Another Hanukkah Miracle?

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Abstract
In Ekhad (2012), a paper on arXiv, "Shalosh B. Ekhad" asks why gaps between consecutive Christmas-in-Hanukkah years are currently always the Fibonacci numbers 2, 3, 5 or 8 years. The answer is not to do with Fibonacci numbers, as shown by the fact that 1 is also a Fibonacci number. The phenomenon is due to the possible lengths in days of periods of 1, 2, 3, etc. years.

Background

The Jewish calendar is lunisolar, meaning that it uses the date of New Moon to fix the starts of months but the Sun and the seasons to fix the starts of years. New Moons are on average about 29½ days apart, so months are always either 29 or 30 days long and usually months alternate between these lengths. This means that a year with 12 months would have about 354 days. However, a cycle of seasons lasts about 365¼ days, so if all years had 12 months the New Year would occur earlier and earlier in the cycle of seasons. This in fact happens in the Islamic calendar, where 67 Muslim years almost exactly equal 65 seasonal cycles. The Jewish calendar avoids this by having some years with 12 months (ordinary years) and some with 13 months (leap years) hence a length of about 384 days. There is a 19-year cycle in which years 3, 6, 8, 11, 14, 17 and 19 are leap.

Originally, the decision of whether a month had 29 or 30 days was based on observation (was the New Moon visible). Whether a year had 12 or 13 months depended on several factors, such as whether the barley crop would be ripe before the festival of Pesach began (and if not, an extra month was needed before Pesach to give more ripening time). Over time, it became more difficult to transmit the calendar information to Jews scattered from Spain to India, so observation was replaced by calculation so that knowledgeable Jews anywhere in the world could work out the calendar for themselves. There is a tradition that this calculation in all its details was mandated as Jewish law by Rabbi Hillel II in 359CE. However, it seems that arguments about the exact details continued until the 10^{th} century; see Stern (2001).

The rules finally agreed were codified by Rabbi Moses Maimonides in the 12^{th} century; see Gandz (1956). They involve calculating the Molad (approximate time of New Moon) of Tishri for each year by assuming that the interval between two New Moons is 29d 12h 44m 3⅓s (the value used by Greek astronomers, which is only about ½ second longer than modern estimates); the Molad of Tishri is 12 or 13 times this interval after the last Molad of Tishri, depending on whether the previous year was ordinary or leap. The New Year (Rosh Hashanah), 1^{st} Tishri, is on the day the Molad falls unless it is postponed by one or two days due to various rules, the most important of which is that this day cannot fall on Sunday, Wednesday or Friday, to avoid various problems with other festivals. Under these rules, ordinary years may have 353, 354 or 355 days, and leap years may have 383, 384 or 385 days. A cycle of 19 years has $12\times12+7\times13 = 235$ months, so the average length of a year is 235/19 times the interval between two New Moons, or about 365.246822 days.

A consequence of these rules was noted by the Persian scholar al-Biruni in 1000CE. He showed that any two years 689,472 years apart must always have the Molad of Tishri at exactly the same time on the same day of the week, so will begin on the same day of the week and have the same number of days; see Sachau (1879) pp.153-4. This means that any definitive calculation of the properties of the Jewish calendar must cover a period of this length.

The mathematical properties of the Jewish calendar have intrigued many mathematicians, Jewish and non-Jewish, over the centuries. For example, C. F. Gauss wrote two papers on the topic: Gauss (1800) and Gauss (1802); see also Burnaby (1901) and Reingold and Dershovitz (2018).

Christmas and Hanukkah

As the Jewish and Gregorian years are of different lengths, the date in the Gregorian calendar of a given Jewish date will vary from year to year. Typically, after a 354-day ordinary Jewish year it will be 365-354 = 11 days earlier in the Gregorian calendar and after a 384-day Jewish leap year it will be 384-365 = 19 days later in the Gregorian calendar. As a result, in the short term, a given Jewish date may during several decades fall on any of about 30 different Gregorian dates, and vice versa. Further, in the long term Jewish dates are on average getting later compared with Gregorian dates. This is because the average length of a Jewish year is about 365.246822 days, but the average for a Gregorian year is 365.2425 days. This difference amounts to over six minutes a year or 4.3 days per thousand years.

At present, the Jewish festival of Hanukkah or Chanukah always falls close to Christmas. It lasts for eight days, and between 1900 and 2099 the Gregorian date of the first day ranges from 28th November to 27th December. Clearly, as Hanukkah lasts for eight days, one of these days coincides with Christmas if and only if the first day is 18th to 25th December inclusive. Ekhad (2012) calls years when this happens Christmas-in-Hanukkah years. He notes that during a period of several millennia, from 1801 to 7390, gaps between such years are always 2, 3, 5 or 8 years. Actually, this is the case since the introduction of the Gregorian calendar in 1582 and, extrapolating that calendar backwards (and if the current rules for the Jewish calendar applied then), it would have been true since 876. Of course, 2, 3, 5 and 8 are all in the Fibonacci series. However, the analysis below shows that this is just a coincidence. Before or after 876 to 7390, the calendar drift means that Christmas-in-Hanukkah years were or will be less frequent, so the gaps between them tend to be bigger. The longer gaps in the periods shortly before and after 876 to 7390 are 11 and 19 years, not the next Fibonacci numbers, 13 and 21.

On the lengths in days of periods of years

The length of a Gregorian year is always 365 or 366 years. Years divisible by four have 366 days, except for century years not divisible by 400 such as 1900 or 2100, and all other years have 365 days. As noted above, Jewish years can have any of six lengths. Further, the lengths of periods of Jewish years measured in days fall into two sets differing by about 30 days, depending on how many leap years the periods contain (unless the period is a multiple of 19 years). These may be called the shorter Jewish periods and the longer Jewish periods. The possible lengths in days for various periods of Jewish years are tabulated by Landau (2001). We thus have the following table of lengths of periods:

Number of years	Length in days		
	Shorter Jewish period	Gregorian period	Longer Jewish period
1	353-355	365-366	383-385
2	707-710	730-731	737-740
3	1092-1094	1095-1096	1121-1124
4	1445-1449	1460-1461	1475-1477
5	1799-1803	1825-1827	1829-1832
6	2184-2187	2190-2192	2214-2217
7	2538-2541	2555-2557	2569-2571
8	2892-2895	2921-2922	2922-2925
9	3276-3279	3286-3288	3306-3309
10	3630-3633	3651-3653	3661-3664
11	4015-4018	4016-4018	4044-4048
12	4368-4372	4382-4383	4399-4402

Possible and impossible intervals

Why cannot two consecutive years both be Christmas-in-Hanukkah years? Consider periods of one year. The lengths in the Jewish calendar must be at least 365-355 = 10 days shorter or 383-366 = 17 days longer than in the Gregorian one. This means that if Christmas falls during Hanukkah in the first year, it must fall well before or well after it in the following year, so the years cannot both be Christmas-in-Hanukkah.

Similarly, gaps of four, seven, 10 and 12 years are impossible. Gaps of two years are just about possible, as a longer Jewish period may be only six or seven days longer than a Gregorian one.

Gaps of six years seem possible, as a Gregorian period may be as little as three days longer than a shorter Jewish one. However, if there is a Christmas-in-Hanukkah after six years, there must also be one half way through the period, so this becomes two gaps of three years. The same is true of gaps of nine years, as a Gregorian period may be as little as seven days longer than a shorter Jewish one, but they become three gaps of three years.

Finally, gaps of more than ten years are impossible during the peak period, as is shown by the fact that gaps of 11 years are possible going by lengths of periods, but in fact none occurs.

It follows that the only possible gaps are 2, 3, 5 and 8 years, as observed.

Festivals other than Hanukkah

With the continual drift of the Jewish calendar relative to the Gregorian, Christmas will eventually fall close to each Jewish festival in turn. The same analysis as above applies to Pesach (Passover), which is also eight days. Succot (Tabernacles) is nine days, but the calculations by "Shalosh B. Ekhad", confirmed by an analysis similar to the above, show that this makes little difference. For a two-day festival, such as Rosh Hashanah (New Year) or Shavuot (Pentecost), gaps between consecutive Christmas-in-festival years must necessarily be greater. They cannot be less than eight years; they may be 11, 19 or more years even at peak times.

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