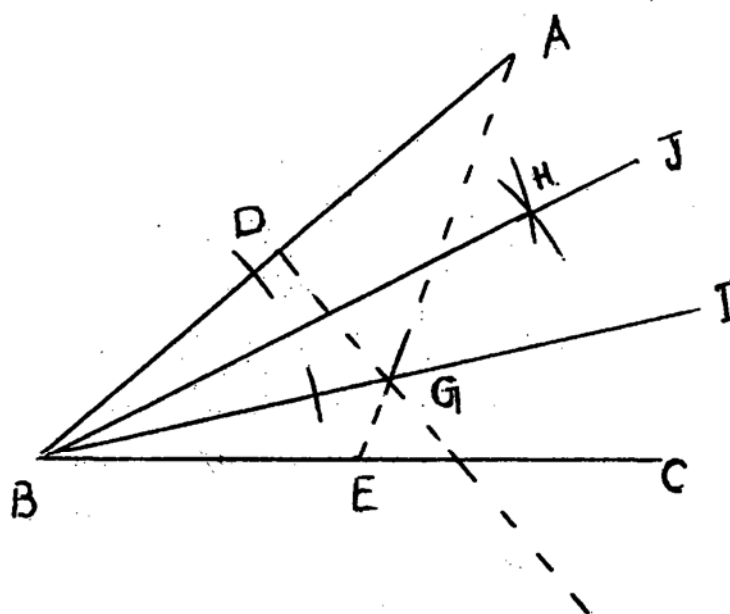


Fig - 30

3. Trisection of an Angle of 50°

An angle ABC of 50° is drawn with equal sides of AB and BC (Fig - 31). D and E are the mid point of AB and BC respectively. A & C , D & C , A & E are joined. The line AE intersects DC at F . B & F are joined and extended to right till it meets AC at G . G and E are joined. GE intersects DC at H . B and H are joined and extended upto I . The angle IBC is $1/3^{\text{rd}}$ of angle ABC . The angle ABI is bisected by conventional method. Finally, we get three equal angle of ABJ , $JB I$ and IBC .

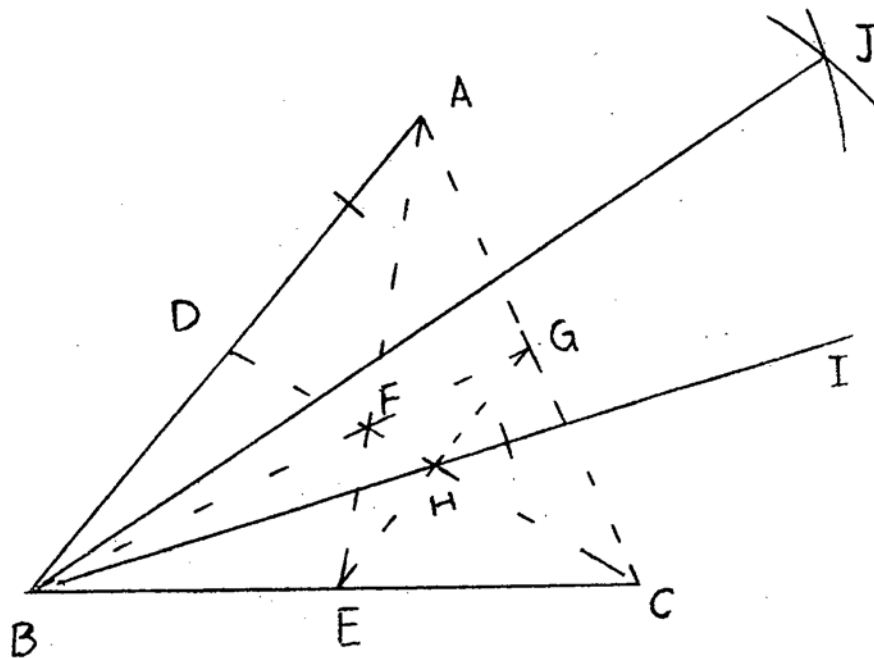


Fig - 31

4. Trisection of an Angle 60°

An equilateral triangle ABC is drawn (fig - 32). The mid points of the sides AB, BC and CA are D, E and F respectively. D & F and A & E are joined. G is the point of intersection of AE and DF and is also the mid point of DF. A line BI is drawn through G. We get an angle A B I which is equal $1/3$ rd of angle A B C. D & C and E & F are joined. These two lines intersect each other at H. A line B J is drawn through H. We get two equal angle bisecting IBC into IBJ and JBC, which are in turn equal to $1/3$ rd angle ABC. Thus we get finally 3 equal angles, ABI, IBJ and JBC each equal to 20° .

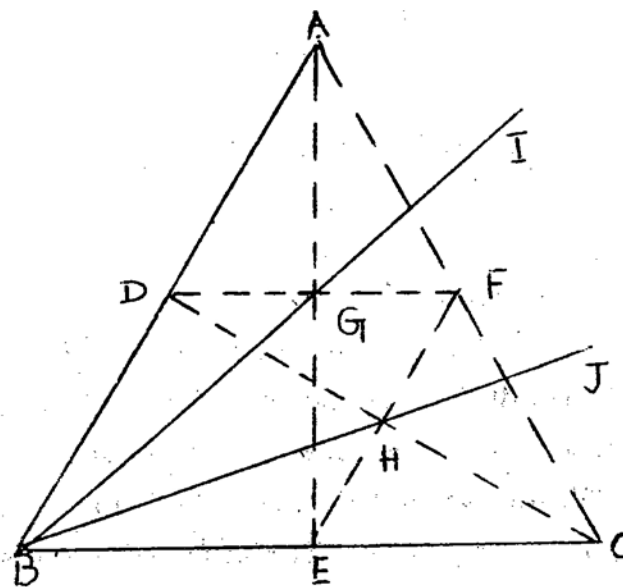


Fig - 32

5. Trisection of an angle of 70°

An angle ABC of 70° is drawn with equal sides of AB and BC (Fig-33). BD left to BC is equal to BC . E , F and G are the mid points of AB , DB and BC respectively. B as the center and radius equal to BC a circle is drawn. F & E are joined and extended which meets the circle at I . I & C are joined. AB when extended downwards meets the circle at H . H & G are joined and extended upwards which meets the circle at J . GJ and IC intersect each other at K . The angle KBC is $1/3^{\text{rd}}$ of the angle of ABC . The angle ABK is bisected conventionally. The angle ABC is finally divided into three equal angles of ABM , MBK and KBC .

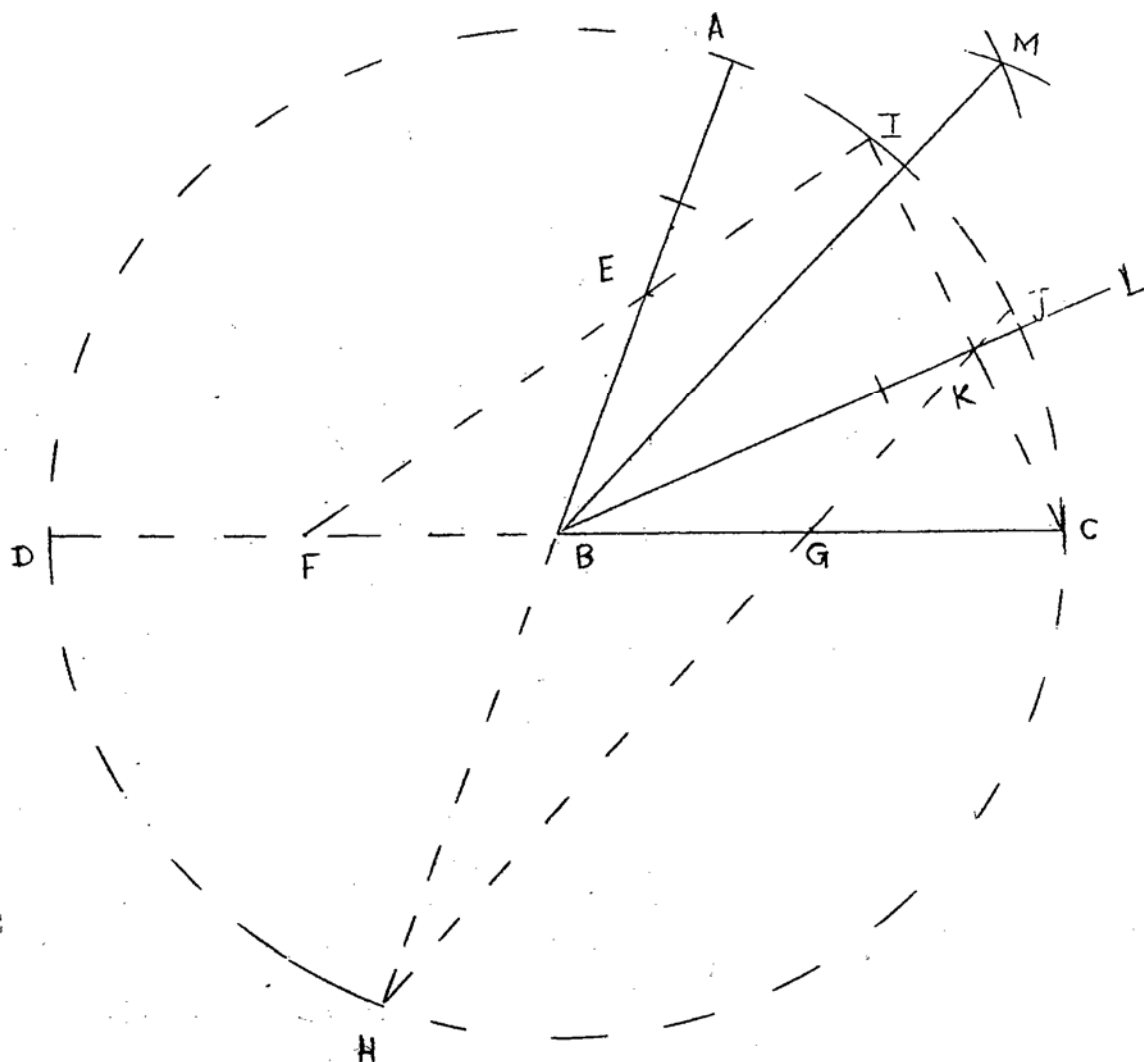


Fig - 33

6. Trisection of an Angle of 80°

An angle ABC of 80° with equal sides of AB and BC is drawn (Fig - 34). A perpendicular DB to BC is drawn at B . C & A are joined and extended upwards such that AC meets right angle at D . E & F are the mid points of AB and BC respectively. D & F are joined. DF intersects AB at G . G & C are joined. A right angle at E to AB is drawn, which intersects AC at H . Second right angle at H to EH is drawn, which intersects GC at K . BI is drawn passing through K . An angle IBC is obtained which is $1/3^{\text{rd}}$ of angle ABC . Angle ABI is bisected in the conventional method to get ABJ and JBI each equal to $1/3^{\text{rd}}$ of angle ABC . Finally we have three equal angles of ABJ , JBI and IBC .

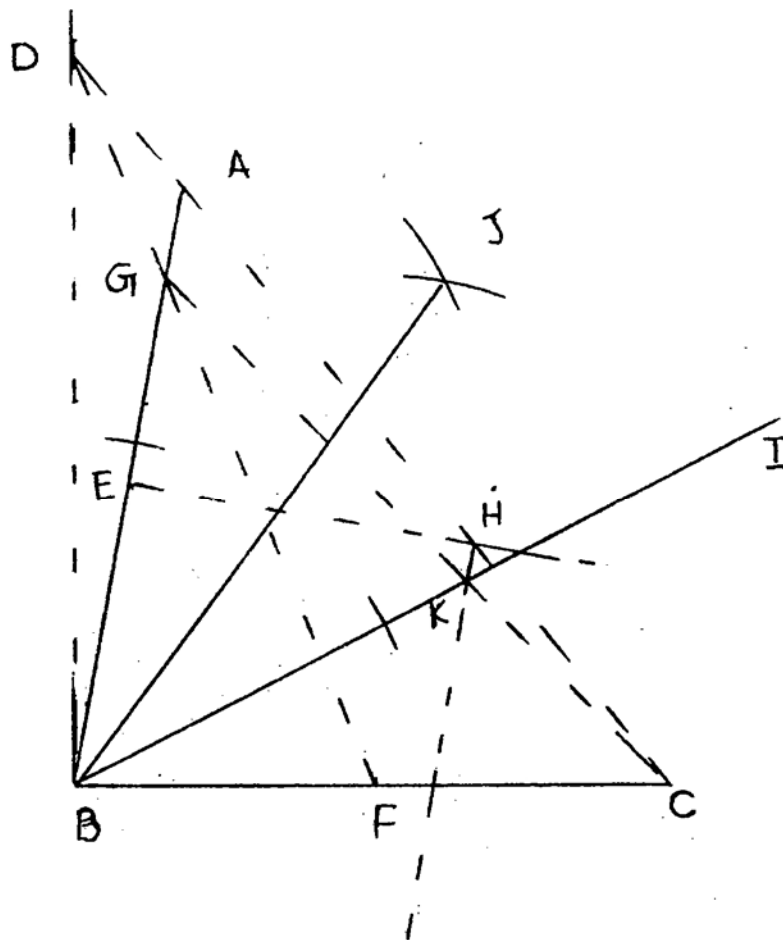


Fig - 34

7. Trisection of an Angle of 90°

A right angle ABC is drawn in which AB and BC are the adjacent sides and AC is the hypotenuse. With the help of a compass, taking B as the centre and arc connecting points A and C is drawn. With the same radius the above arc is intersected by two smaller arcual lengths one from A and the second from C . These two intersecting points on the arc are D and E . When D and E are joined to B we get the angle ABC trisected into $ABE = EBD = DBC = 30^\circ$ each. (Fig .35)

9. Trisection of an angle of 110°

An angle ABC of 110° with two equal sides AB and BC is drawn (Fig 37). A line BD equal to BC is drawn left to B . E & F are the mid points of BD and BC respectively. B as the centre and radius equal to BC a circle is drawn. AB extended downwards which meets the circle at H . G is the mid point of BH . H & F are joined and extended upwards which meets the circle at J . Similarly G & F are joined and extended upwards which meets the circle at I . E & I are joined. EI and FJ intersect each other at K . BL is drawn through K . The angle LBC is $1/3^{\text{rd}}$ of the angle ABC . Angle ABL is bisected conventionally and finally we get three equal angles of ABM , MBL and LBC .

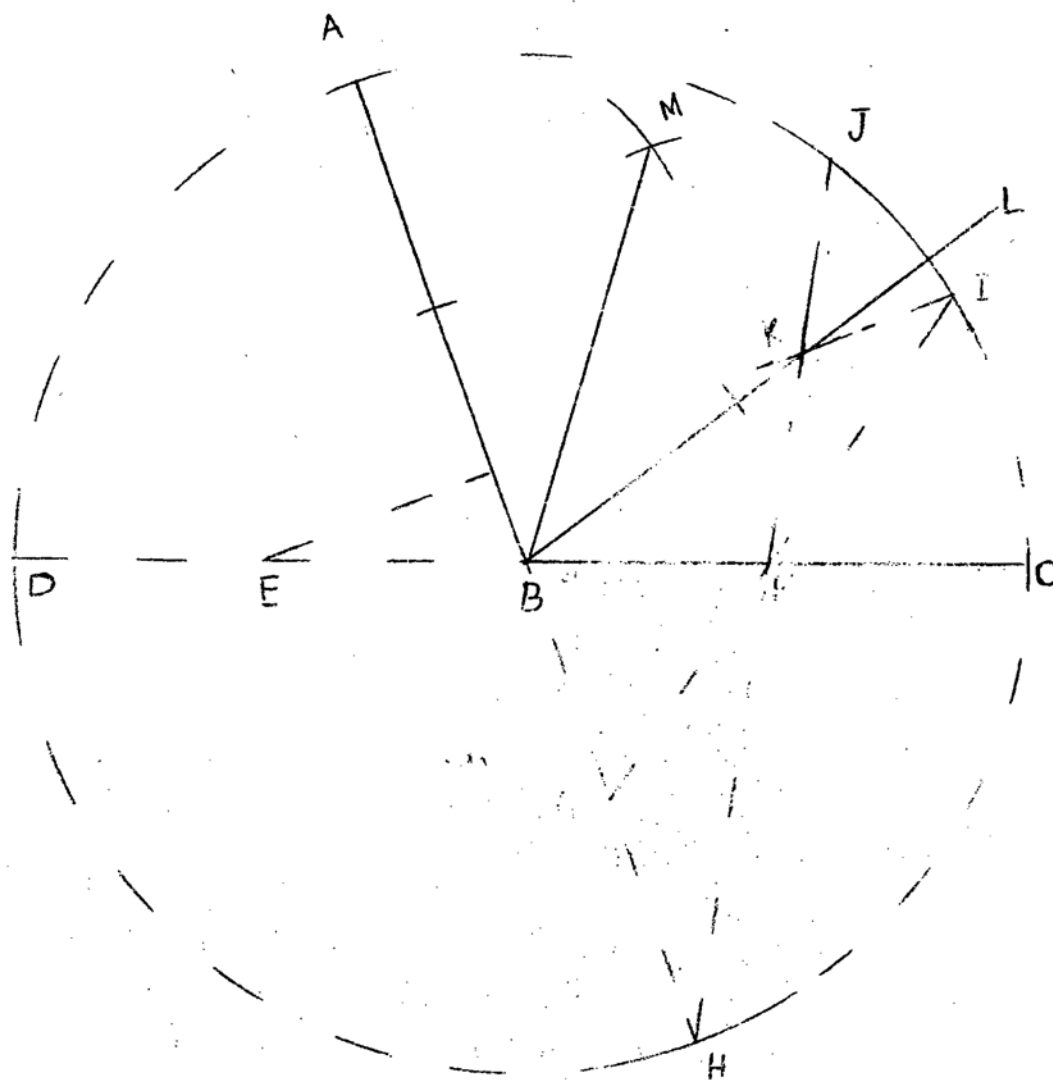


Fig - 37

10. Trisection of an angle of 120°

An angle ABC with 120° is drawn (Fig 38). A line from B is extended to its left upto D such that $BC = BD$. E and F are the mid points of BD and AB respectively. A semi circle with center B and radius equal to BC is drawn.

A line DG is drawn passing through the mid point F which meets the circle at G . E & G are joined. Similarly, A & C are joined. The line EG intersects AC at H . A line BJ passing through H results in a new angle of ABJ which is $1/3^{\text{rd}}$ of the angle ABC . The angle JBC is bisected with the conventional method. We get two angles JBI & IBC which are in turn equal to $1/3^{\text{rd}}$ of angle ABC . Finally the angle ABC is trisected and we get three equal angles of ABJ , JBI and IBC .

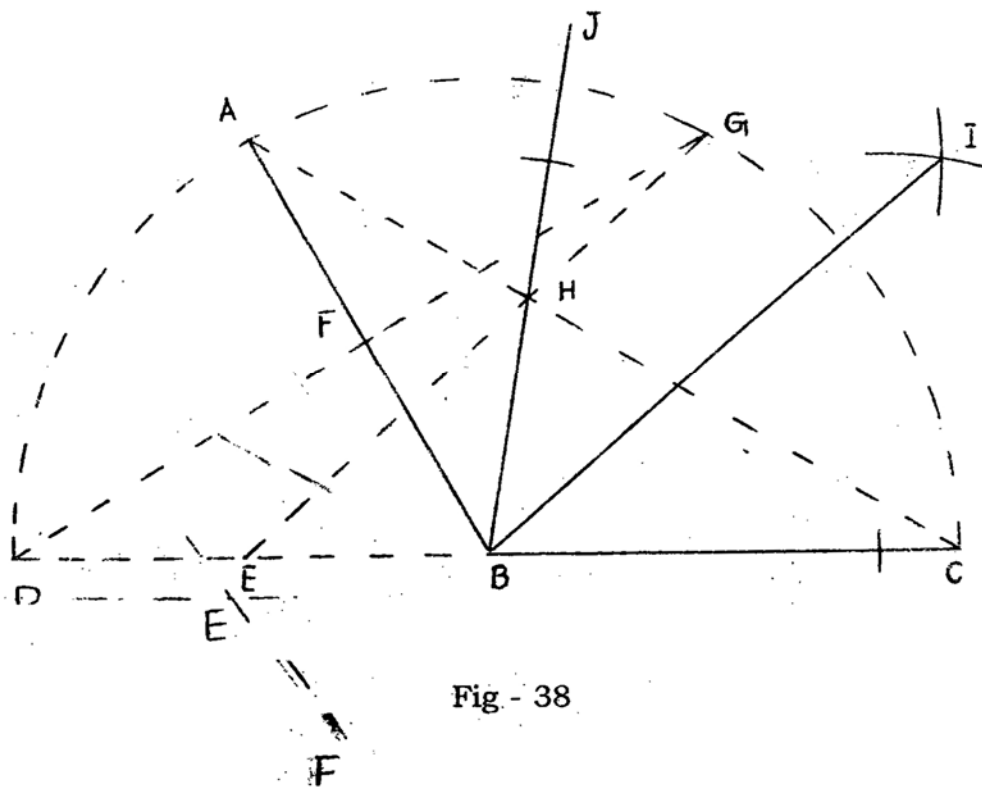


Fig - 38

11. Trisection of an angle of 130°

An angle ABC of 130° with the two equal sides AB and BC is drawn (Fig - 39). A line BD equal to BC is drawn left to B . B as the centre and of radius equal to BC a semi circle is drawn. E , F & G are the mid points of AB , DB and BC respectively. D & E are joined and extended which meets the circle at H . H & G are joined. F & E are joined and extended which meets the circle at I . I & C are joined. IC and HG intersect each other at J . BK is drawn through J which trisects the angle ABC . The angle ABK is bisected by conventional method. Finally, we get three equal angles, ABL , LBK and KBC .

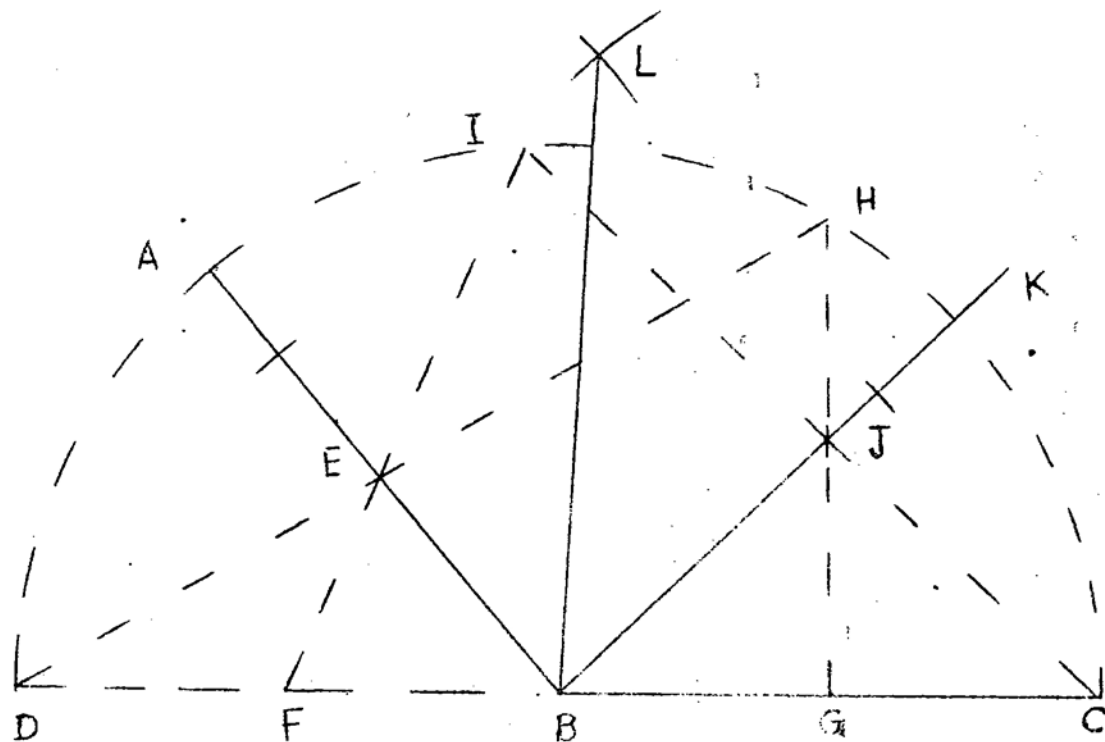


Fig - 39

12. Trisection of an angle of 140°

An angle ABC of 140° with two equal sides of AB and BC is drawn (Fig - 40). DB and CE are half the length of BC. F is the mid point of BC. B and F as the centres, of radius equal to BC two semicircles are drawn which intersect each other at G. G & E are joined. GE intersects one of the circle at H. Taking B and H as the reference points a line from H is drawn which meets the other semicircle at I. A & I are joined. AI intersects GE at J. BK is drawn through J to get an angle KBC which is $1/3^{\text{rd}}$ angle of angle ABC. The angle ABJ is bisected by conventional method. Finally we get three equal angles of ABL, LBK and KBC.

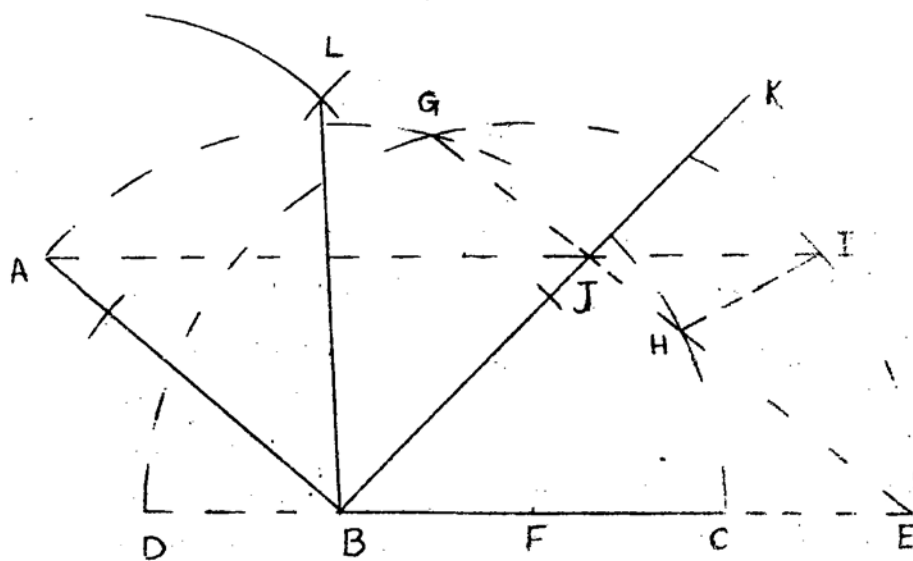


Fig - 40

13. Trisection of an angle of 150°

An angle ABC of 150° with two equal sides AB and BC is drawn (Fig -41). A line BD equal to the length of BC is drawn from B to left. B as its centre and of radius equal to BC , a semicircle is drawn. E is the midpoint of BD . A & E are joined and is extended upto F such that AF is equal to AB . A line FG is drawn passing through B . GBC is an angle equal to $1/3^{\text{rd}}$ of angle ABC . The angle ABG is bisected by conventional method. Finally, we get three equal angles, ABH , HBG and GBC each equal to 50° .

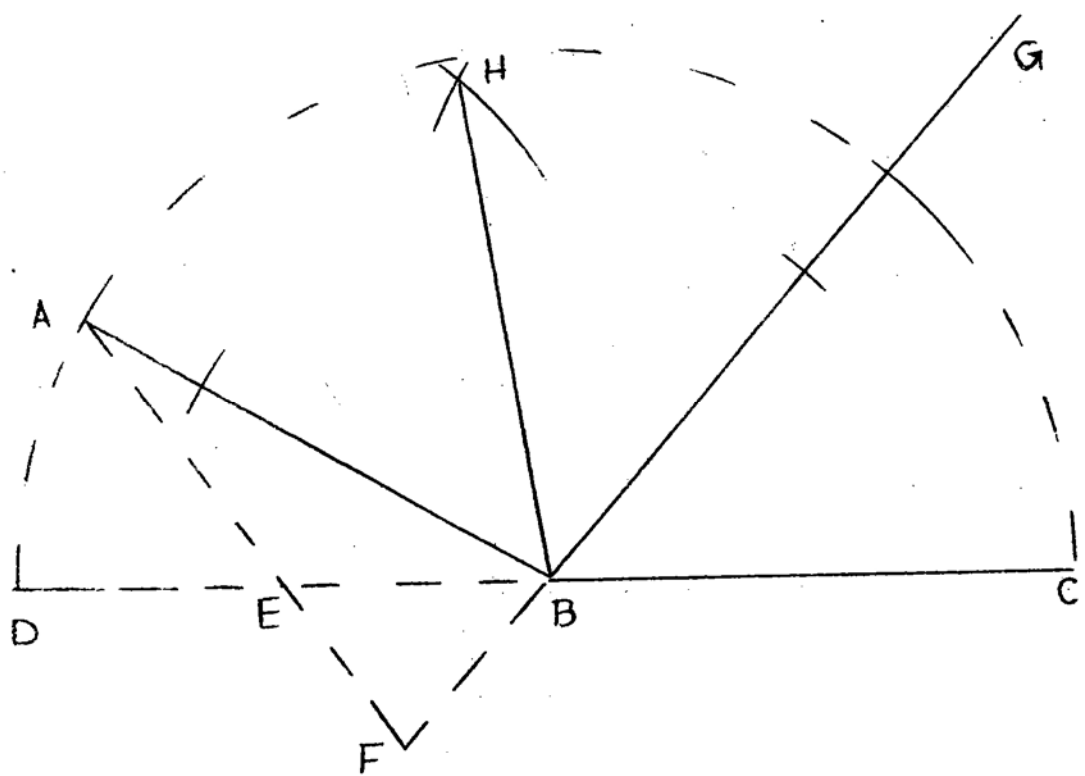


Fig - 41

14 Trisection of an angle of 160°

An angle of ABC of 160° at B is drawn with two equal sides AB & BC (Fig-42). From B a line BD equal to BC is extended to its left. B , as the center and of radius equal to BC , a semicircle is drawn. E & F are the midpoints of BD and AB respectively.

E & F are joined and extended till EF meets the circle at G . A & C are joined. AC intersects EG at H . A line DI is drawn passing through H which in turn meets circle at I . B & I are joined to get an angle IBC which is equal to an angle $1/3^{\text{rd}}$ of angle ABC . Angle ABI is bisected in the conventional way to get finally three equal angles of ABJ , $JB I$ and IBC .

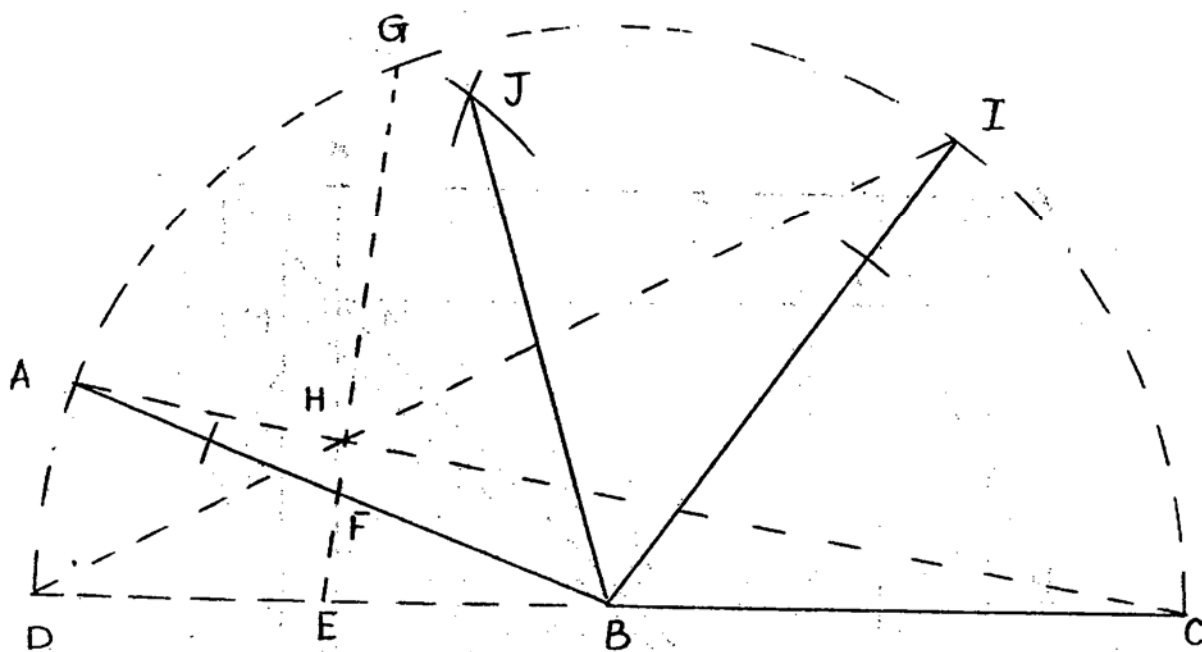


Fig - 42

24. DUPLICATION OF THE CUBE

A Geometric Construction

A new formula for the arrival of the required side of the duplicated cube is proposed and is $\sqrt{\left(\frac{31+14\sqrt{2}}{32}\right)a^2}$ where a is the given side of a cube.

METHOD:

A square ABCD of a given side is drawn. A circle is inscribed with it. Two diagonals AC and BD are drawn. The diagonals intersect the circle at E, F, G, and H. When four parallel lines through the points E, F, G & H to the four sides are drawn, we get four smaller squares for example, one square is K H J D. One parallel line through EH is I J which is equal to AD. The sum of the lengths of IE, EH and HJ is equal to the side AD. HM is half the length of HJ.

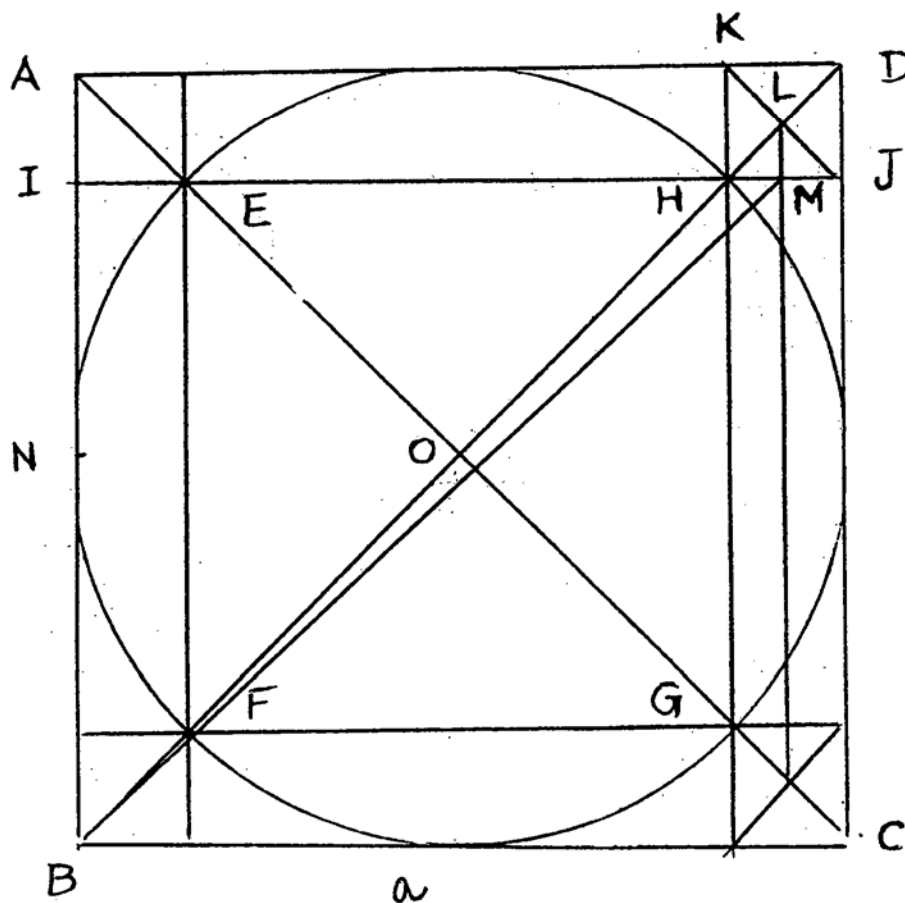


Fig - 43

$$IE=IA = HJ = \frac{\sqrt{2}a - 1a}{2\sqrt{2}}$$

$$EH = \frac{\sqrt{2}a}{2}$$

$$IE+EH+HJ = a \text{ (side of the square)}$$

$$HM = HJ/2 = \frac{\sqrt{2}a - 1a}{4\sqrt{2}}$$

$$IA + IN + NB = a \text{ (side of the square)}$$

$$IN = \frac{\sqrt{2}a}{4}$$

$$NB = \frac{a}{2}$$

When we join BM we get a right angled triangle IBM. IB and IM are the two sides and BM is the hypotenuse.

$$IM = IE + EH + HM$$

$$IB = IN + NB$$

$$MB = \sqrt{(IE+EH+HM)^2 + (IN+NB)^2}$$

$$= \sqrt{\left(\frac{\sqrt{2}a - 1a}{2\sqrt{2}} + \frac{\sqrt{2}a}{2} + \frac{\sqrt{2}a - 1a}{4\sqrt{2}}\right)^2 + \left(\frac{\sqrt{2}a}{4} + \frac{a}{2}\right)^2}$$

$$= \sqrt{\left(\frac{31+14\sqrt{2}}{32}\right)^2 a^2}$$

MB is the hypotenuse which is also the required side of the cube double that with the side BC

25. The list of methods to compute π

S.No.	Article	Method No.	π Value
CIRCUMFERENCE METHODS			
1.	An accurate approximation to mathematical π	1	3.1464466.....
		2	3.1464466.....
		3	3.1464466.....
2.	-do - another method	4	3.1464466.....
		5	3.1464466.....
		6	3.1464466.....
3.	-do - some more methods	7	3.1464466.....
		8	3.1464466.....
		9	3.1464466.....
4.	-do- multi-circle method	10	3.1464466.....
5.	-do- Siva method	11	3.1464466.....
		12	3.1464466.....
6.	-do- Isosceles right angled triangle method	13	3.1464466.....
7.	Approximation of π value an intuitive method	14	3.1464466.....
		15	3.1464466.....
		16	3.1464466.....
8.	CC Approach to approximate π value	17	3.1464466.....
9.	Pythagorean approach to compute π	18	3.1464466.....
10.	Circle formation method to compute π	19	3.1464466.....
11.	Computation of π through tangents	20	3.1464466.....
		21	3.1464466.....
12.	A simple method to compute π	22	3.1464466.....
13	Square - minus - corner area method to compute π	23	3.1464466.....
14.	Triangle - from - circle method to compute π	24	3.1464466.....

26. THE LIST OF FORMULAE FOR THE CIRCUMFERENCE (WHOLE / PART) OF A CIRCLE

$$1) \quad 3d + \frac{\left(d - \frac{\sqrt{2}d}{2}\right)}{2}$$

$$2) \quad 4a - \left(\frac{\sqrt{2}}{4} a + \frac{1}{2} a\right)$$

$$3) \quad 7 \frac{d}{2} - \frac{\sqrt{2}}{4} d$$

$$4) \quad 2a + \left[a - \frac{\left(a - \frac{\sqrt{2}a}{2}\right)}{2} \right] + \left[\frac{\sqrt{2}a}{2} - (\sqrt{2}a - a) \right]$$

$$5) \quad 2a + \left[a - \frac{a - \frac{\sqrt{2}a}{2}}{2} \right] + \left[a - \frac{\sqrt{2}a}{2} \right]$$

$$6) \quad 2a + \frac{\sqrt{2}a}{2} + \left[a - \frac{\sqrt{2}a}{2} \right] + \left[\frac{a - \frac{\sqrt{2}a}{2}}{2} \right]$$

$$7) \quad d + d\sqrt{2} + 5 \cdot \left[\frac{d}{2} - \left(d \frac{\sqrt{2}}{4}\right) \right]$$

Where **a**, the side of the superscribed square = **d**, the diameter of the inscribed circle

$$8) \quad d - \frac{3\sqrt{2}}{2} + 7 \left[\frac{d - \frac{d\sqrt{2}}{2}}{2} \right]$$

$$9) \quad 2d + d \frac{\sqrt{2}}{2} + 3 \left[\frac{d - \frac{d\sqrt{2}}{2}}{2} \right]$$

$$10) \quad d \left(4 \frac{\sqrt{2}}{4} + 4 \frac{\sqrt{2}-1}{\sqrt{2}} + \frac{\sqrt{2}-1}{2\sqrt{2}} + \sqrt{2}-1 \right)$$

$$11) \quad 4 \left\{ a - \left[\frac{a + \left(\frac{a\sqrt{2}-1a}{2} \right)}{4\sqrt{2}} \right] \right\}$$

$$12) \quad 4a - \left[\frac{a + \left(\frac{a\sqrt{2}-1a}{2} \right)}{\sqrt{2}} \right]$$

$$13) \quad a - \left(\frac{\frac{a}{2} + \frac{\sqrt{2}a}{4}}{4} \right) = \frac{\pi d}{4} = \frac{\pi a}{4}$$

$$14) \quad 2a - \left\{ \frac{\frac{\sqrt{2}a}{2} + \frac{\frac{\sqrt{2}a-1a}{2}}{\sqrt{2}}}{2} \right\} = \frac{\pi d}{2} = \frac{\pi a}{2}$$

$$15) \quad 2a - \left\{ \frac{\frac{\sqrt{2}a}{2} + \frac{\sqrt{2}a-1a}{2\sqrt{2}}}{2} \right\} = \frac{\pi d}{2} = \frac{\pi a}{2}$$

Where **a**, the side of the superscribed square = **d**, the diameter of the inscribed circle

$$16) \quad 2a - \left\{ \frac{\sqrt{2}a + \left(\frac{\sqrt{2}a - 1a}{\sqrt{2}} \right)}{4} \right\} = \frac{\pi d}{2} = \frac{\pi a}{2}$$

$$17) \quad \frac{\sqrt{2}a}{4} \times \frac{\sqrt{2}(14-\sqrt{2})}{16} = \frac{\pi d}{8} = \frac{\pi a}{8}$$

$$18) \quad 2 \left\{ \frac{\sqrt{2}(14-\sqrt{2})}{16 \times a} \right\} \times \sqrt{\left(\frac{a}{2} \right)^2 - \left\{ \left(\frac{\sqrt{\frac{\sqrt{2}-1}{2\sqrt{2}}}}{2} \right) a \right\}^2} \times \left\{ \sqrt{\frac{\sqrt{2}-1}{2\sqrt{2}}} \right\} a = \frac{\pi d}{8} = \frac{\pi a}{8}$$

$$19) \quad 4a - \left\{ \frac{\sqrt{2}a - 1a}{2\sqrt{2}} + \frac{\sqrt{2}a - 1a}{2} + \sqrt{2} \left(\frac{\sqrt{2}a - 1a}{2\sqrt{2}} + \frac{\sqrt{2}a - 1a}{2} \right) \right\} = \pi d = \pi a$$

$$20) \quad \left(\frac{t_1+t_2+t_3+t_4}{=4t} \right) - \left(\frac{2t + \sqrt{2}t}{4} \right) = \pi t = \pi a = \pi d$$

$$21) \quad \left(\frac{t_1+t_2+t_3+t_4}{=4t} \right) - \left\{ \left(1 + \frac{\sqrt{2}t - t}{2t} \right) \left(\frac{\sqrt{2}t}{2} \right) \right\} = \pi t = \pi a = \pi d$$

$$22) \quad 4a - \left(\frac{a + \sqrt{2}a}{2\sqrt{2}} \right) = \pi d = \pi a$$

$$23) \quad 4a - \frac{a}{4 \left(\sqrt{2} \times \frac{\sqrt{2}-1}{2} \right)} = \pi d$$

$$24) \quad 3\sqrt{3}a - 3 \left(\sqrt{3}a - \frac{7a}{6} + \frac{\sqrt{2}a}{12} \right) = \pi d$$

Where a , the side of the superscribed square = d , the diameter of the inscribed circle

**27. THE LIST OF FORMULAE FOR THE AREA
(WHOLE / PART) OF A CIRCLE**

$$25). \quad 2r^2 + \left[r^2 \left(1 + \frac{\sqrt{2}-1}{2\sqrt{2}} \right) \right]$$

$$26). \quad 4r^2 - \left[\frac{r^2}{4 \frac{\sqrt{2}-1}{\sqrt{2}}} \right]$$

$$27). \quad a^2 - \left\{ \left(\frac{1}{2} \times \frac{\sqrt{2}a}{2} \times \frac{\sqrt{2}a}{4} \right) \left(\frac{50+\sqrt{2}}{8} \right) \right\} = \frac{\pi d^2}{16} = \frac{\pi a^2}{16}$$

$$28). \quad \frac{a^2}{4} - 2 \left\{ \frac{\frac{a^2}{16}}{8 \left(\frac{\sqrt{2}-1}{\sqrt{2}} \right)} \right\} = \frac{\pi d^2}{16} = \frac{\pi a^2}{16}$$

$$29). \quad \left(\frac{1}{2} \times \frac{\sqrt{2}a}{2} \times \frac{\sqrt{2}a}{4} \right) + 2 \left\{ \frac{\frac{a^2}{32}}{1 + \frac{\sqrt{2}-1}{2\sqrt{2}}} \right\} = \frac{\pi d^2}{16} = \frac{\pi a^2}{16}$$

$$30). \quad a^2 - 8 \left[\frac{\left(\frac{\sqrt{2}a - 1a}{2\sqrt{2}} \right)^2}{2} + \left\{ \frac{\frac{a^2}{16}}{3 + 2\sqrt{2}} \times \left(\sqrt{2} + \frac{\sqrt{2}-1}{4\sqrt{2}} \right) \right\} \right]$$

Where **a**, the side of the superscribed square = **d**, the diameter of the inscribed circle

$$31). \quad 8 \left\{ \frac{1}{2} \left(\frac{\sqrt{2}a}{4} \right)^2 + \frac{1}{2} \times \frac{\sqrt{2}a}{4} \times \frac{\sqrt{2}a - 1a}{2\sqrt{2}} + \left(\frac{14 - 9\sqrt{2}}{128} \right) a^2 \right\} = \frac{\pi a^2}{4}$$

$$32). \quad 6 \left\{ \frac{1}{2} \left(\frac{a}{2} \times \sqrt{\left(\frac{a}{2} \right)^2 - \left(\frac{a}{4} \right)^2} \right) + a^2 \left(\frac{14 - (\sqrt{2} + 6\sqrt{3})}{96} \right) \right\} = \frac{\pi d^2}{4} = \frac{\pi a^2}{4}$$

$$34). \quad \left(\frac{1}{2} \times \frac{\sqrt{2}a}{2} \left(\frac{a}{2} + \frac{\sqrt{2}a}{4} \right) \right) + 2 \frac{a^2}{32} \left(1 + \frac{2 - \sqrt{2}}{4} \right) + 2 \left(\frac{42 - 11\sqrt{2}}{128} \right) a^2$$

$$38). \quad a^2 - \left[\left\{ 4 \left(\frac{2a - \sqrt{2}a}{4} \right)^2 \right\} + 8 \left\{ \frac{\left(\frac{2a - \sqrt{2}a}{4} \right) \left(\frac{\sqrt{2}a}{4} \right) \left(\sqrt{2} + \frac{2 - \sqrt{2}}{8} \right)}{2 + 2\sqrt{2}} \right\} \right]$$

$$39). \quad a^2 - \frac{a^2}{16 \times \sqrt{2} \times \frac{\sqrt{2} - 1}{2}} = \frac{\pi d^2}{4} = \frac{\pi a^2}{4}$$

$$40). \quad \left[\frac{1}{2} \times \sqrt{3}a \times \frac{3a}{2} \right] - 3 \left[\left(\frac{\sqrt{3}a}{2} \times \frac{a}{2} \right) - \left(\frac{7a}{12} \times \frac{a}{2} \right) + \left(\frac{\sqrt{2}a}{24} \times \frac{a}{2} \right) \right] = \frac{\pi a^2}{4}$$

Where **a**, the side of the superscribed square = **d**, the diameter of the inscribed circle

$$33). \quad \frac{1}{2} \left(\sqrt{a^2 - \left(\frac{a}{2}\right)^2} \right) \left(\frac{3a}{4}\right) + 2 \left\{ \frac{1}{2} \left(\frac{\frac{a}{4} \times \left(\sqrt{a^2 - \left(\frac{a}{2}\right)^2}}{2} \right)^2 \right) + a^2 \left(\frac{14 - (\sqrt{2} + 3\sqrt{3})}{48} \right) + a^2 \left(\frac{14 - (\sqrt{2} + 6\sqrt{3})}{96} \right) \right\} = \frac{\pi d^2}{4} = \frac{\pi a^2}{4}$$

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36).

$$\left[\left\{ \frac{\frac{\sqrt{2}a}{4} \times \frac{\sqrt{2}a}{4}}{2} \right\} + \left\{ \frac{\frac{\sqrt{2}a}{4} \times \frac{2a - \sqrt{2}a}{4}}{2} \right\} + 2 \left\{ a \frac{\sqrt{\frac{2 - \sqrt{2}}{4}}}{2} \times \frac{a}{2} - \sqrt{\left(\frac{a}{2}\right)^2 - \left(a \sqrt{\frac{2 - \sqrt{2}}{4}}\right)^2} \right\} + 2 \left\{ \frac{14 - (\sqrt{2} + 16 \sqrt{2 - \sqrt{2}})}{256} \right\} a^2 \right]$$

$$= \frac{\pi d^2}{4} = \frac{\pi a^2}{8}$$

Where a, the side of the superscribed square = d, the diameter of the inscribed circle

37).

$$\left\{ \frac{1}{2} \times \frac{\sqrt{2}a}{4} \times \left(\frac{a}{2} + \frac{\sqrt{2}a}{4} \right) \right\} + \left\{ \frac{\frac{\sqrt{2}a}{4} \times \frac{2a - \sqrt{2}a}{4}}{2} \right\} \quad (1)$$

$$\left[\frac{a \sqrt{\frac{2 - \sqrt{2}}{4}}}{2} + 2 \left\{ \frac{\frac{a}{2} - \sqrt{\left(\frac{a}{2} \right)^2 - \left(a \sqrt{\frac{2 - \sqrt{2}}{4}} \right)^2}}{2} \right\} \right] + \quad (3)$$

$$+ 2 \left\{ \frac{14 - (\sqrt{2} + 16 \sqrt{2 - \sqrt{2}})}{256} a^2 \right\} + \left\{ \left(\frac{42 - 11\sqrt{2}}{128} \right) a^2 \right\} + \left\{ \frac{a \sqrt{\frac{2 - \sqrt{2}}{4}} \times \sqrt{\left(\frac{a}{2} \right)^2 - \left(a \sqrt{\frac{2 - \sqrt{2}}{4}} \right)^2}}{2} \right\} + \left\{ \left(\frac{14 - 9\sqrt{2}}{128} \right) a^2 \right\} + \quad (4)$$

(5)

(6)

(7)

$$+ \left\{ \frac{\frac{a}{2} \times \frac{a}{2}}{2} \right\} + 2 \left\{ \frac{a^2}{32} \left(1 + \frac{\sqrt{2} - 1}{2\sqrt{2}} \right) \right\} + \left\{ \frac{\frac{\sqrt{2}a}{4} \times \frac{\sqrt{2}a}{4}}{2} \right\} + \left\{ \frac{a^2}{32} \left(1 + \frac{\sqrt{2} - 1}{2\sqrt{2}} \right) \right\} \quad (8)$$

(9)

(10)

(11)

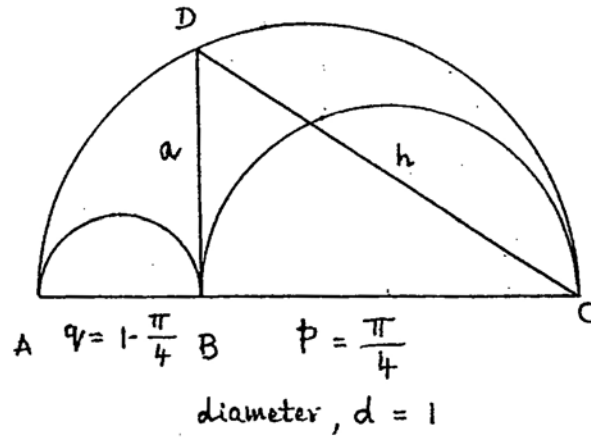
$$= \frac{\pi d^2}{4} = \frac{\pi a^2}{4}$$

Where **a**, the side of the superscribed square = **d**, the diameter of the inscribed circle

28. DURGA METHOD OF SQUARING A CIRCLE

(Theoretical Approach)

Theoretically the Durga Method of Squaring a Circle can be represented in the following way.



Applying the **Altitude Theorem** we can calculate the altitude $a = BD$.

$$a^2 = q \times p = \left(1 - \frac{\pi}{4}\right) \frac{\pi}{4}$$

Applying **Pythagorean Theorem** to the triangle DBC we can obtain $h = CD$ which is equal to the length of the required side of the square whose area is equal to the area of the circle (whose diameter is $d=1$).

$$h^2 = a^2 + p^2$$

$$\begin{aligned} h &= \sqrt{a^2 + p^2} = \sqrt{\left(1 - \frac{\pi}{4}\right) \frac{\pi}{4} + \left(\frac{\pi}{4}\right)^2} \\ &= \frac{1}{2} \sqrt{\pi} \end{aligned}$$

31. RANDOM TRIANGLES and the Proposed π value

Prof. Trevor Easingwood of Derby Lonsdale College of Higher Education, Derby had published his article entitled "Random Triangles" in **The Mathematical Gazette**, 1981, page 245 and in which he derived a formula $(1/2) - \pi/8$ for the area of a curvature entity (shaded area in the diagram) QRS. QRS is $1/2$ corner entity located in between the inscribed circle of diameter equal to 2 (Fig -1).

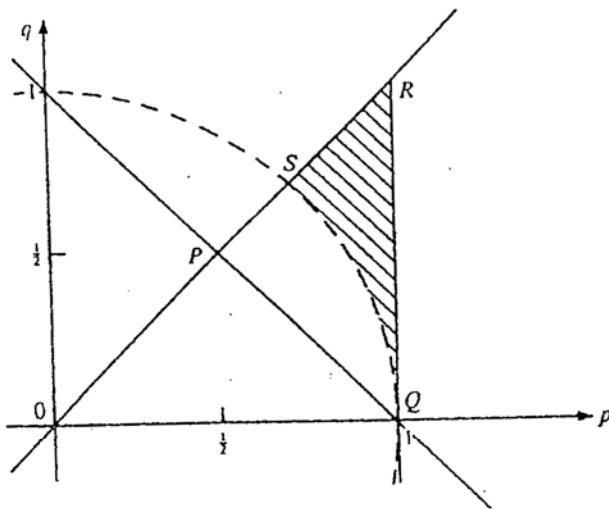


Figure - 1

The author in his article entitled "Area of a Circle and Mathematical π (page No., 26) has divided **exactly** a square - circle nexus into 32 parts of two dimensions. 16 parts of one dimension called C_1 entity and the remaining 16 parts of another dimension called C_2 entity. Formulae for C_1 and C_2 are derived on the basis of their parameters (Fig-2).

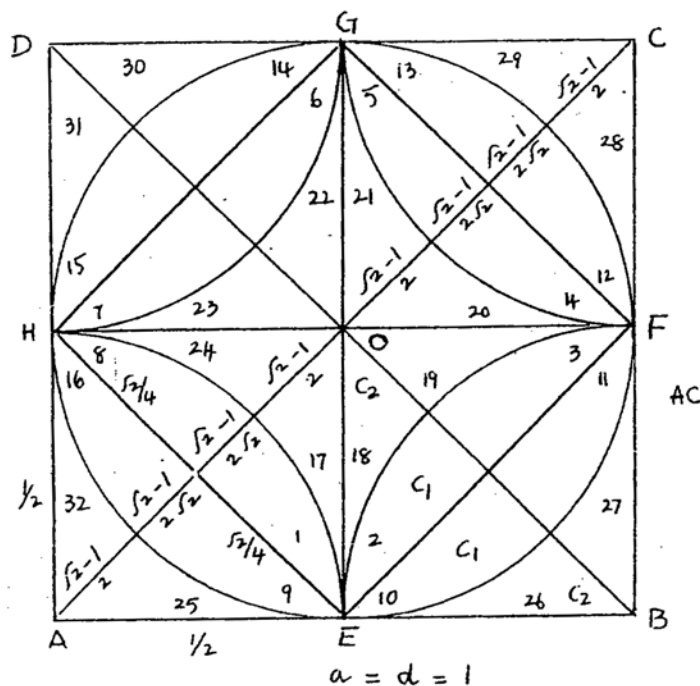


Figure - 2

By area and lengths of every parameter, C_2 is **identical** to QRS (shaded area) of Easingwood. The only difference between the two formulae of QRS and C_2 is that the Easingwood's formula is expressed in terms of π and not in terms of its parameters.

$$\text{QRS} = \frac{1}{2} - \frac{\pi}{8} = C_2 = \frac{\frac{a^2}{32}}{4 \left(\frac{\sqrt{2}-1}{\sqrt{2}} \right)}$$

Where the diameter, d (of QRS) is equal to 2 = the side of the superscribed square a (of C₂).

$$\text{If } \frac{1}{2} - \frac{\pi}{8} = \frac{\frac{4}{32}}{4 \left(\frac{\sqrt{2}-1}{\sqrt{2}} \right)}$$

$$\pi = (14-\sqrt{2})/4$$

The number 3.1415926 if considered as equal to π it is impossible to convert the number exactly in terms of the parameters of QRS (=C₂). It is for the mathematicians therefore, to decide either the proposed number $(14-\sqrt{2})/4$ based on parameters of the QRS entity as right or the present number 3.1415926.... which is **incompatible** with the parameters of QRS (=C₂) as right. Further, **in terms of $\sqrt{2}$, π which is equal to $(14-\sqrt{2})/4$ is exact.**

32. CIRCLE - CONNECTION FOR THE ARCHIMEDEAN INSCRIBED POLYGON

Archimedes inscribed a polygon of **96 sides** with the circle and gave perimeter of the polygon as equal to $3 \frac{10}{71} = 3.140845....$ When the diameter of the superscribed circle is 1 unit, the perimeter = π value. **Prof. Constantine Karapappoulos of University of Patra, Patra, Greece** has suggested a formula for the Archimedian minimum as equal to $3 + \frac{1}{3} \left(1 - \frac{\sqrt{3}}{3} \right)$ in the range of $3 \frac{10}{71}$ to $3 \frac{10}{70}$

The proposed formula for $\pi = (14 - \sqrt{2})/4$ gives the circumference of the circle as equal to $3 + \frac{1}{2} \left(1 - \frac{\sqrt{2}}{2} \right)$

$$\underbrace{3 + \left\{ \frac{1}{3} \left(1 - \frac{\sqrt{3}}{3} \right) \right\}}_{\text{Perimeter of Polygon}} \cdot \underbrace{\left\{ \frac{3\sqrt{3}(\sqrt{2}-1)}{2\sqrt{2}(\sqrt{3}-1)} \right\}}_{\text{Circle connecting link}} = 3 + \frac{1}{2} \left(1 - \frac{\sqrt{2}}{2} \right) = \frac{14 - \sqrt{2}}{4}$$

= Circumference of Circle

$\frac{3\sqrt{3}(\sqrt{2}-1)}{2\sqrt{2}(\sqrt{3}-1)}$ is obtained by the calculation method. One need not set aside it on the reason that the above factor is based on the calculation method. This factor or circle connecting link can be viewed as a **clue** or a guiding factor for further analysis. On close observation one finds that the denominator and the numerator of the link are **symmetrical** and give us a right relationship between the inscribed polygon and the superscribed circle.

34. AN ALGEBRAIC, ONE SURD FORMULA FOR π NUMBER

Starting from John Wallis (1650) and ending with the latest infinite series of David Bailey, Peter Borwein and Simon Plouffe many mathematicians had considered π **number as transcendental** and gave many formulae to compute π .

$$1 \quad \frac{\pi}{2} = \frac{2.2.4.4.6.6.8...}{1.3.3.5.5.7.7...} \quad \text{(of John Wallis)} \\ 1665 \text{ AD}$$

$$2 \quad \pi = \sum_{n=0}^{\infty} \left(\frac{4}{8n+1} - \frac{2}{8n+4} - \frac{1}{8n+5} - \frac{1}{8n+6} \right) \left(\frac{1}{16} \right)^n$$

(of David Bailey, Peter Borwein and Simon Plouffe)
1996 AD

From 42 methods and some **with proofs** the π number has been computed and is **exactly equal** to $3 + \frac{1}{2} \left(1 - \frac{\sqrt{2}}{2} \right)$ and this can be simplified as equal to $(14-\sqrt{2})/4$ or $3 + \frac{2-\sqrt{2}}{4}$. As one gets an algebraic equation with integral coefficients for it the status of transcendency of π number is **questioned**.

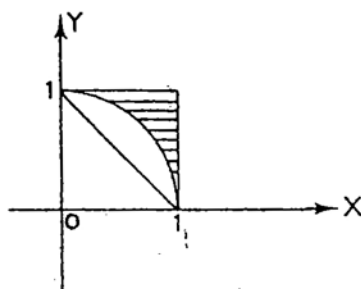
The π number can be expressed in terms of a surd $\sqrt{2}$ only and are as follows:

$$1. \quad (\sqrt{2} \times \sqrt{2}) + \left(\frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}} \right) + \left(\frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}} \right) + \frac{(\sqrt{2} \times \sqrt{2}) - \sqrt{2}}{\sqrt{2} \times \sqrt{2} \times \sqrt{2} \times \sqrt{2}}$$

$$2. \quad \frac{(\sqrt{2} \times \sqrt{2}) + (\sqrt{2} \times \sqrt{2}) + (\sqrt{2} \times \sqrt{2}) + (\sqrt{2} \times \sqrt{2}) + (\sqrt{2} \times \sqrt{2}) + (\sqrt{2} \times \sqrt{2}) + (\sqrt{2} \times \sqrt{2}) - \sqrt{2}}{\sqrt{2} \times \sqrt{2} \times \sqrt{2} \times \sqrt{2}}$$

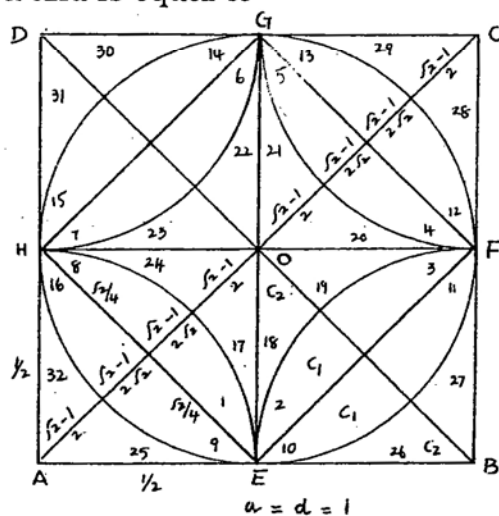
35. PROBABILITY OF ACUTE - ANGLED TRIANGLES AND THE PROPOSED π VALUE.

Prof. C.R. FLETCHER of the University College of Wales, Department of Pure Mathematics, ABERYSTWYTH, had proposed a formula in terms of π for the shaded area in the following diagram and the article was published in THE MATHEMATICAL GAZETTE (1971, December, Page No. 422):



The Formula for the area of the shaded area proposed was equal to $1 - \frac{1}{4} \pi$ when the diameter is equal to 2.

From the following diagram another formula has been proposed by the author for the same area of Fletcher based on the **parameters** of it. The formula is for the same area and is equal to



$$\frac{\frac{a^2}{32}}{2 \left(\frac{\sqrt{2}a - a}{\sqrt{2}a} \right)}$$

a , is the side of the superscribed square = d , the diameter of the inscribed circle. As the both the entities are identical the two formulae are equal.

$$\text{If } 1 - \frac{1}{4} \pi = \frac{\frac{a^2}{32}}{2 \left(\frac{\sqrt{2}a - a}{\sqrt{2}a} \right)} = \frac{\frac{4}{32}}{2 \left(\frac{\sqrt{2} - 1}{\sqrt{2}} \right)} \quad \text{Where } a = d = 2$$

$$\pi = \frac{14 - \sqrt{2}}{4}$$

36. CIRCLE MINUS PETALS = SQUARE METHOD IN SUPPORT OF PROPOSED π VALUE

A square ABCD with side a , and two diagonals AC & BD, each of $\sqrt{2}a$ are drawn. O is the intersection of two diagonals. H, L, M and N are the mid points of the sides of the square. Taking H, L, M and N as centres and $OC (= \frac{\sqrt{2}a}{2})$ as the radius a circle is drawn which overscribes the square. The diameter of it is $\sqrt{2}a$.

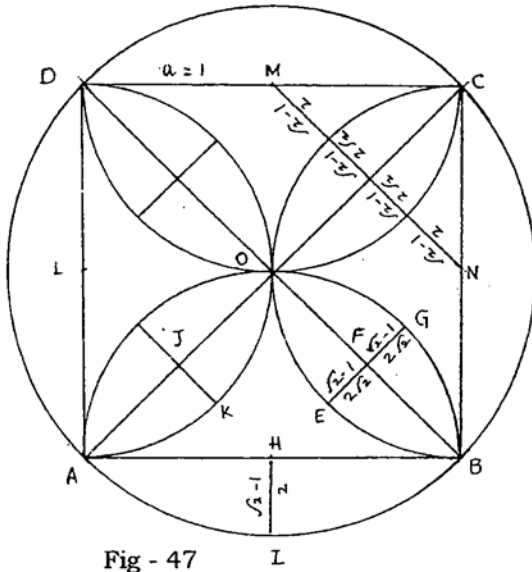


Fig - 47

$$1. \quad AB = a = 1$$

$$2. \quad AO = \frac{\sqrt{2}a}{2} = \frac{\sqrt{2}}{2}$$

$$3. \quad EG = 2 \left(\frac{\sqrt{2} - 1}{2 \sqrt{2}} \right)$$

$$4. \quad MN = \frac{\sqrt{2}a}{2} = 2 \left(\frac{\sqrt{2} - 1}{2} \right) + 2 \left(\frac{\sqrt{2} - 1}{2 \sqrt{2}} \right)$$

$$5. \quad HI = \frac{\sqrt{2} - 1}{2}$$

The area of each petal has been calculated ($4 \times C_1$ entity) in the earlier methods.

$$6. \quad \text{Area of each petal} = 4 \left\{ \frac{a^2}{32} \left(1 + \frac{\sqrt{2}a - a}{2a \sqrt{2}} \right) \right\}$$

$$7. \quad \text{Area of circle} = \frac{\pi d^2}{4} = \frac{14 - \sqrt{2}}{4} \times \frac{(\sqrt{2}a)^2}{4} \quad \text{Where diameter, } d = \sqrt{2}a$$

$$8. \quad \text{Area of AH BI} = \frac{a^2}{8} \left(1 + \frac{\sqrt{2} - 1}{2 \sqrt{2}} \right)$$

$$9. \quad \text{Area of AJOK} + \text{OFBG} = \text{Area of AHBI}$$

10. The difference between the area of the circle and the sum of the areas of four petals is equal to the area of the inscribed square.

Area of the circle - area of 4 petals = area of the square

$$\left\{ \frac{14 - \sqrt{2}}{4} \times \frac{(\sqrt{2}a)^2}{4} \right\} - 16 \left\{ \frac{a^2}{32} \left(1 + \frac{\sqrt{2} - 1}{2 \sqrt{2}} \right) \right\} = a^2$$

37. "A Pi - less AREA" OF J.V. NARLIKAR AND THE COMPUTATION OF π

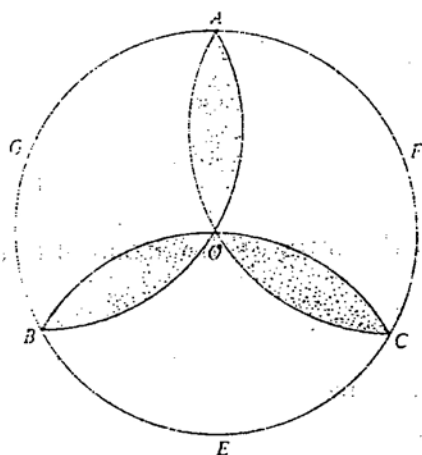
INTRODUCTION: π value has been computed from the following methods and the value obtained by every method is $(14-\sqrt{2})/4$.

I.	Direct & Indirect methods	
	1. Area and Circumference methods	= 42 methods
II.	Supporting Methods	
	2. Hippocratean crescents	= 6 methods
	3. Method enshrined in the Encyclopaedia Britannica	= 1 method
	4. Random triangles of Prof. Trevor Easingwood & Acute - angled triangles method of Prof. C.R. Fletcher	= 2 methods
	5. Circle - petal = Square method	= 1 method
		52 methods

Thus from the earlier 52 methods the value obtained is equal to $(14-\sqrt{2})/4$ for π .

Yet another supporting method is now proposed and submitted to the world of mathematics.

PROF. J.V. NARLIKAR ** of Dept. of Applied Mathematics, University College, CARDIFF, had given a formula for the area of a petal in terms of π and was published in **THE MATHEMATICAL GAZETTE** (1981, March, Page No. 32).



"The three-petal lotus shown in the figure is one which every child, learning to use the compass, is taught to draw. What is the area of the **unshaded** region within the circles although surrounded on all sides by arcs of circles, the answer is independent of π – as the simple calculation shows below.

** Present Address: Director & Homi Bhabha Professor, Inter - University Center for Astronomy and Astrophysics, Post Bag 4, Ganesh Khind, PUNE 411 007, India.

7. Formula for the area of the petal according to Prof. J.V. Narlikar is

$$\left(\frac{\pi}{3} - \frac{\sqrt{3}}{2} \right) a^2 \quad \text{where } a, \text{ is the radius } 2a, \text{ is the diameter.}$$

8. Formula for the same area of the petal **based on its parameters** proposed by the author from his earlier methods is

$$\left\{ \frac{(10 + 2\sqrt{3}) - (\sqrt{6} + 2\sqrt{2})}{12} \right\} a \left\{ \frac{2 - \sqrt{3}}{4} \right\} a$$

Where diameter = a

When the diameter = 2a the above formula is modified as shown below:

$$\left\{ \frac{(10 + 2\sqrt{3}) - (\sqrt{6} + 2\sqrt{2})}{12} \right\} 2a \left\{ \frac{2 - \sqrt{3}}{4} \right\} 2a$$

$$\text{If } \left\{ \frac{\pi}{3} - \frac{\sqrt{3}}{2} \right\} a^2 = \left\{ \frac{(10 + 2\sqrt{3}) - (\sqrt{6} + 2\sqrt{2})}{12} \right\} 2a \left\{ \frac{2 - \sqrt{3}}{4} \right\} 2a$$

$$\pi = \frac{14 - \sqrt{2}}{4}$$

The mathematicians of the world are requested to try if their $\pi = 3.1415926...$ can be obtained based on the parameters of the petal !

38. RANDOM TRIANGLES OF TREVOR EASINGWOOD AND COMPUTATION OF π

I. INTRODUCTION:

Prof. Trevor Easingwood had published an article entitled **Random Triangles** in **THE MATHEMATICAL GAZETTE** (1981, page.247) and gave a formula in terms of π i.e. $\frac{\pi}{3} - \frac{\sqrt{3}}{4}$ for the area OST shown in the following figure.

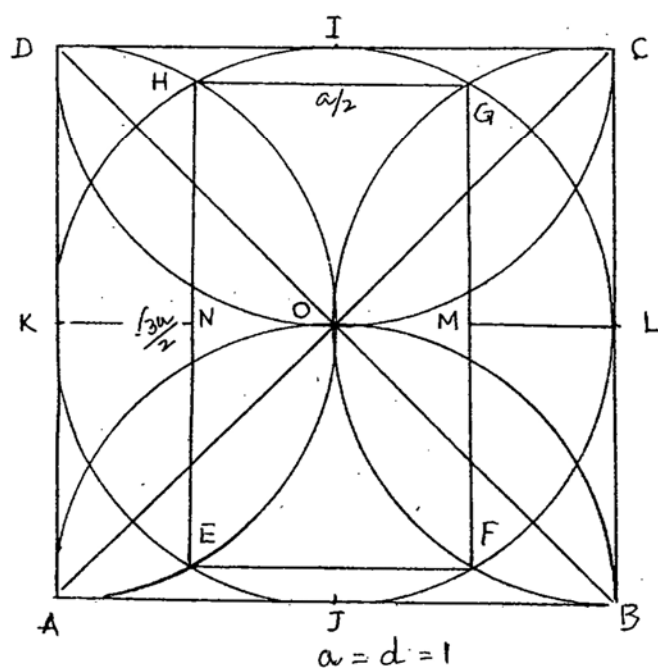


Fig - 49

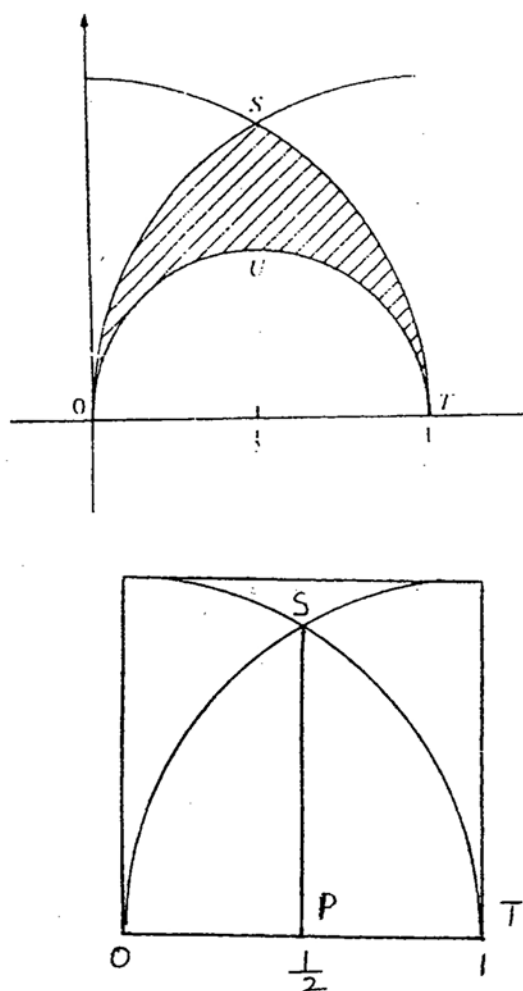


Fig - 50

The author from his earlier methods and on the guidance of the fig.49. (MGL) now proposes a formula for the same area OST **on the basis of its parameters**. For its correctness the following **proof** is given. **The sum of areas of all the constituent entities of the square is exactly equal to the area of the square** (fig.49) .

$$\text{Area of the Square ABCD} = \underbrace{\text{KDI} + \text{ICL} + \text{LBJ} + \text{JAK}}_{4 \text{ Corner Areas}} + \underbrace{\text{HIG} + \text{EFJ}}_{2 \text{ Smaller Curvature Entities}} + \underbrace{\text{MGL} + \text{MFL} + \text{EKN} + \text{HKN}}_{4 \text{ Larger Curvature Entities}} + \underbrace{\text{EFGH}}_{\text{Rectangle}}$$

$$a^2 = 4 \left\{ \frac{\frac{a^2}{8}}{8 \left(\frac{\sqrt{2}-1}{\sqrt{2}} \right)} \right\} + 2 \left\{ \frac{14 - (\sqrt{2} + 6\sqrt{3})}{96} \right\} a^2 + 4 \left\{ \left(\frac{28\sqrt{3} - (18 + 2\sqrt{6})}{288} \right) a \times \frac{\sqrt{3}a}{2} \right\} + \left\{ \frac{\sqrt{3}a}{2} \times \frac{a}{2} \right\}$$

II. Proposed Formula

Easingwood's formula for the area of OST = areas of PST + PSO of Fig. 50
the Easingwood's formula was derived with $a=2$.

$$\text{If } \frac{\pi}{3} - \frac{\sqrt{3}}{4} = 2 \left\{ \left(\frac{28\sqrt{3} - (18 + 2\sqrt{6})}{288} \right) a \times \frac{\sqrt{3}a}{2} \right\} = 2 \left\{ \left(\frac{28\sqrt{3} - (18 + 2\sqrt{6})}{288} \right) 2 \times \frac{\sqrt{3} \times 2}{2} \right\}$$

$$\text{Then } \pi = (14 - \sqrt{2})/4$$

III. Another Proof for $\pi = (14 - \sqrt{2})/4$

By subtracting the 4 corner areas from the square ABCD (fig. 49) we can get the exact area of the inscribed circle. And the ratio of the exact area of the circle to its radius squared will also give an exact value of π .

$$\pi = \frac{\text{Square} - 4 \text{ corner areas}}{\text{radius} \times \text{radius}} = \frac{\pi r^2}{r^2} = \frac{a^2 - 4 \left\{ \frac{\frac{a^2}{8}}{8 \left(\frac{\sqrt{2}-1}{\sqrt{2}} \right)} \right\}}{\left(\frac{a}{2} \right)^2} = \frac{14 - \sqrt{2}}{4}$$

IV. Easingwood's Second formula in terms of π for the area OUTS (56th Method to Compute π)

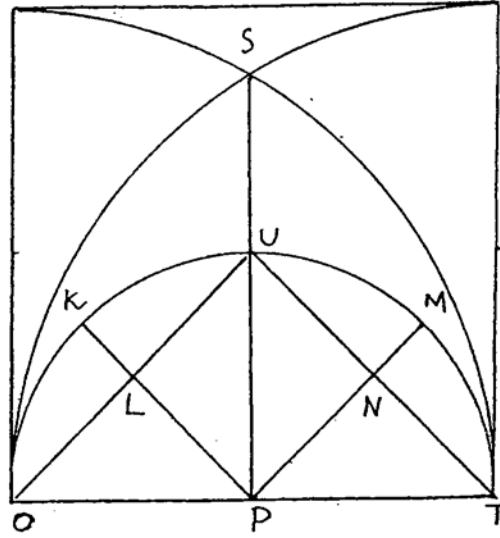


Fig - 51

$$1. OP = PT = PK = PU = PM = a/2 \quad \text{where diameter} = a$$

$$2. MN = KL = \left(\frac{\sqrt{2} - 1}{2\sqrt{2}} \right) a$$

$$3. OU = TU = \frac{\sqrt{2}a}{2}$$

$$4. \text{Area of MNT} = \text{MNU} = \text{OKL} = \text{KLU}$$

$$= \frac{a^2}{32} \left(1 + \frac{\sqrt{2} - 1}{2\sqrt{2}} \right)$$

$$5. PL = PN = \frac{\sqrt{2}a}{4}$$

$$6. \text{Area of OPU triangle} = \frac{1}{2} ab = \frac{1}{2} \times \frac{\sqrt{2}a}{2} \times \frac{\sqrt{2}a}{4} = \frac{a^2}{8}$$

$$7. \text{Area of Semicircle OKUMT} = \text{OKU} + \text{UMT} + \text{OPU} + \text{UPT}$$

$$4 \left\{ \frac{a^2}{32} \left(1 + \frac{\sqrt{2} - 1}{2\sqrt{2}} \right) \right\} + 2 \left(\frac{a^2}{8} \right) = \left(\frac{14 - \sqrt{2}}{32} \right) a^2$$

8. Area OST - area OUT = area OUTS (shaded area in the Easingwood's page -102)

$$2 \left\{ \left(\frac{28\sqrt{3} - (18 + 2\sqrt{6})}{288} \right) 2a \times \frac{\sqrt{3} \times 2a}{2} \right\} - \left(\frac{(14 - \sqrt{2})}{32} \right) a^2$$

where $a = 1$

9. Proposed formula for the area of OUTS =

$$\left\{ \frac{46\sqrt{3} - (2 + 5\sqrt{6} + 5\sqrt{2})}{192} \right\} a \left\{ \frac{\sqrt{3} - 1}{4} \right\} a$$

$$\text{where SU (Fig. 51)} = \left(\frac{\sqrt{3} - 1}{4} \right) a$$

where $a = 2$

10. Easingwood's formula for OUTS is

$$\left(\frac{\pi}{3} - \frac{\sqrt{3}}{4} \right) - \frac{\pi}{8}$$

$$\text{If } \left(\frac{\pi}{3} - \frac{\sqrt{3}}{4} \right) - \frac{\pi}{8} = \left\{ \frac{46\sqrt{3} - (2 + 5\sqrt{6} + 5\sqrt{2})}{192} \right\} a \left\{ \frac{\sqrt{3} - 1}{4} \right\} a$$

where $a = 2$

$$\text{then } \pi = \frac{14 - \sqrt{2}}{4}$$

39. A PERTINENT QUESTION ON THE π VALUE

In the computation of π so far, 56 methods have been proposed and submitted to the world of Mathematics. All are based on the parameters of the concerned entities (Present π value equal to 3.1415926.... was based on the parameters of the inscribed polygon with the circle as its limit. The method of infinite series is either totally unrelated to the parameters of the circle or if related only to the right angles such as arctan series etc.)

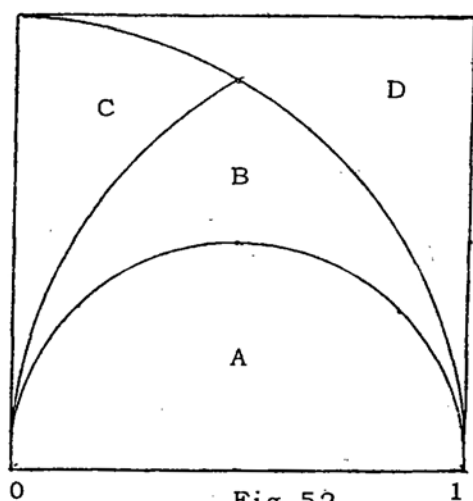


Fig-52

Prof. Trevor Easingwood (Derby) has suggested formulae for the areas of A, B & D, Prof. C.R. Fletcher (Aberystwyth) for the area of D and Prof. Stephen Ainely (Hilmarton, Calne, Wilts) for the area of C. All formulae are in terms of π . The sum of the areas of all the four constituent entities is equal to the area of the square i.e. 1.

$$\underbrace{\frac{\pi}{8}}_A + \underbrace{\left\{ \left(\frac{\pi}{3} - \frac{\sqrt{3}}{4} \right) - \frac{\pi}{8} \right\}}_B + \underbrace{\left\{ \left(\frac{\pi}{6} + \frac{\sqrt{3}}{4} \right) - \frac{\pi}{4} \right\}}_C + \underbrace{\left\{ 1 - \frac{1}{4}\pi \right\}}_D = 1$$

$$A = 4 \left\{ \frac{1}{32} \left(1 + \frac{\sqrt{2}-1}{2\sqrt{2}} \right) \right\} + 2 \left(\frac{1}{8} \right)$$

$$B = 2 \left\{ \frac{46\sqrt{3} - (2 + 5\sqrt{6} + 5\sqrt{2})}{192} \right\} - 2 \left\{ \frac{\sqrt{3}-1}{4} \right\}$$

$$C = \left(\frac{14-\sqrt{2}}{4 \times 6} + \frac{\sqrt{3}}{4} \right) - \left(\frac{14-\sqrt{2}}{4 \times 4} \right)$$

$$D = \frac{\frac{4}{8}}{8 \left(\frac{\sqrt{2}-1}{\sqrt{2}} \right)}$$

The areas of A, B & D are based on the parameters and C is based on the proposed π value as this entity is surrounded on two sides by two arcs. It is also clear that the proposed π for C tallies with the other three areas.

The areas of A,B,C, & D are calculated 1. by applying the present π value equal to 3.1415926.... and 2. by the guidance of the parameters of concerned entities and 3. with the proposed π value (for C). Though the individual values for the four entities vary between the two approaches but their total value is same i.e. 1.

Table

	A	B	C	D		TOTAL
1. As per $\pi = 3.1415926$	0.392699	+ 0.2214858	+ 0.1712133	+ 0.2146019	=	1.
2. Based on the parameters of the concerned entities	0.3933056	+ 0.2224969	+ 0.1708088	+ 0.2133884	=	0.999999
3. $\pi = (14-\sqrt{2})/4$	on simplification				=	1.

DISCUSSION

The two values do not coincide. It is a well-known fact from the time of C.L.F Lindemann that π is a transcendental number. It is an approximation. Though many decimal places were obtained based on the geometrical constructions, all were without proofs. Values obtained from geometrical constructions and infinite series were never deferred. Many methods were invented later and all leading to 3.1415926.... including Simpson's rule known for its approximation.

The geometrical constructions adopted by the author are very simple and based on the composite systems of circle, square, triangle etc. Exact area of the circle has been calculated. Hippocratean crescents were given values. We accept Hippocratean crescents for the reason that they are in accordance with the Euclid's **Elements** : The Hippocratean crescents are solved because of three reasons : 1. Parameters of crescents are taken for calculations. 2. proposed $\pi = (14-\sqrt{2})/4$ is applied 3. π is an algebraic number.

Based on the parameters of three petals of Prof. J.V. Narlikar (Cardiff, now in India) and four petals of Prof. K. Capell (Queensland 4067, Australia) and from the π -less areas, the proposed π value has been computed. We have seen in the beginning of this article that the formulae which are given for the formulae of Easingwood, Fletcher also helped in the derivation of the proposed π value.

For two thousand five hundred years the Hippocratean crescents remained unsolved though π value was known from the days of Archimedes (250 B.C.). His π value equal to 3.14 is right. For the 3rd decimal place onwards the later mathematicians were misled. Thus we have Hippocratean crescents on one hand and π number 3.1415926.. on the other hand. We can not claim both as right. The one which has no proof has to be probed further and come with crucial evidence for its continued acceptance. For the present, the proposed π value equal to $(14-\sqrt{2})/4$ has proof and is in total agreement with the crescents.

The values shown in the Table do not tally each other. **Do you accept the values based on the parameters of the concerned entity or valued obtained with an approximate π number?**

$\pi = 3.1415926...$ is cent per correct, when, if it is applied to the inscribed polygon with the circle as its limit and not to the circle.

To conclude the author requests the inquisitive mathematicians to analyse the proposed geometrical constructions and get analytical proof for it or applied the principles of higher Mathematics in the interest of finding the truth of π value.

40. 57th METHOD TO COMPUTE π

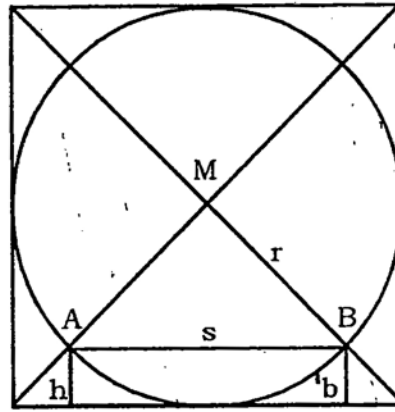


Fig-53

1. Diameter = $d = 1$, radius = $r = \frac{1}{2}$
2. The length of chord AB = $s = \frac{\sqrt{2}}{2}$
3. The height of the segment = $h = \frac{\sqrt{2} - 1}{2\sqrt{2}}$
4. The arcual length of the segment = $b = \frac{\pi d}{4}$ Quarter Circumference = $\frac{\pi}{4}$
5. The area of a segment of a circle = A

$$A = \frac{br}{2} - \frac{s(r-h)}{2} = \left\{ \frac{\frac{\pi}{4} \times \frac{1}{2}}{2} - \frac{\frac{\sqrt{2}}{2} \left(\frac{1}{2} - \frac{\sqrt{2}-1}{2\sqrt{2}} \right)}{2} \right\}$$

The area of a segment is calculated as the difference of the areas of the sector and the triangle AMB (fig.53)

b) Proposed formula based on the parameters of the segment

$$A = \frac{d^2}{16} \left\{ 1 + \left(\frac{\sqrt{2}}{2} \times \frac{\sqrt{2}-1}{2\sqrt{2}} \times \sqrt{2} \right) \right\}$$

So $a = b$, **General formula = proposed formula**

where $b = \frac{\pi d}{4} = \frac{\pi}{4}$ is not known.

$$\left\{ \frac{\frac{\pi}{4} \times \frac{1}{2}}{2} - \frac{\frac{\sqrt{2}}{2} \left(\frac{1}{2} - \frac{\sqrt{2}-1}{2\sqrt{2}} \right)}{2} \right\} = \frac{1}{16} \left\{ 1 + \left(\frac{\sqrt{2}}{2} \times \frac{\sqrt{2}-1}{2\sqrt{2}} \times \sqrt{2} \right) \right\}$$

where $d = 1$

$$\text{then } \pi = (14 - \sqrt{2})/4$$

41. SUMMARY OF π

1. $\pi = 3.141592653589793238462643 \dots$ An infinite non repeating decimal fraction.
2. For practical purposes $\pi \approx 22/7$ or 3.14 or 3.1416.
3. **Euclid** never calculated π . He did prove the theorem that if A is the area of a circle of radius r and A' is the area of the circle of radius r' then $A/r^2 = A'/r'^2$. This number A/r^2 may be called π for short ; hence we have the familiar formula $A = \pi r^2$
4. π range of Archimedes is $3 \frac{10}{71} < \pi < 3 \frac{1}{7}$
5. $\pi \epsilon p y \epsilon \tau p o s$ (= perimeter)
6. For the first time π was used for the ratio of the circumference to the diameter by **William Jones** in a publication in 1706.
7. **Euler** adopted the symbol π in 1737 (for general use)
8. **Heinrich Lambert** showed that π is irrational in 1767.
9. **C.L.F. Lindemann** showed that π is a transcendental number in 1882.
10. $b = \left\{ 2r^2 - r(4r^2 - a^2)^{1/2} \right\}^{1/2}$ is the side of the regular inscribed polygon having twice the number of sides.
11. The first scientific attempt to compute π , however, seems to be that of **Archimedes** (240 BC). The method of computing π by regular inscribed and circumscribed polygons is known as the Classical method of computing π .
12. **James Gregory** in 1667 issued his "*Vera Circuli et Hyperbolae quadratura*" in which he showed how the areas of the circle and hyperbola could be obtained in the form of infinite convergent series. He inferred that the quadrature of a circle was impossible.
13. The latest formula in the infinite series for the computation of π is :

$$\sum_{n=0}^{\infty} \left(\frac{4}{8n+1} - \frac{2}{8n+4} - \frac{1}{8n+5} - \frac{1}{8n+6} \right) \left(\frac{1}{16} \right)^n$$

14. **Dr. Kanada**, University of Tokyo calculated 206,158,430,000 decimal digits of π .
15. π can be defined as :

$$2 \int_0^1 \frac{dx}{\sqrt{1-x^2}}$$

16. In non-Euclidean geometry π also occurs in some formulas but is no longer the ratio of the circumference to the diameter of a circle, for the ratio is not a constant in non-Euclidean geometry.
17. As early as 1714 an Italian mathematician derived the formula.

$$\pi = \frac{e}{\sqrt{\log e^i \times \log e^i}}$$

18. Another surprising formula with π is $e^{i\pi} = -1$
19. π is as irrational as $\sqrt{2}$
20. **Tsu, Chung - Chih's** (480 A.D.) formula for $\pi = 355 / 113 = 3.1415929$ (from 113355)
21. **Aryabhata - 1** (b.476 AD) - his formula for π "Tenth Sloka of *Ganita*" in *Aryabhatiya* states

चतुरधिकं शतमष्टगुणं द्वाषष्टिस्तथा सहस्राणाम् ।
अयुतद्वयविष्कम्भस्यासन्नो वृत्तपरिणाहः ॥

Hundred plus four, multiplied by eight and added to sixty two thousand is the nearly approximate measure of the circumference of a circle of diameter twenty thousand.

$$\pi = \text{Circumference} / \text{diameter} = 62832 / 20000 = 3.1416$$

22. **Bhaskaracharya-II** (b.1114 AD) on nearly precise circumference of a circle of diameter 1250 units. *Lilavathi*, a treatise on arithmetic and mensuration by Bhaskaracharya-II states :

व्यासे भनन्दाग्नि हते विभक्ते ।
खबाणसूर्यैः परिधिः स सूक्ष्मः ॥

Three thousand nine hundred twenty seven multiplied by the diameter and divided by one thousand two hundred and fifty is the nearly precise circumference of the circle.

23. $\pi = 3.1415926 \dots$ has no proof. Millions of mathematicians were born and gone after Archimedes, but unfortunately no mathematician could give a single proof. However, we have 367 different proofs for the Pythagorean proposition. It is clear therefore that $\pi = 3.1415926 \dots$, its transcendency and the impossibility of squaring a circle are all **false**.

* * * *

42. SARASWATI * METHOD OF TRANSFORMATION OF A SQUARE INTO A CIRCLE.

In 800 B.C. an Indian mathematician **Baudhayana** had transformed a square into a circle approximately. The same method was taught by the later mathematicians : Apastamba, Katyana, Manava etc.

In the **Durga method of squaring a circle**, a required **side** of a square ($CD = h$) has been arrived at whose length is $\frac{1}{2} \sqrt{\pi}$. The area of the square with side equal to $\frac{1}{2} \sqrt{\pi}$ is equal to the area of a given circle. An **accurate** geometrical construction with straight edge and compass has been done for squaring a circle.

With the same method by adding one more parameter (EF) to the diagram of the Durga method we can now get a required **radius** also (FC) whose circle has an area equal to the area of the given square (Fig.2)

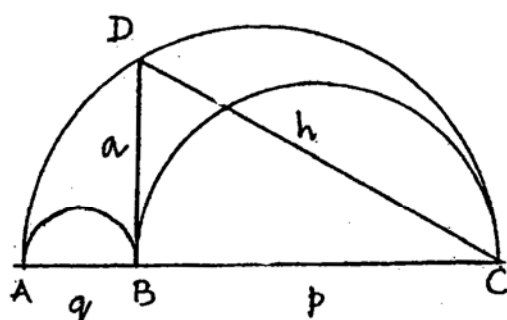


Fig - 1

Durga Method

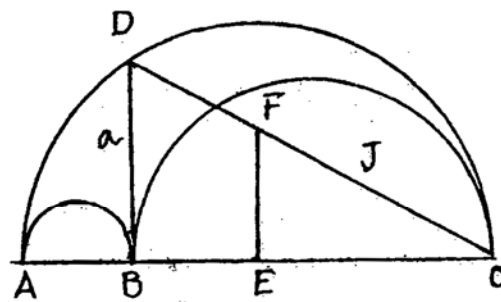


Fig - 2

Saraswathi Method

Area of Square = area of circle

$$x^2 = \pi r^2$$

$$x, \text{ side} = 1, r, \text{ radius} = \frac{1}{2}$$

$$x^2 = 1 = \pi \times \frac{1}{2} \times \frac{1}{2} = \frac{\pi}{4}$$

$$x = \sqrt{\frac{\pi}{4}} = \frac{1}{2} \sqrt{\pi}$$

Area of Circle = area of Square

$$\pi r^2 = x^2$$

$$x, \text{ side} = 1$$

$$r^2 = \frac{x^2}{\pi}$$

$$r = \sqrt{\frac{x^2}{\pi}}$$

$$\text{when } x = 1$$

$$r = \sqrt{\frac{1}{\pi}}$$

* Mother Saraswati is the goddess of Learning.

1. AC = diameter = side = 1
2. BC = p = $\frac{\pi}{4}$ (can be obtained exactly)
3. AB = q = 1 - p = $1 - \frac{\pi}{4}$
4. BD = a (on applying **Altitude theorem** we can get a)

$$a^2 = p \cdot q = \frac{\pi}{4} \left(1 - \frac{\pi}{4}\right)$$

5. On applying **Pythagorean theorem** we can get DC (=h) from the right angled triangle DBC.

$$h^2 = a^2 + p^2$$

$$h = \sqrt{a^2 + p^2}$$

$$\sqrt{\frac{\pi}{4} \left(1 - \frac{\pi}{4}\right) + \left(\frac{\pi}{4}\right)^2} = \frac{1}{2} \sqrt{\pi}$$

= required **side**

where $\pi = (14 - \sqrt{2})/4$

1. E is the mid point of AC = EC = $\frac{1}{2}$
2. BC = $\frac{\pi}{4}$
3. BE = BC - EC = $\frac{\pi}{4} - \frac{1}{2}$
4. EF is the perpendicular line to AC at E.
5. FC = J = required **radius**.

In the fig.1 h is equal to $\frac{1}{2}\sqrt{\pi}$ which is the required side of the square for a given circle. When another perpendicular line to AC is drawn at E, it meets DC at F dividing h into two parts DF and FC.

$$DC = h, \quad FC = J$$

$$\text{When } BC = \frac{\pi}{4} \quad \text{then } DC = \frac{1}{2} \sqrt{\pi}$$

Since the triangles DBC and FEC are similar we can get FC.

$$FC = \frac{EC}{BC} \times DC = \frac{\frac{1}{2}}{\frac{\pi}{4}} \times \frac{1}{2} \sqrt{\pi} = \sqrt{\frac{1}{\pi}}$$

Required **radius**, $r = \sqrt{\frac{1}{\pi}}$

$$\text{Area of the circle} = \pi r^2 = \pi \times \sqrt{\frac{1}{\pi}} \times \sqrt{\frac{1}{\pi}} = 1 \text{ Sq. Unit}$$

Thus, from the same method we can get **both** the required **side** of the square (for a given circle) as well as required **radius** of the circle (for a given square).

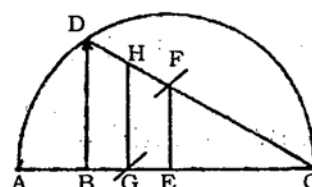
Discussion

As this method gives the **exact** area of a circle i.e. 1sq. unit from the given square of its side, 1 unit, we can say that this is possible only when π is an algebraic number. **Is it possible, if π were to be a transcendental number, as mathematicians contend, to get an area of a circle equal to 1sq. unit by calculation and / or by construction?**

4.3. LAKSHMI METHOD OF EQUIVALANCE BETWEEN A SQUARE AND A CIRCLE IN PERIMETER

The perimeter of a square with unit side is 4. If 4 is the circumference of a circle the **required radius** of it is $2/\pi$

$$2\pi r = 4, \quad r = \frac{4}{2\pi} = \frac{2}{\pi}$$



From Saraswathi diagram

1. With the centre C, draw an arc with a length equal to FC, which intersects AC at G

$$\text{So } FC = GC = \sqrt{\frac{1}{\pi}}$$

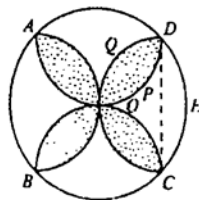
2. A perpendicular on AC at G, intersects DC at H, which is the **required radius** equal to $HC = \frac{2}{\pi}$

3. From similar triangles DBC and HGC we can obtain HC

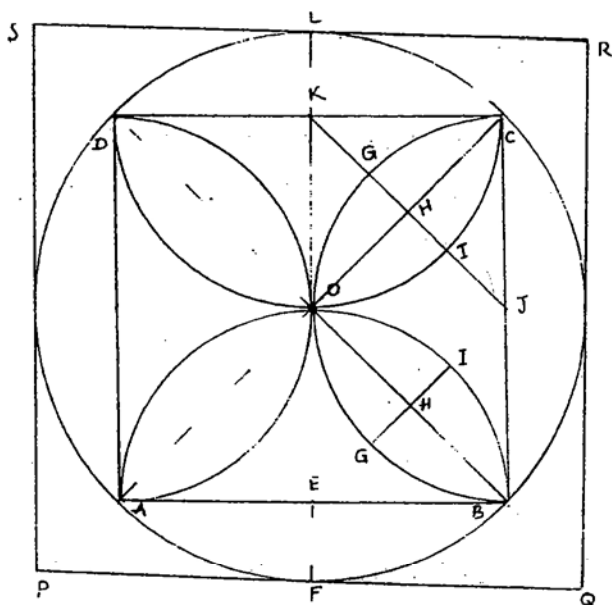
$$HC = \frac{GC}{BC} \times DC = \frac{\sqrt{\frac{1}{\pi}}}{\frac{\pi}{4}} \times \frac{1}{2} \sqrt{\pi} = \frac{2}{\pi}$$

ARITHMETICAL SUBMISSION TO THE CAPELL'S PROPOSITION

The following is the extract from the article "From π - less area to π " by Prof. K. Capell, Dept. of Mathematics, University of Queensland, St. Lucia, Queensland 4067, Australia, published in The Mathematical Gazette (1984, pages 283 & 284).



"..... that the area of the segment DCE in Fig. is equal to the area of one full petal OPDQ. It follows that the unshaded area within the circle ABCD is equal to that of the square ABCD".



Arithmetical submission

The petal OGC I has four subsegments - OGH, GHC, CHI and OHI. The segment AEBF has two subsegments AEF and BEF.

1. Side AB = diameter = $d = 1$
2. Side PQ = diameter = $D = \sqrt{2}$
3. $OH = \sqrt{2}/4$, $GH = \frac{\sqrt{2} - 1}{2\sqrt{2}}$;

$$\left\{ 1 + \left(\frac{\sqrt{2}}{4} \times \frac{\sqrt{2} - 1}{2\sqrt{2}} \times 2\sqrt{2} \right) \right\} = \frac{6 - \sqrt{2}}{4}$$

$$4. \text{ Area of subsegment OGH} = \frac{d^2}{32} \times \frac{6 - \sqrt{2}}{4}$$

$$5. \text{ Area of petal} = \frac{1}{32} \times \frac{6 - \sqrt{2}}{4} \times 4 = x$$

$$6. \text{ Area of subsegment AEF} = \frac{D^2}{32} \times \frac{6 - \sqrt{2}}{4}$$

$$7. \text{ Area of segment AEBF} = \frac{2}{32} \times \frac{6 - \sqrt{2}}{4} \times 2 = y$$

$$8. \text{ Area of circle with the diameter } D = \frac{\pi D^2}{4} = \frac{14 - \sqrt{2}}{4} \times \frac{2}{4} = z$$

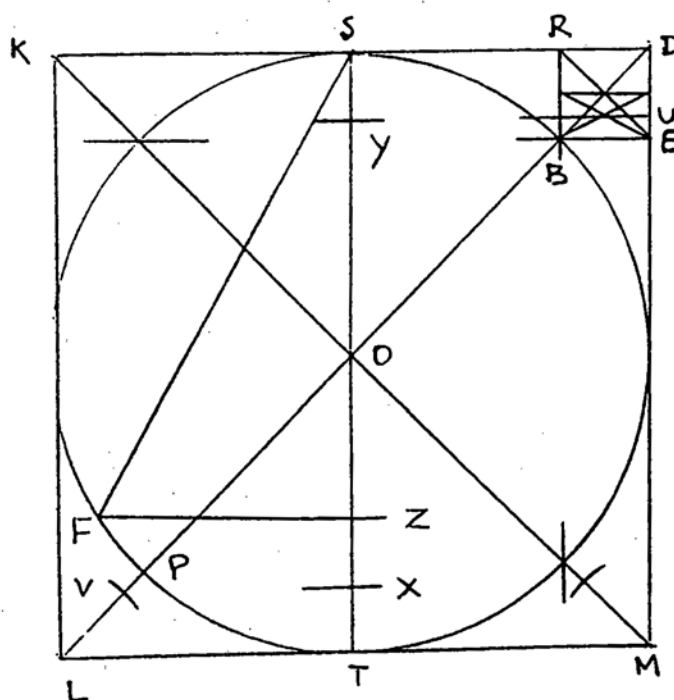
$$9. \text{ Area of the 4 petals} = \text{Area of the 4 segments} = 4x = 4y \\ = 4 \left\{ \frac{1}{32} \times \frac{6 - \sqrt{2}}{4} \times 4 \right\} = 4 \left\{ \frac{2}{32} \times \frac{6 - \sqrt{2}}{4} \times 2 \right\}$$

$$10. \text{ Area of the circle} - 4x = \text{area of the square ABCD} = 1$$

Discussion:

1. $\pi = 3.1415926$ is an approximation.
2. Professor Capell's proposition is absolutely correct.
3. To calculate the area of the circle $\pi = (14 - \sqrt{2})/4$ is adopted.
4. $\pi = 3.1415926$ does not suit the proposition of Prof. Capell.
5. Since proposed π suits the Prof. Capell's proposition, $\pi = (14 - \sqrt{2})/4$ is also absolutely correct.

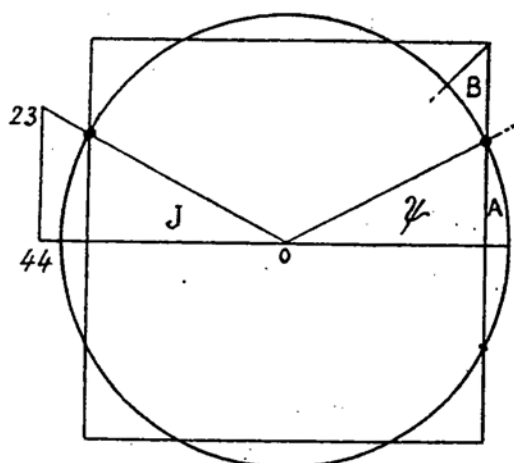
SQUARING A CIRCLE



Draw a square and two diagonals. Taking O as centre inscribe a circle. BEDR is second square. Draw two diagonals, twice, to divide DE into 4 parts. Draw a parallel line to KD, from U which intersects ST at Y. LV = DE. Draw a parallel line to LM, from V which intersects ST at X. $XY = \frac{1}{4}$ th of circumference. $XY = ZS$. Draw a perpendicular on ST at Z which intersects circumference at F. Join FS which is equal to the side of the square whose area is equal to the area of the inscribed circle.

1. $LM = PB = 1$
2. $DE = LV = \frac{1}{4}(2 - \sqrt{2})$
3. $UE = \frac{1}{16}(2 - \sqrt{2})$
4. $SY = \frac{3}{16}(2 - \sqrt{2})$
5. $XT = \frac{1}{4\sqrt{2}}(2 - \sqrt{2})$
6. $XY = ST - (SY + XT) = \frac{1}{16}(14 - \sqrt{2})$

2978. Approximate quadrature of the circle

Dec. 1961
Page 330

In the above figure the areas of the square and the circle are equal if the areas J and B are equal, and therefore if $\cos^2 \psi = \frac{\pi}{4}$; that is

$$\tan \psi = .5227232\dots;$$

but

$$23/44 = .5227272\dots$$

and so we obtain a good approximation by taking $\tan J = 23/44$.

6 Munmouth St., London W.C. 2

LOUIS LOYNES

Editorial Note

The Editor would welcome information on the history of this construction.

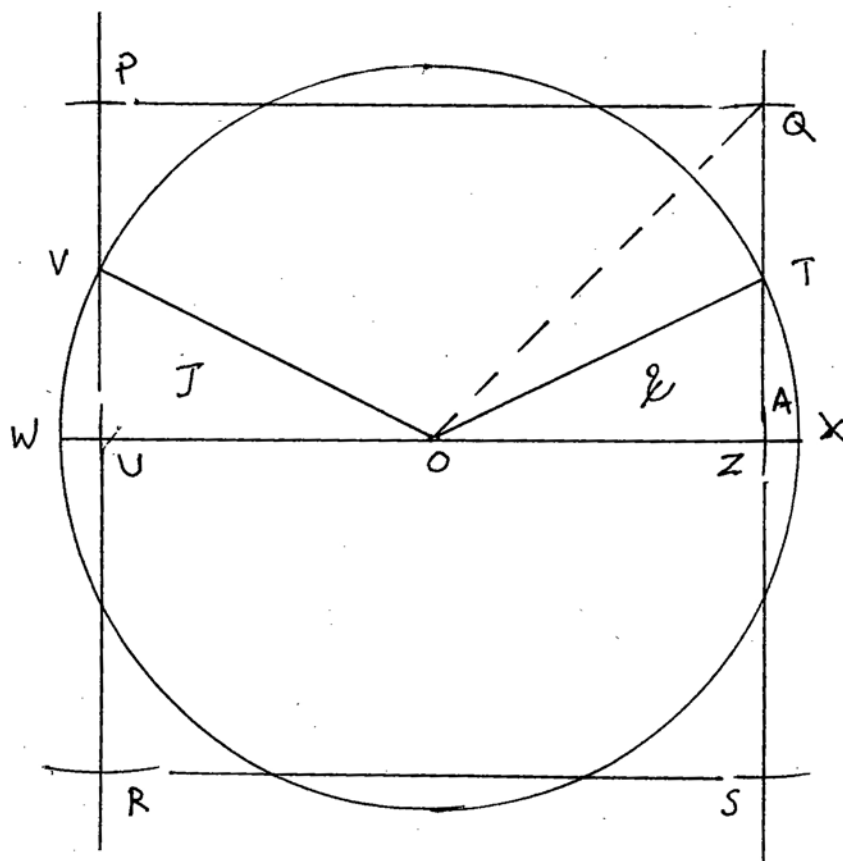


Figure 2 explained

1. PRSQ = Square, Side = RS = UZ = $\frac{1}{4}\sqrt{14-\sqrt{2}} = 0.8869113$
 2. RS of Fig.2 = FS of Fig 1.
 3. FS can be obtained applying altitude theorem.
 4. $FZ = \sqrt{(TZ)(ZS)}$, $FS = \sqrt{(FZ)^2 + (ZS)^2} = 0.8869113 = \frac{\sqrt{14-\sqrt{2}}}{4}$
 5. $OU = OZ = \frac{1}{2}UZ = \frac{1}{8}\sqrt{14-\sqrt{2}} = 0.4434556$
 6. XW = diameter of the circle = 1, OV = OT = radius = $\frac{1}{2}$
 7. $ZT = \sqrt{OT^2 - OZ^2} = 0.2309701$
 8. $\pi = \frac{1}{4}(14-\sqrt{2})$ is adopted.
 9. Area of ΔQOZ = Area of sector BOX

$$= \frac{1}{2}(OZ \times ZQ) = \frac{1}{2}\left(\frac{1}{8}\sqrt{14-\sqrt{2}}\right)^2$$

$$= \frac{45}{360} \times \frac{14-\sqrt{2}}{16}$$
- Thus the areas A and B in the Fig.2 are equal.
10. $\cos^2 \Psi = \frac{\pi}{4}$, $\cos = \left(\frac{OZ}{OT}\right)^2 = \left(\frac{\frac{1}{8}\sqrt{14-\sqrt{2}}}{\frac{1}{2}}\right)^2 = \frac{14-\sqrt{2}}{16}$
 11. $\tan \Psi = \frac{ZT}{OZ} = \frac{0.2309701}{0.4434556} = 0.5208427$
 $\tan J = \frac{UV}{OU} = \frac{0.2309701}{0.4434556} = 0.5208427$

As $\tan \psi = \tan J$, the areas of the square and the circle, are equal exactly.

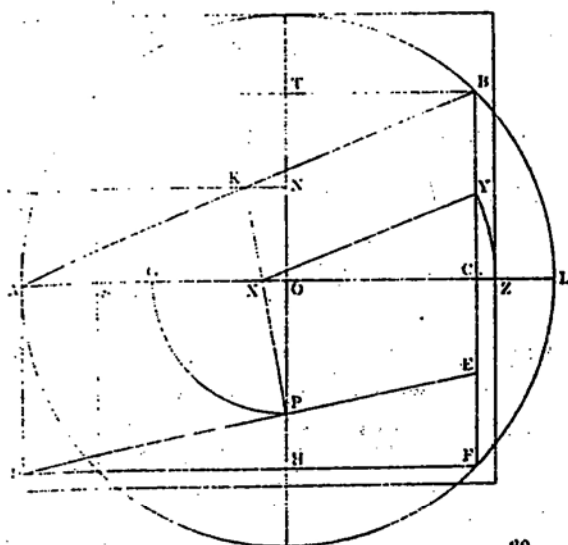
A Geometrical Construction for $\sqrt{\pi}$

1970
Feb.

MATHEMATICAL NOTES

59

3250. A geometrical look at $\sqrt{\pi}$



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THE MATHEMATICAL GAZETTE

$$SC = \sqrt{2}(1.414214) = TH$$

$$AC = 1 + \frac{\sqrt{2}}{2} (1.707107)$$

$$BC = \frac{\sqrt{2}}{2} (0.707107) = TO = OG$$

$$AB = \sqrt{(2 + \sqrt{2})} (1.847759)$$

$$KN = AO(1) - \frac{AC}{2} = \frac{2 - \sqrt{2}}{4} (0.146447)$$

$$NP = \frac{1}{2} + \frac{\sqrt{2}}{4} (0.853553)$$

$$\left(\frac{KN}{NP}\right) = 0.171573$$

$$OP(0.5) \cdot \frac{KN}{NP} = XO(0.085787)$$

$$AO(1) - XO = AX(0.914213)$$

$$OC + XO = XC(0.792891)$$

$$XY \parallel AB \left(\frac{AB}{AC} = 1.482392\right)$$

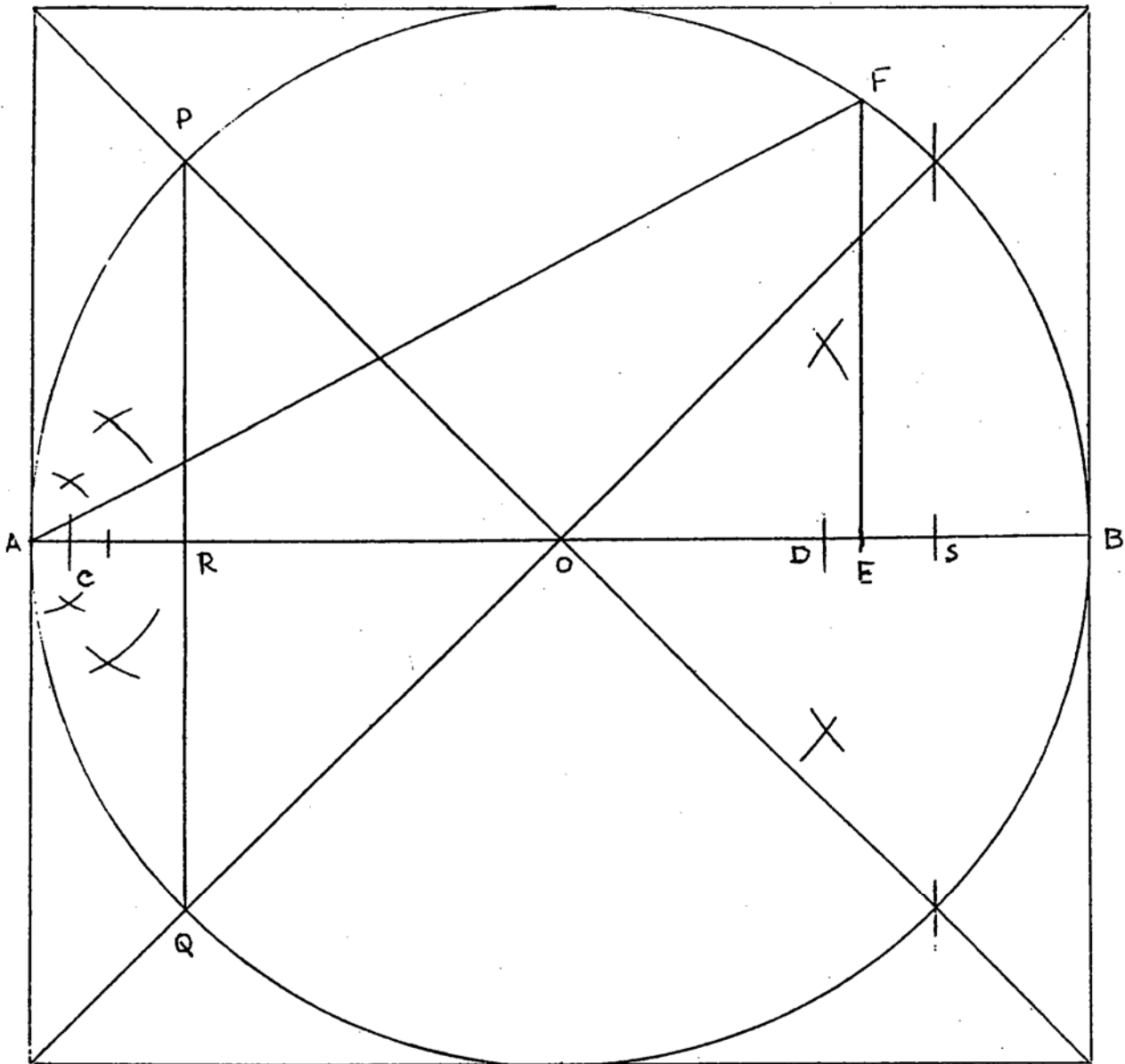
$$XC \cdot \frac{AB}{AC} = XY(0.858222) = XZ$$

$$AX + XZ = AZ(1.772435)$$

$$AZ = \sqrt{\pi} = 0.00061$$

74 Rowcayton Avenue,
Rowcayton, Conn.

CROCKETT JOHNSON



1. $AB = 2$, $PQ = RS = \sqrt{2}$, $AR = SB = \frac{1}{2} (AB - RS) = 0.2928932$,
2. Bisect AR , twice, to get $AC = 0.0732233$,
3. $AO = OB = 1$, Bisect OB to get $OD = DB = 0.5$, $AD = 1.5$,
4. $AD = CE$, $AE = AC + CE = 0.732233 + 1.5 = 1.5732233 = \frac{\pi}{2}$
5. $EB = AB - AE = 2 - 1.5732233 = 0.4267767$,
6. Draw a perpendicular to AB at E which intersects circumference at F . Apply altitude theorem to get EF .
 $EF = \sqrt{AE \times EB} = 0.8193991$
7. Join FA . $FA = \sqrt{(AE)^2 + (EF)^2} = 1.7738225 = \sqrt{\pi}$

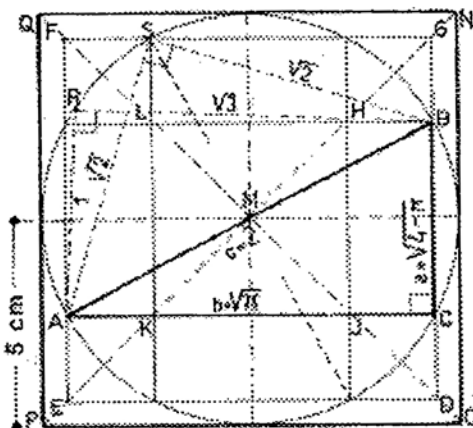
Dr.-Ing. Helmut Sander
 Hauffstrasse 28
 34125 Kassel, Germany

German version 

The purely geometrical squaring of the circle with straightedge and compass is possible only within the tolerance of an approximation. But knowing the value of the irrational number π of the circle ($\pi = 3,14159265 \dots$), it is possible to transform it as a *line* or rather as a *shape* of a circle or a square. The number of the shape of a circle is πr^2 , the number of the shape of the square is $(\text{root-}\pi r^2)^2 = (r[\text{root-}\pi])^2$. If the radius of the circle is $r = 1$, the shape of the circle and the shape of the square DEFG **equal** π . In the following illustration (Figure 1), the *line of the unit* is 5 cm. The square of the unit therefore amounts $5^2 = 25 \text{ cm}^2$. The larger square NOPQ goes up to $2^2 = 4$ units (or $10^2 = 100 \text{ cm}^2$). This means that the shapes of circle ($= \pi$) and square ($= \pi$) are smaller than this larger square.

The shape of the square DEFG $= \pi$ observes the Pythagorean theorem $a^2 + b^2 = c^2$ for right triangles, because the two shapes (square DEFG and the surrounding shapes) add up to the square-shape NOPQ. The formula is:

$$[5(\text{root-}\pi)]^2 + [5(\text{root-}4 - \pi)]^2 = 25 (\pi + 4 - \pi) = 100.$$



The inner circle of the larger square includes the right triangle:

$$ABC = a:b:c = \text{root}(4 - \pi) : \text{root}(\pi : 2).$$

The Pythagorean theorem confirms the equation $(4 - \pi) + \pi = 4$, and is also applicable to "irrational" right triangles. If you move the point C on its semi-circle between A and B, then there are demonstrated *ad infinitum* all possible right triangles. The sum of the squared catheti in every case equals 4 (for instance the triangle ABS: $2(\text{root-}2)^2 = 4$; the triangle ABR: $1^2 + (\text{root-}3)^2 = 4$), because *all* plane triangles are touching a revolving circle, their forms are represented in the picture of the squaring of the circle. It is a really "cosmological arrangement", because we are able to project the optical vaulted sky on to a planar celestial chart. On the other hand the circle with the

A geometrical ensemble to generate the squaring of the circle by Helmut Sander for the NN.

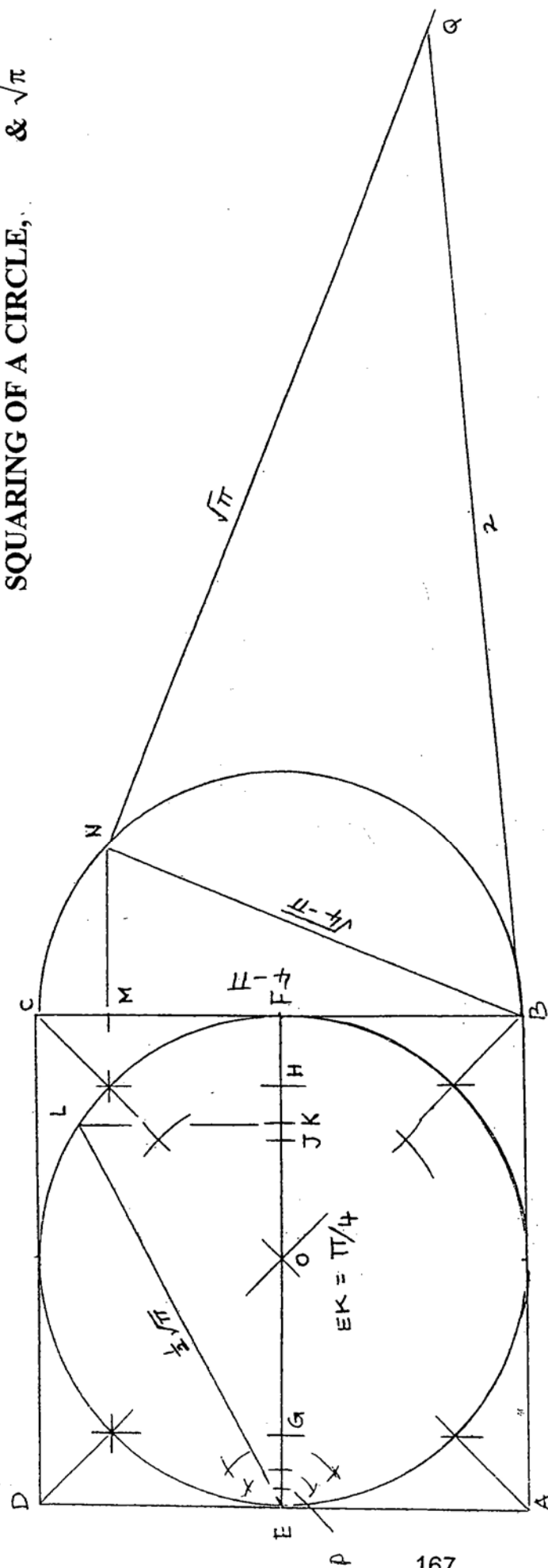
radius 1 is the inner circle of a virtual right triangle with the sides 3:4:5. We have a given shape of the square in our design of 100, and in so far we could double this proportion 3:4:5 to 6:8:10 and integrate it into the circle: $6^2 + 8^2 = 10^2$ ($36 + 64 = 100$).

Here we are able to identify a connection to the infinite system of the so-called "primitive Pythagorean triples", which include all *natural* and all *positive* real numbers.

The many various imaginable triangles in the circle like ABC can be put together (as a pair; two identical axes) either to symmetrical rectangles or to deltoids (one identical axis), if you superimpose the triangle ABC over the axis AB. Rectangles and deltoids belong to the symmetrical polygons of the circle. The shapes of the polygons, ACBR and ACBS for instance, are nonsymmetrical. Two of the equal rectangles can be put together (by rotation of 90°) to form the arms of a Greek Cross. Its outer corners are the polygonal points of an inscribed symmetrical octagon on the circle with the radius 1. Octagons have a centered symmetry with eight axes. The shape of the inner square of the cross HJKL equals the shape of the slim frame between NOPQ and DEFG. In consequence of this, the shape with the value π is equal to the shape of the larger frame between NOPQ and HJKL. This relationship is valid in every case. Greek Crosses make it easier to make out the relationships of many shapes and numbers within all plane triangles, also between other polygons and concerning the squaring of the circle. The most important aids (for the squaring of the circle) are the *Strahlensätze* (intercept theorems) and the evident Pythagorean theorem for right triangles. These aids are valid with regards to all plane triangles, including the famous irrational "Fermat triangles" (Pierre de Fermat 1601 - 1665). This hypothesis provided geometrical explanations in correspondence to the theory of numbers. A special case is the centered symmetrical hexagon (Star of King David) with the sides equal to 1, the most simple Fermat-equation $1^n + 1^n = 2$ and the equilateral triangle, the so-called "Eye of God".

Translated by Gert Sperling

SQUARING OF A CIRCLE, & $\sqrt{\pi}$



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1. ABCD = Square, $AB = EF = BC = 1$
2. Inscribe a circle
3. $GH = \frac{\sqrt{2}}{2}$, $EG = HF = \frac{2 - \sqrt{2}}{4}$
4. Bisect EG twice. $EP = \frac{2 - \sqrt{2}}{16}$
5. $EO = OF = \frac{1}{2}$, Bisect OF, $OJ = JF = \frac{1}{4}$
6. $EJ = EO + OJ = \frac{3}{4}$
7. $EJ = PK$, $EK = EP + PK = \frac{14 - \sqrt{2}}{16} = \frac{\pi}{4}$
8. $KF = EF - EK = \frac{2 + \sqrt{2}}{16}$
9. Draw perpendicular to EF at K which intersects circumference at L
10. Apply altitude theorem to get $KL = \frac{\sqrt{26 + 12\sqrt{2}}}{16}$ Join EL.
11. Apply Pythagorean theorem to get $EL = \frac{\sqrt{14 - \sqrt{2}}}{4}$ = side of the square whose area is equal to the area of the inscribed circle.
12. $EG = CM = \frac{2 - \sqrt{2}}{4}$. $BM = BC - CM = \frac{2 + \sqrt{2}}{4}$
13. Draw semicircle on BC. Apply altitude theorem to get $MN = \frac{\sqrt{2}}{4}$
14. Join BN = $\frac{\sqrt{2 + \sqrt{2}}}{2}$. Draw perpendicular to BN at N. $BQ = 2$
15. $NQ = \sqrt{\pi} = \sqrt{\frac{14 - \sqrt{2}}{4}}$

A NON-CONVENTIONAL FORMULA TO CALCULATE THE AREA OF A CIRCLE

(First article of this research work)

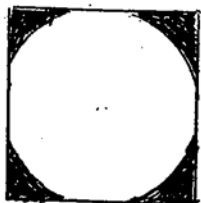
ABSTRACT

Pi and K are two different constants. To derive K, two methods were adopted. In the first method, a square and an inscribed circle were drawn on a paper, cut and weighed. The ratio between the two Q/C is K, 1.2712921... In the second method, caps of containers were chosen and encircled with a thread and measured of their lengths. The ratio between the perimeters of square and circle Scf/Ccf is K, 1.2727253... Formulae to calculate the area $\left(\frac{4}{k}r^2\right)$ and the circumference $\left(\frac{8}{k}r\right)$ of a circle and the surface area $\left(\frac{16}{k}r^2\right)$ and the volume $\left(\frac{16}{3k}r^3\right)$ of a sphere were proposed. As $\frac{4}{k}$ is π the classical constant π too can be approximately by experimental methods and the results are 3.1464051... and 3.142862...

A traditional formula to calculate the area of a circle is πr^2 where π is 3.1415926536..., r is the radius of a circle.

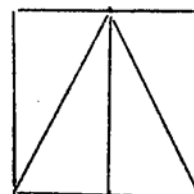
It has been thought for some time to find out a different formula with the known relationship between the area of a square and the area of a circle.

It is a fact that there is one similarity between a square and the inscribed circle. The sides of a square and the diameter of a circle are same. Hence, a relationship constant, k is calculated between the two¹.



The area of a square = Q
(Quadrilateral)
The area of a circle = C
 $Q/C = k$ which is constant

¹ It is presumed by the author that such a relationship between the inscribed triangle in the square and square might have helped the early mathematicians in the arrival of the formula to calculate the area of a triangle as $\frac{1}{2} ab$



The areas of a square a circle can be calculated using the conventional formulae a^2 and πr^2 respectively. The value thus obtained for k is 1.2732395...

To arrive at the value of k the author has adopted two different methods – a physical method and a mathematical method. In the first method the part of the paper that is square and the part of the paper that is circle are weighed in a chemical balance. By this method the value for k is 1.2712921 (accurate weighing and selection of paper made of uniform density may give k value much more nearer to 1.2732395...)

Two kinds of paper are chosen for weighing and the values are given below.

Table I

Sl. No.	Paper used	Weight of the area of square (Q) grams	Weight of the area of circle (C) grams	Q/C = k	Average k value
A. CARDBOARD					
1.	17.1 cm	3.101	2.435	1.2735112	
2.	24.7 cm	6.531	5.105	1.2793339	
B. WHITE PAPER					
3.	21.6 cm	2.759	2.154	1.2808727	
4.	10.6 cm	0.647	0.517	1.2514506	1.2712921

In the second method, to calculate the value of k four caps of containers of different sizes are chosen. The caps are encircled with a thread touching the rim/base of them. Such threads are straightened and measured of their lengths. It is repeated three times and average taken. The k values are calculated and are detailed in the Table below. The average k value thus obtained is 1.2727253 which may be much more nearer to 1.2732395... if correct measurements of the circumference are taken.

Table II

Sl. No.	Sides of a Square/ diameter of a circle (cms)	Circumference of the circle (cms) (Ccf)				Perimeter of the square (cms)(Scf)	$k = \frac{Scf}{Ccf}$
		1	2	3	Average		
1.	12.6	38.7	38.7	38.8	38.733	50.4	1.3012160
2.	9.7	30.8	30.8	30.7	30.766	38.8	1.2611324
3.	7.0	22.1	21.9	22.1	22.033	28.0	1.2708210
4.	6.1	19.3	19.4	19.5	19.400	24.4	1.2577319
					Average k value =		1.2727253

Scf = Square Perimeter, Ccf = Circle Circumference

Using the value of k one can calculate the area of a circle.

The sides of a square (a)

The area of a square (Q)

The diameter of circle (d)

The radius of a circle (r)

The are of a circle (C)

The relative constancy between Q and C (K)

When **a** is equal to **d**

the formula proposed for calculation of the area of a circle is $C = \frac{d^2}{k}$

When $d = 2r$

the formula can be rewritten as $C = \frac{4}{k} \times r^2$

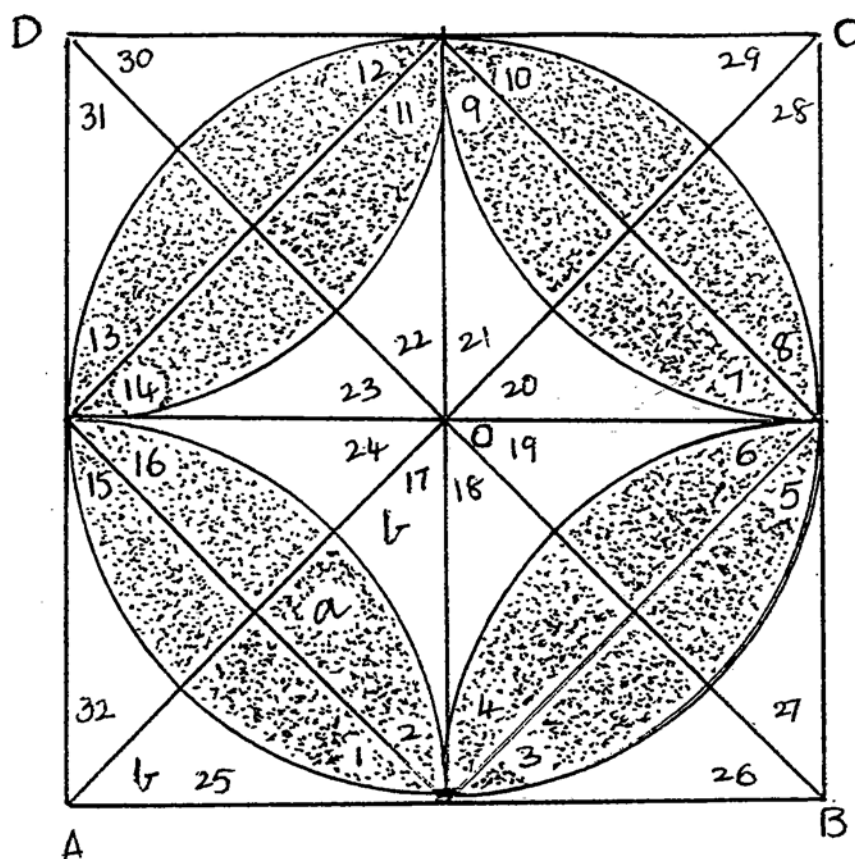
Similarly, the formula proposed for the calculation of circumference of

a circle is $\frac{8}{k} \times r$ or $\frac{4d}{k}$

On verification it has been found that $\frac{4}{k} = \pi$

The relationship constant is 1.9098592... when the volumes and surface areas of a cube and a sphere are taken into consideration.

SEGMENTAL AREAS IN THE SQUARE



ABCD = Square AB = 1

Inscribe a circle. Diameter = 1, Radius = $\frac{1}{2}$. Divide the circle-square system into 32 segments. 1 to 16 are 'a' segments. 17 to 32 are 'b' segments.

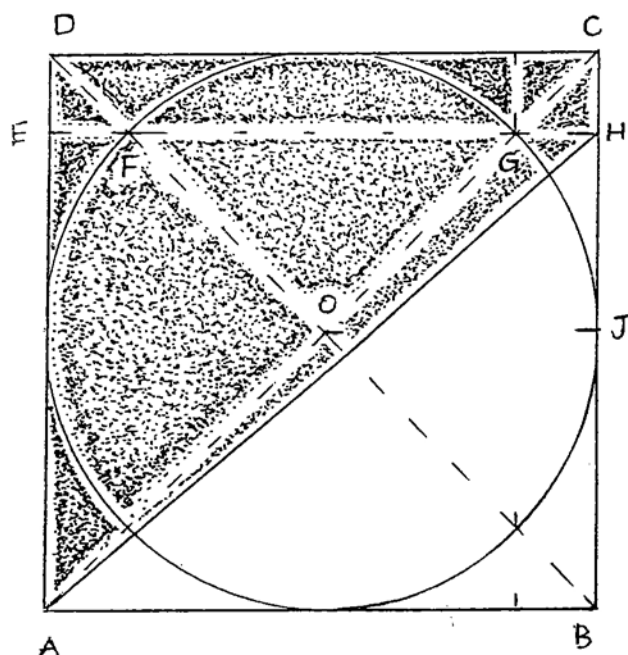
$$\text{Area of segment a} = \frac{6 - \sqrt{2}}{128} = 0.035826456....$$

$$\text{Area of segment b} = \frac{2 + \sqrt{2}}{128} = 0.026673543...$$

$$\text{Area of the square} = 16a + 16b = 1$$

$$\text{Area of the circle} = 16a + 8b = 0.7866116.....$$

$$\text{Pi} = \frac{\text{Area of circle}}{\text{radius} \times \text{radius}} = \frac{1}{4}(14 - \sqrt{2}) = 3.1464466$$



ABCD = Square

AB = 1

Draw two diagonals. Inscribe a circle Diameter = 1

OF = OG = radius = BJ = JC = 0.5

ΔFOG , FG = Hypotenuse

$$= \frac{\sqrt{2}}{2} = 0.7071067...$$

EF = GH

$$= \frac{2 - \sqrt{2}}{4} = 0.1464466...$$

Fig.2

$$EF + FG + GH = \frac{2 - \sqrt{2}}{4} + \frac{\sqrt{2}}{2} + \frac{2 - \sqrt{2}}{4} = 1$$

$$GH = CH = 0.1464466 \dots$$

$$HJ = \frac{\sqrt{2}}{4} = 0.3535533 \dots$$

Area of the square = 1

Area of ΔHAB , AB = 1, HB = BJ + HJ = 0.8535533.....

$$\frac{1}{2}(AB \times HB) = 0.426776695 \dots$$

Area of ADCH = Square - area of ΔHAB = 0.5732233.....

Area of 16 a's in Fig.1 = 0.5732233..... = Area of ADCH

Area of 16 b's in Fig.1 = 0.4267766..... = Area of ΔHAB

The areas of segments 'a' and 'b' thus calculated are correct. Hence π

derived from the Siva method is equal to $\frac{1}{4}(14 - \sqrt{2})$ is also correct.

Area of ΔHKB

$$KB = AK = 0.5$$

$$HB = BJ + HJ = 0.8535533..... = \frac{2+\sqrt{2}}{4}$$

$$\text{Area} = \frac{1}{2}(KB \times HB) = 0.2133883..... = \frac{2+\sqrt{2}}{16}$$

Area of KADCH = Area of the square – area of ΔHKB

$$1 - 0.2133883..... = 0.7866116.....$$

Area of the inscribed circle = πr^2

If $\pi = 3.1415926.....$, area of circle = 0.7853981.....

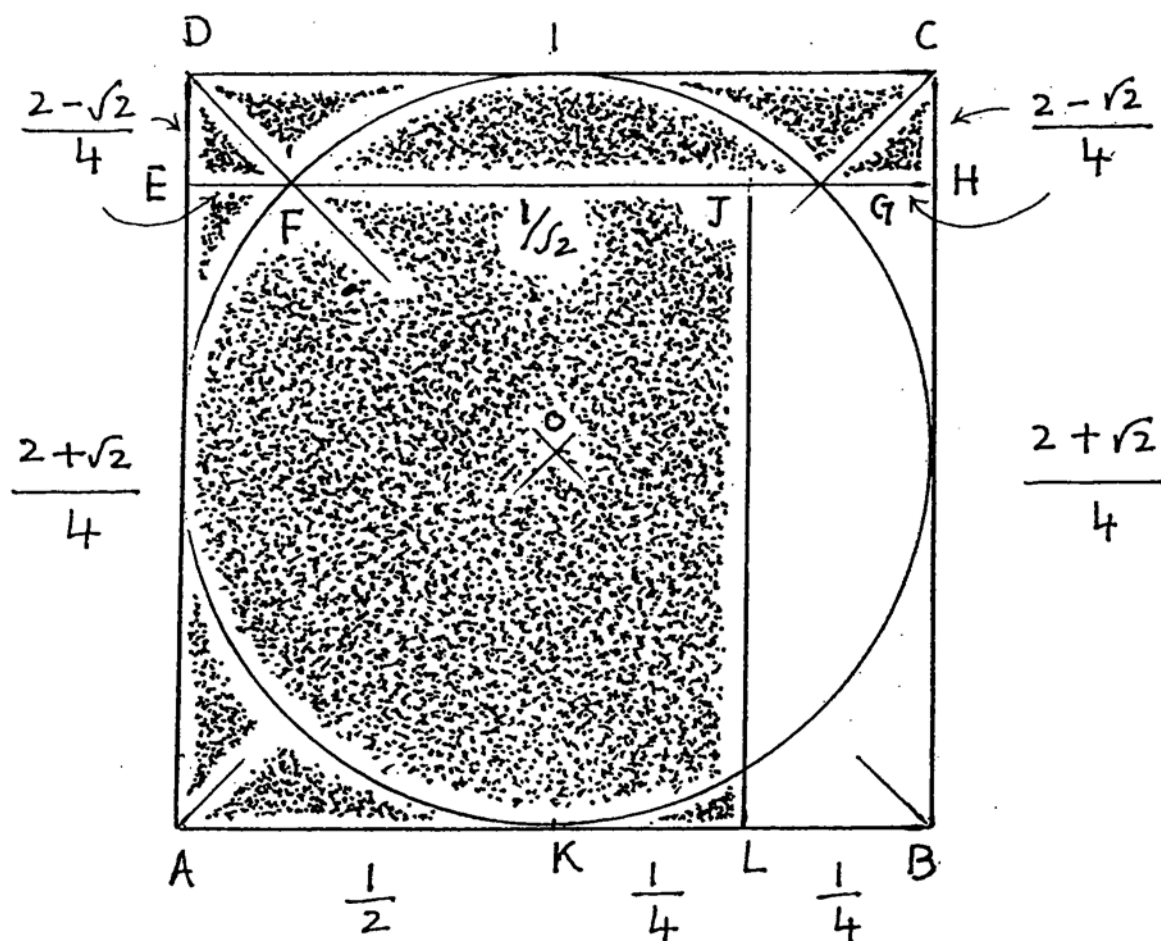
$$\begin{aligned}\pi &= \frac{1}{4}(14 - \sqrt{2}) = 3.1464466....., \text{ area of circle} = 0.7866116..... \\ &= \frac{1}{16}(14 - \sqrt{2})\end{aligned}$$

EH, a parallel line to DC is created on account of the intersection of circumference with the two diagonals. Anything that is created subsequently using 'H' point will similarly establish right relationship between circle and square. Area of KADCH is in total agreement with that of the circle whose area is computed with the proposed π . Hence

$\frac{1}{4}(14 - \sqrt{2})$ is true π value.

$$\text{Circumference} = \text{BADCH} = 1+1+1+ \frac{2-\sqrt{2}}{4} = \frac{14-\sqrt{2}}{4}$$

CIRCLE IN RECTANGLES



$$1. \text{ Area of JLBH} = \frac{1}{4} \left(\frac{2+\sqrt{2}}{4} \right) = \frac{2+\sqrt{2}}{16}$$

$$2. \text{ Area of CDEH} = 1 \left(\frac{2-\sqrt{2}}{4} \right) = \frac{2-\sqrt{2}}{4}$$

$$3. \text{ Area of JEAL} = \frac{3}{4} \left(\frac{2+\sqrt{2}}{4} \right) = \frac{6+3\sqrt{2}}{16}$$

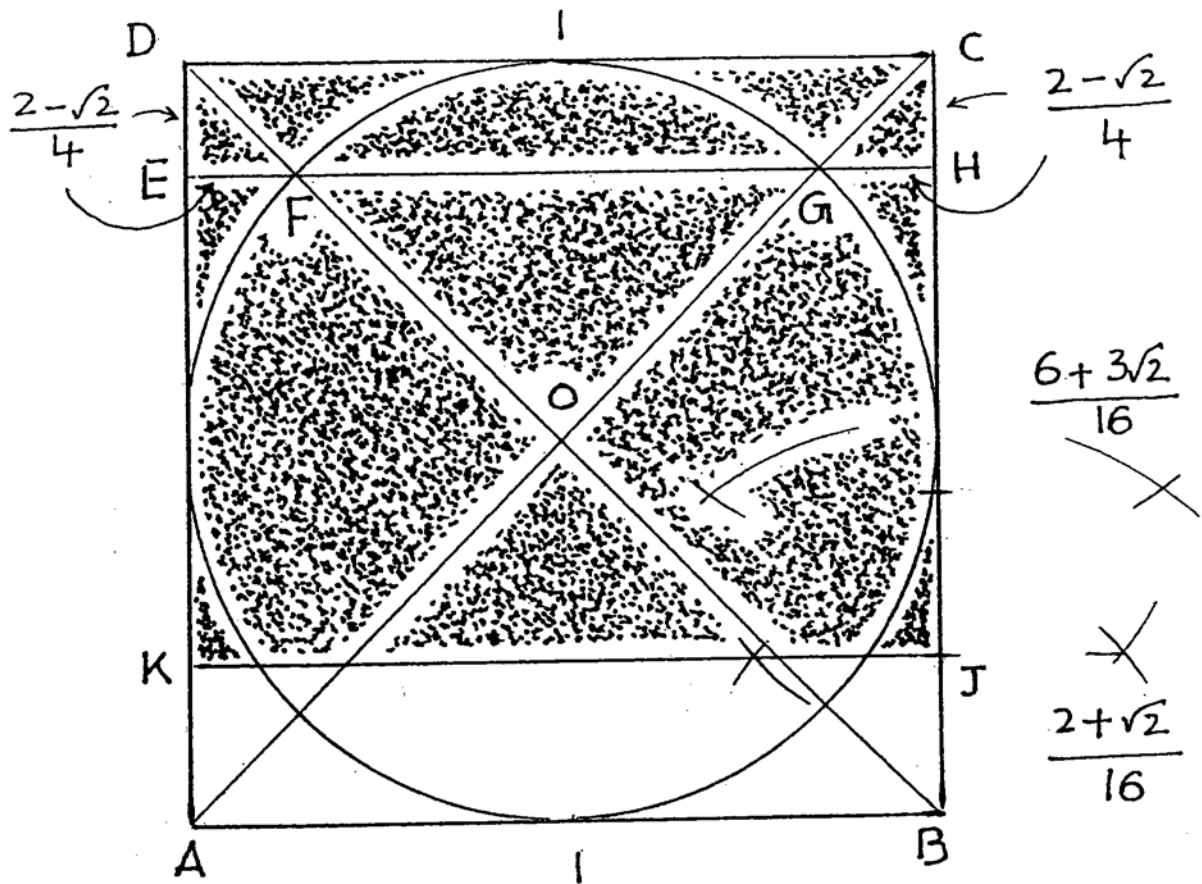
$$4. \text{ Area of ABCD} = 1$$

$$5. \text{ Area of 2 + 3 = area of circle} = \frac{2-\sqrt{2}}{4} + \frac{6+3\sqrt{2}}{16} = \frac{14-\sqrt{2}}{16}$$

$$\text{Side} = \text{diameter} = 1$$

$$\text{So } \pi = \text{area} / \text{radius} \times \text{radius} = \frac{14-\sqrt{2}}{4}$$

CIRCLE IN RECTANGLE



$$HB = \frac{2+\sqrt{2}}{4}$$

Bisect HB twice

$$\text{Area of KJCD} = 1 \left(\frac{14-\sqrt{2}}{16} \right) = \frac{14-\sqrt{2}}{16}$$

$$\text{Area of KABJ} = 1 \left(\frac{2+\sqrt{2}}{16} \right) = \frac{2+\sqrt{2}}{16}$$

$$\text{Area of ABCD} = 1$$

$$\text{Side} = \text{diameter} = 1$$

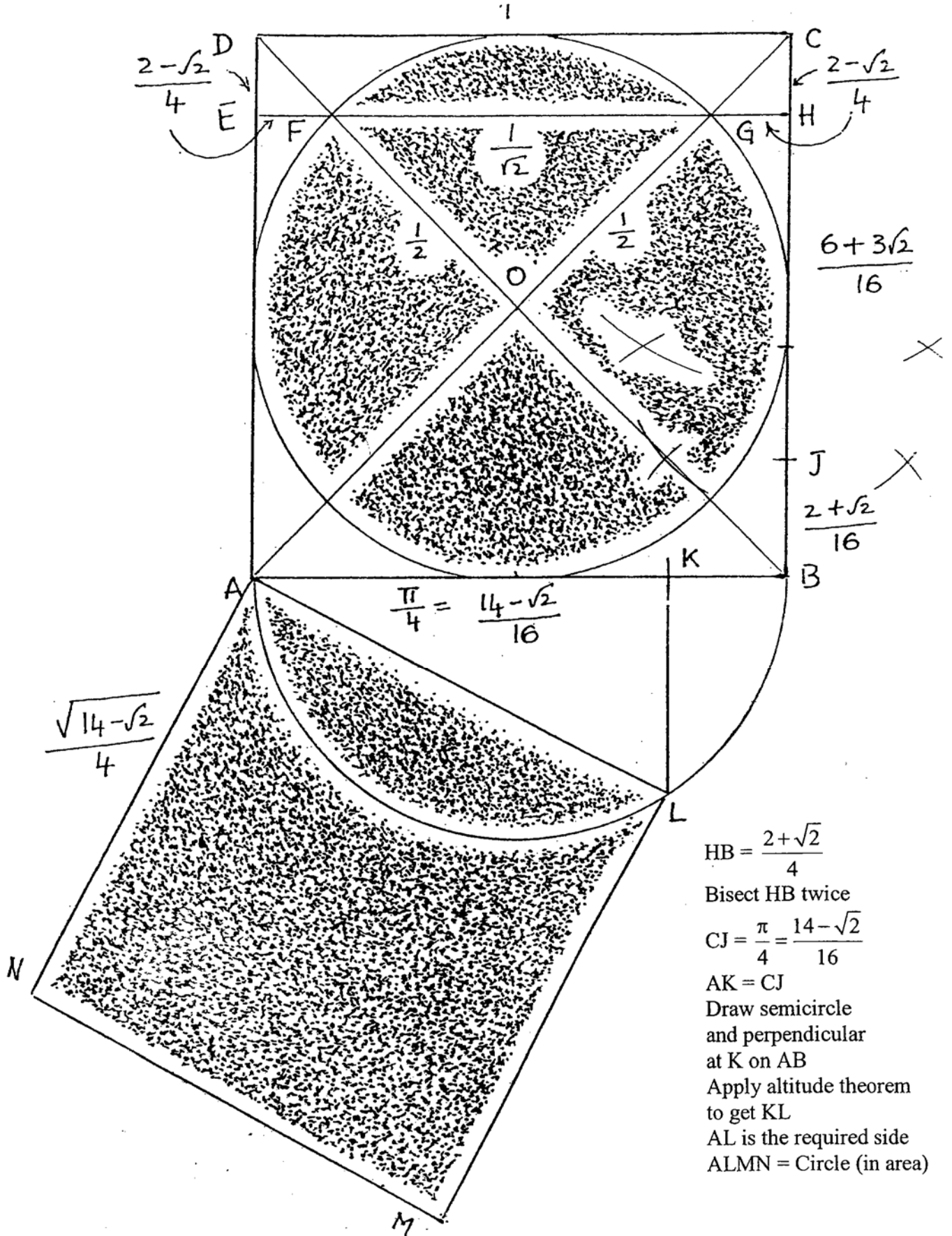
$$\text{If } \pi = 3.1415926 \dots \text{Area} = 0.78539815 \dots$$

$$\pi = 3.1464466 \dots \text{Area} = 0.7866116 \dots$$

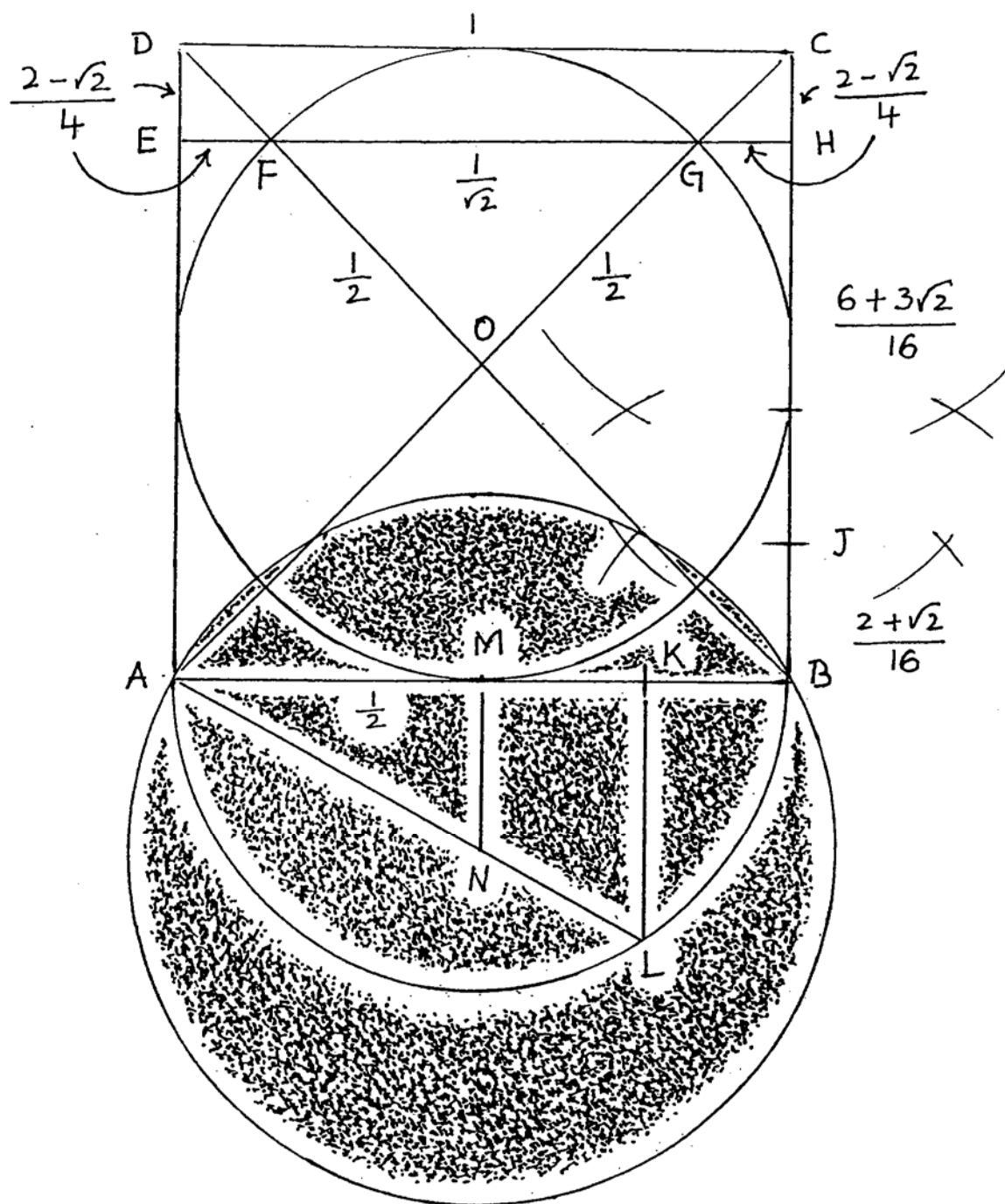
$$= \frac{1}{16} (14-\sqrt{2})$$

$\therefore$ Area of KJCD = area of circle

SQUARING OF CIRCLE



CIRCLE OF UNIT AREA



$$HB = \frac{2 + \sqrt{2}}{4}$$

Bisect HB twice.

$$CJ = \frac{\pi}{4} = \frac{14 - \sqrt{2}}{16}$$

$$AK = CJ$$

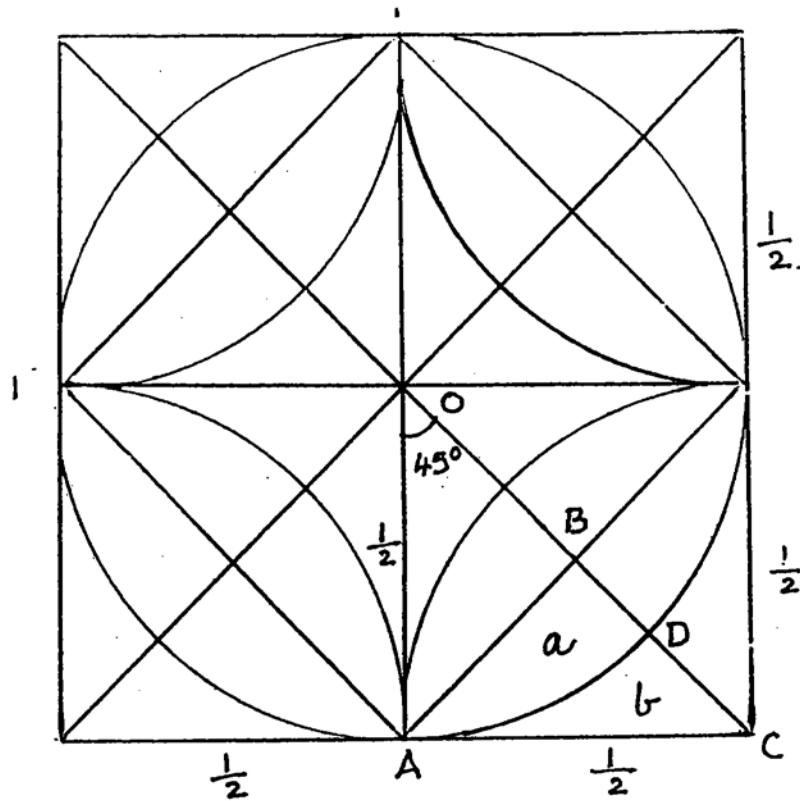
Draw semicircle and perpendicular at K on AB. Apply altitude theorem to get KL.

Apply Pythagorean theorem to get AL. $AL = \frac{\sqrt{14 - \sqrt{2}}}{4}$. $AB = 1$. $AM = MB = \frac{1}{2}$.

Draw perpendicular at M on AB. Apply the concept of similar triangles to get AN

which is the required radius (equal to $\sqrt{\frac{1}{\pi}} = \sqrt{\frac{4}{14 - \sqrt{2}}}$) of the circle whose area is 1.

COMPUTATION OF SEGMENTAL AREAS (an Elementary Approach)



$$1. \quad BD = \frac{2 - \sqrt{2}}{4}$$

$$CD = \frac{\sqrt{2} - 1}{2}$$

$$AB = OB = \frac{\sqrt{2}}{4}$$

$$2. \quad \text{Area of } \triangle ABC = \frac{1}{16}$$

$$3. \quad \text{Area of } a + b = \frac{1}{16}$$

$$4. \quad \text{Area of } \triangle OAC = \frac{1}{8}$$

$$\triangle OAB = \frac{1}{16}$$

$$\text{Sector } OAD \approx \frac{1}{10}$$

$$5. \quad \text{Area of segment 'a'} = OAD - OAB = \frac{3}{80} = 0.0375$$

$$\text{Area of segment 'b'} = OAC - OAD = \frac{1}{40} = 0.025$$

$$6. \quad \text{'a' is larger than 'b'. So, 'a' is greater than half of } \triangle ABC \left(\frac{1}{16} \right).$$

$$\text{'b' is smaller than 'a'. So, 'b' is lesser than half of } \triangle ABC \left(\frac{1}{16} \right).$$

$$7. \quad \text{Half of } \triangle ABC = \frac{1}{32} = y$$

$$8. \quad \text{Let } a - y = s, \quad y - b = t; \quad s = t$$

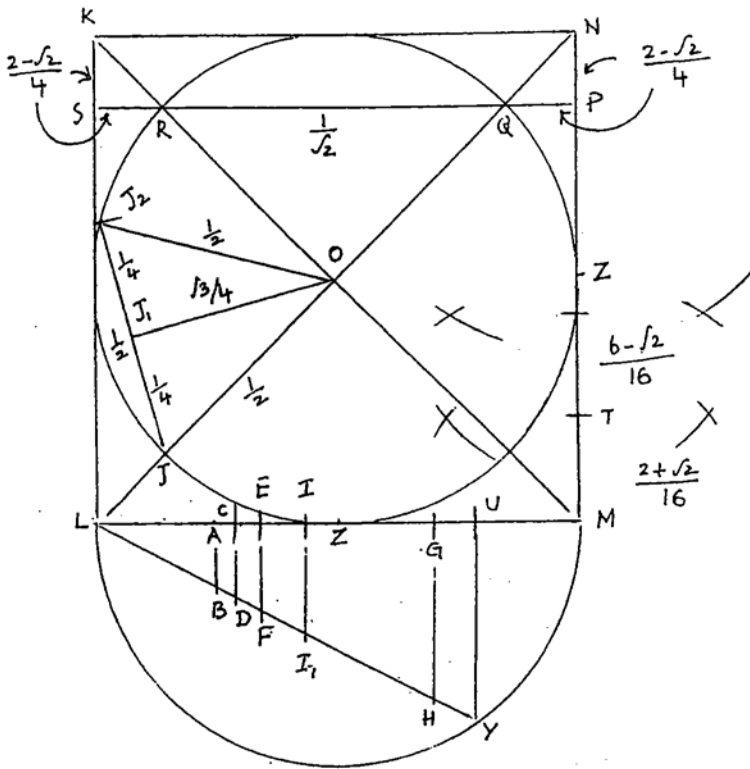
$$9. \quad \text{Let us assume } s = \frac{BD \times AC}{16} = \frac{2 - \sqrt{2}}{128}; \quad t = \frac{CD \times AB}{16} = \frac{2 - \sqrt{2}}{128}$$

$$10. \quad a - y = s; \quad a = s + y = \frac{6 - \sqrt{2}}{128}$$

$$11. \quad y - b = t; \quad b = y - t = \frac{2 + \sqrt{2}}{128}$$

$$12. \quad \therefore a + b = \triangle ABC$$

CIRCLE OF DIFFERENT DIMENSIONS



$$LM = 1, LZ = \frac{1}{2}, LA = \frac{1}{4}$$

$$NT = LU = \frac{1}{16}(14 - \sqrt{2})$$

$$LY = \frac{\sqrt{14 - \sqrt{2}}}{4}$$

Apply similar triangles
(AST) to get LB

$$LB = \frac{LA}{LU} \times LY$$

$$= 0.281877135.....$$

= required radius to

1. Surface area of sphere = 1

$$LC = LB$$

$$LD = \frac{LC}{LU} \times LY = 0.317818842.... = \text{required radius}$$

2. Circumference = 2

$$PZ = \frac{\sqrt{2}}{4}; PZ = LE$$

$$LF = \frac{LE}{LU} \times LY = 0.398634321.... = \text{required radius}$$

3. Area of the circle = $\frac{1}{2}$

$$LO = \frac{\sqrt{2}}{2}; LG = LO; LH = \frac{LG}{LU} \times LY = 0.797268756.... = \text{required radius}$$

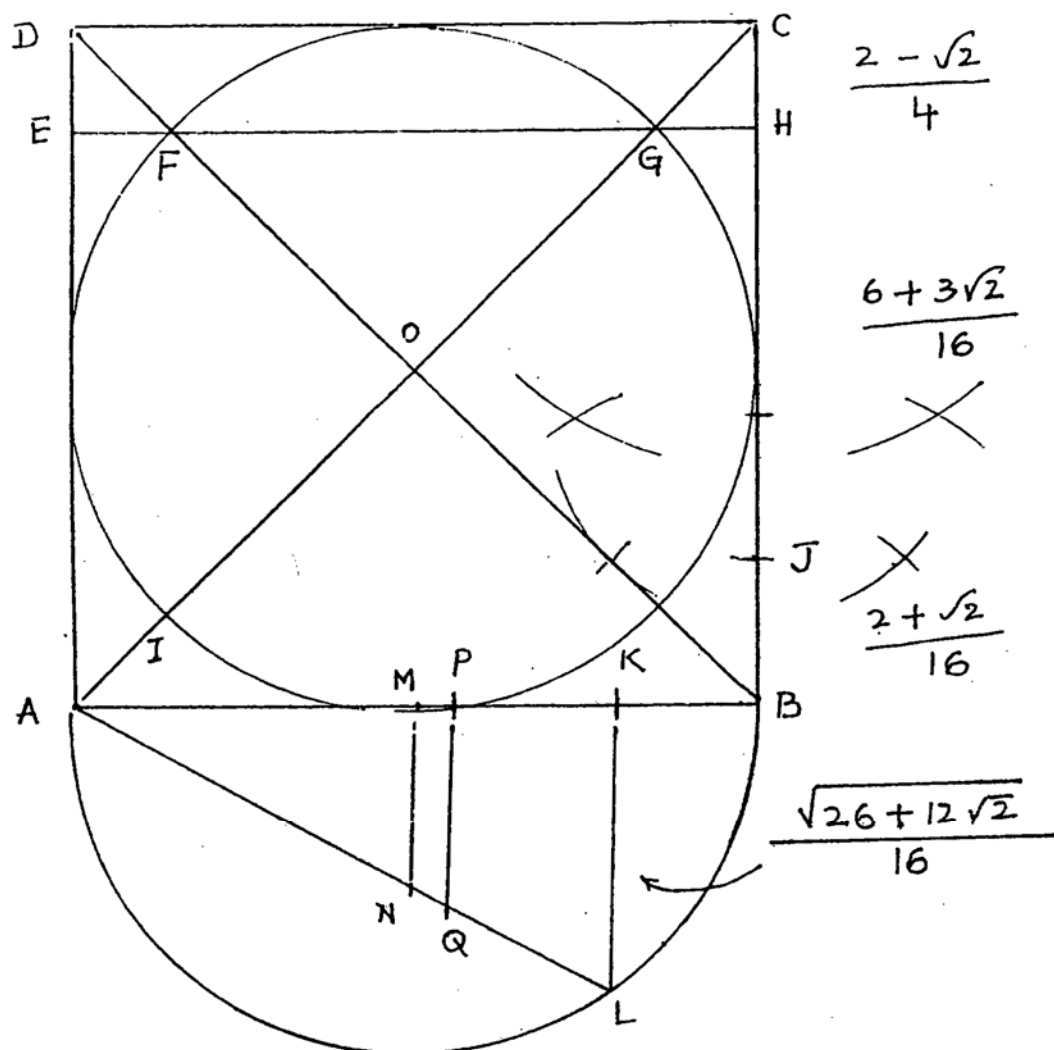
4. Area of the circle = 2

$$OJ = JJ_2 = OJ_2 = \frac{1}{2}; OJ_1 = \frac{\sqrt{3}}{4}$$

$$OJ_1 = LI = \frac{\sqrt{3}}{4}; LI_1 = \frac{LI}{LU} \times LY = 0.488225466.... = \text{required radius}$$

5. Area of the circle = 0.75

Proof for the exact interrelationship between a square and its inscribed circle or unit side of the square from $\pi/4$.



$$AB = \text{Side} = IG = \text{diameter} = 1 \quad OF = OG = \frac{1}{2}$$

$$\triangle FOG. \quad FG = \frac{\sqrt{2}}{2}, \quad CH = \frac{2 - \sqrt{2}}{4} = GH = EF = DE$$

$$\text{Bisect HB twice.} \quad CJ = \frac{14 - \sqrt{2}}{16} = \frac{\pi}{4}$$

$CJ = AK$. Draw semi-circle on AB with centre M . Draw perpendicular on AB at K . Join KL . Apply altitude theorem to get KL . Join AL . Apply Pythagorean theorem to get AL .

$$AL = \frac{\sqrt{14 - \sqrt{2}}}{4} \quad AM = MB = \frac{1}{2}$$

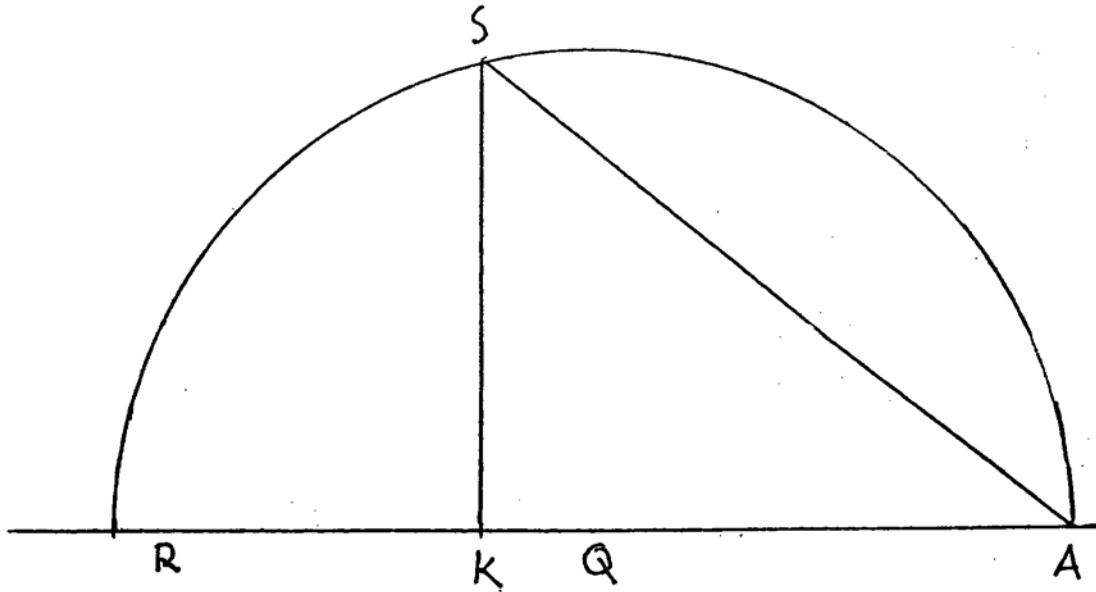


Fig. 2

Draw perpendicular on AB at M which intersects AL at N. Apply similar triangles to get AN. $AN = \frac{AM \times AL}{AK} = \frac{2}{\sqrt{14-\sqrt{2}}}$. $AN = AP$.

Draw perpendicular on AB at P which intersects AL at Q. Apply similar triangles to get AQ.

$$AQ = \frac{AP \times AL}{AK} = \frac{8}{14-\sqrt{2}} \quad AR = 2AQ = \frac{16}{14-\sqrt{2}}$$

$$AK = CJ = \frac{14-\sqrt{2}}{16} = \frac{\pi}{4} \quad KR = AR - AK = \frac{29+14\sqrt{2}}{8(14-\sqrt{2})}$$

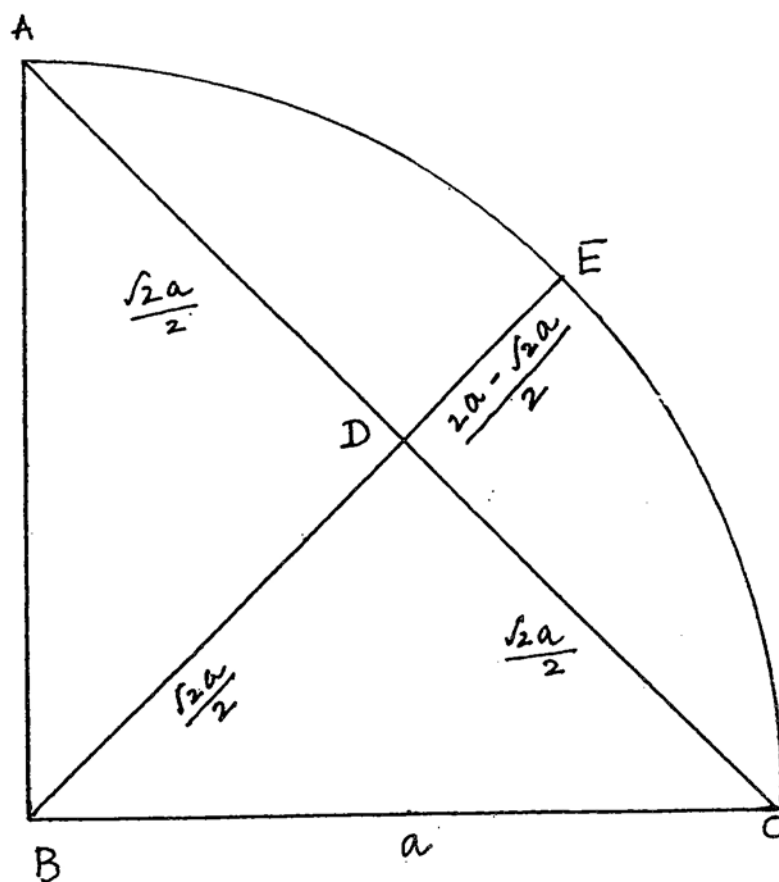
Draw perpendicular on AR at K, which intersects circumference at S.

Apply altitude theorem to get KS. $KS = \sqrt{\frac{29+14\sqrt{2}}{128}}$. Join AS. Apply

Pythagorean theorem to get $AS = \sqrt{AK^2 + KS^2}$

$AS = 1 = \text{Side of the square of Fig. 1}$

PYTHAGOREAN WAY OF CALCULATING ARCUAL LENGTH



$$AB = BC = a, AC = \sqrt{2}a$$

$$BD = \frac{\sqrt{2}a}{2}, DE = \frac{2a - \sqrt{2}a}{2}$$

$$AEC = \text{arc} = \frac{\text{Circumference}}{4}$$

where radius = a

Pythagorean theorem

$$AB^2 + BC^2 = AC^2$$

$$BD < AB ; \text{ But } BE = AB$$

Using the length DE arcual length can be obtained. An increased length of AB called AB_1 is necessary. AB_1 can be obtained by the following relation.

$$AB_1 = a + \left\{ \left(\frac{2a - \sqrt{2}a}{2} \right) \left(\frac{7\sqrt{2} - 9}{8 - 4\sqrt{2}} \right) \right\}$$

$$AB_1 = BC_1$$

Applying Pythagorean theorem

$$= AB_1^2 + BC_1^2 = AEC^2$$

Fig.2:

1. ΔABC . Draw an arc AEC with centre B and with radius AB.
 $AB = BC = BE = a$
2. $AC = \sqrt{2} a$. $BD = AD = DC = \frac{\sqrt{2}a}{2}$
3. $DE = BE - BD = \frac{2a - \sqrt{2}a}{2}$
4. Bifurcate DE twice of get $FE = \frac{2a - \sqrt{2}a}{8}$
5. $FG = \frac{a}{2}$. $GE = FE + FG = \frac{6a - \sqrt{2}a}{8}$
6. $BG = BE - GE = \frac{2a + \sqrt{2}a}{8}$
7. Bifurcate BG to get $BH = HG = \frac{2a + \sqrt{2}a}{16}$
8. $HE = HG + GE = \frac{14a - \sqrt{2}a}{16}$
9. $HE = BJ$
10. Draw a perpendicular on AB at J, which intersects AC at K.
 $BJ \times \sqrt{2} = KC$
11. $KC = AB_1$
12. $KC = CL = \frac{14\sqrt{2}a - 2a}{16}$
13. $KL = AEC$ arcual length
14. $KC^2 + CL^2 = KL^2$

EXACTNESS IN THE INTERCONVERSION BETWEEN CIRCLE AND SQUARE

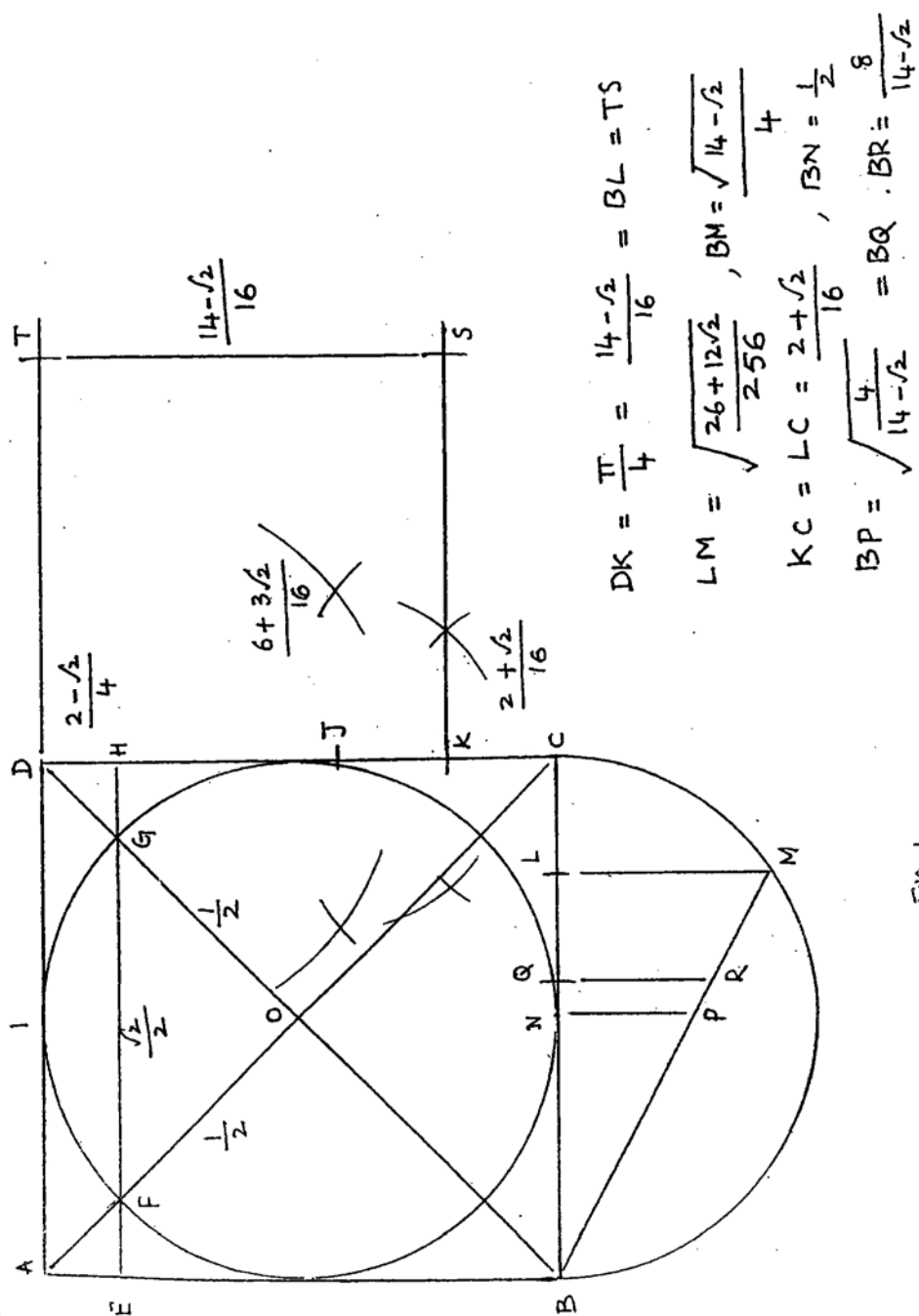


Fig. 1

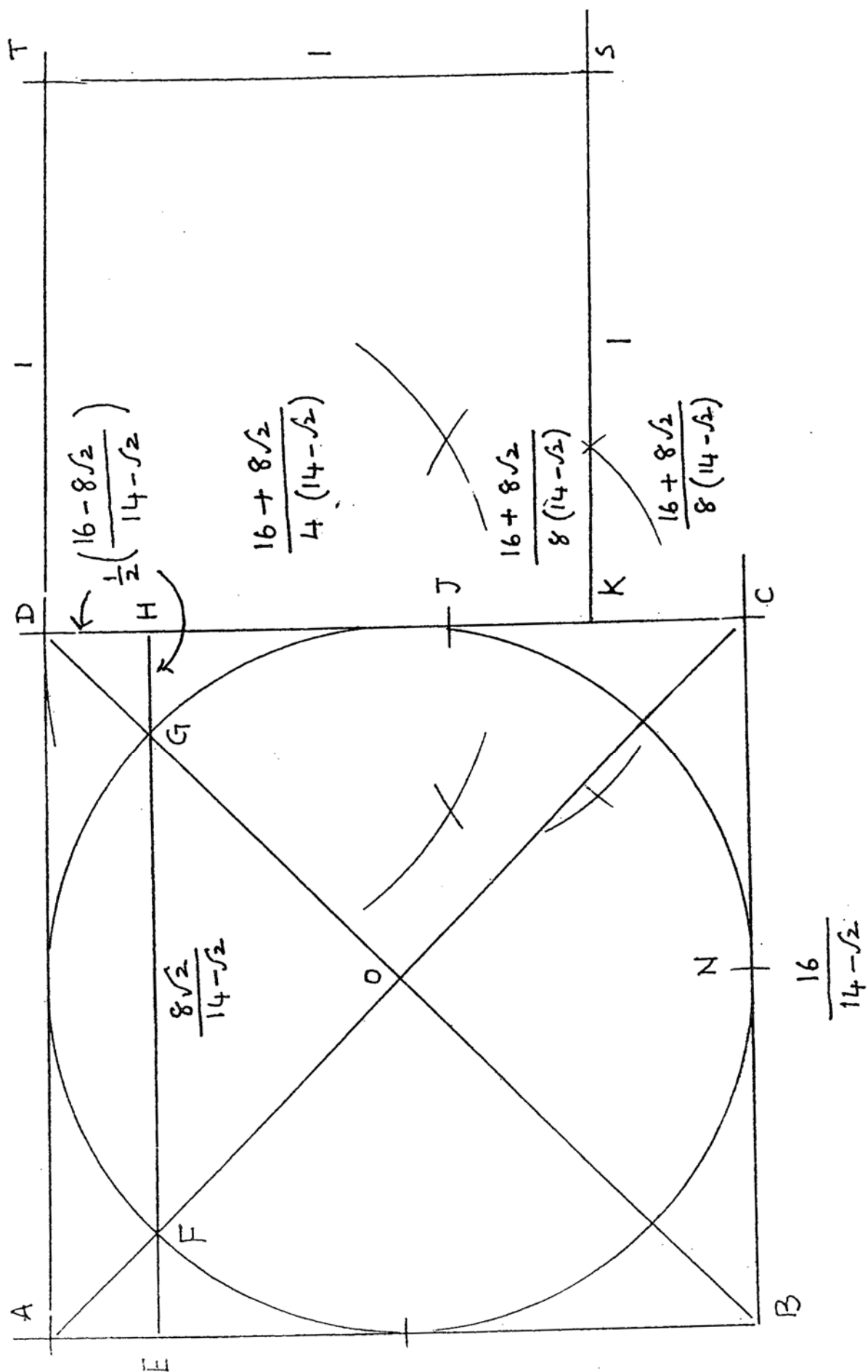


fig. 2

Fig.1

1. ABCD = Square. AB = 1, OF = OG = $\frac{1}{2}$
2. FG = Hypotenuse = $\frac{\sqrt{2}}{2}$. AE = EF = GH = DH

$$DH = \frac{2-\sqrt{2}}{4}, \quad HC = \frac{2+\sqrt{2}}{4}$$

Bifurcate HC twice. $DK = \frac{\pi}{4} = \frac{14-\sqrt{2}}{16}$

3. DK = BL. BN = NC = $\frac{1}{2}$

Draw semicircle on BC.

Draw perpendicular on BC at L which intersects semicircle at M.

Join LM. Join BM. Apply altitude theorem to get LM.

4. Apply Pythagorean theorem to get BM.

5. BN = $\frac{1}{2}$. Apply concept of similar

triangles to get $BP = \frac{BM \times BN}{BL}$

6. BQ = BP. Apply concept of similar triangles to get $BR = \frac{BM \times BQ}{BL}$

$$7. BR = \frac{8}{14-\sqrt{2}}$$

8. KSTD = CBEADH = circumference = π

Fig. 2

1. BR of Fig.1 = BN of Fig.2 = $\frac{8}{14-\sqrt{2}}$

2. Side BC = $\frac{16}{14-\sqrt{2}}$, Square = ABCD.

3. Inscribe circle. BN = OF = radius.

4. ΔFOG . FG = Hypotenuse = $\frac{8\sqrt{2}}{14-\sqrt{2}}$

5. EF = GH = DH = AE = $\frac{1}{2}(AD - FG)$

$$= \frac{1}{2} \left(\frac{16 - 8\sqrt{2}}{14 - \sqrt{2}} \right)$$

6. HC = DC - DH

$$= \left(\frac{16}{14 - \sqrt{2}} \right) - \frac{1}{2} \left(\frac{16 - 8\sqrt{2}}{14 - \sqrt{2}} \right)$$

$$= \frac{16 + 8\sqrt{2}}{2(14 - \sqrt{2})}$$

7. Bifurcate HC twice to get HJ, JC,

$$JK, KC. \quad KC = \frac{16 + 8\sqrt{2}}{8(14 - \sqrt{2})}$$

8. DK = DH + HJ + JK = 1

9. KSTD = CBEADH = Circumference of the inscribed circle = Perimeter of square ABCD square of Fig.1.