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Shalosh B. Ekhad: a computer credit for mathematicians

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Abstract
With the advent of personal computers, mathematicians and PC team up with each other in the scientific sphere, wherein one sheds light on the significance of the other. However, hitherto PC rarely appears in publications to meet with a fair share of identification, and even its unique contribution has only been flirted with. For this underrepresented minority case in academia, Shalosh B. Ekhad is duly credited with his remarks "the computer helps so much and so often" in sociocultural paradigms. With the pull of academic accession, this article examines the perduring dark story of the Zeilberger–Ekhad theorem with extensive visualizations. This scientometrical case study is expected to provide a unique opportunity to re-scrutinize the whole story, discourage bias in later accounts that may appear, and uncover some unfolded motivations in human–machine cooperation scenarios.

Keywords Shalosh B. Ekhad · Scientific impact · Publication ethics · Authorship credit · Human–machine cooperation

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Introduction

After the 1936 envisioning of Alan Turing and the ensuing debut of personal computer (PC) (Turing 1936), the long-heralded human–machine cooperation scenarios have been constantly refashioned by computers for humans (Crandall et al. 2018; Morell 1996; Silver et al. 2018; Waldrop 2001). Although the original story of personal computing is still pending further discovery, the overwhelming reality is that PCs are always empowering our scientific activities, mediating human-to-human communication and changing human–machine relationships in a profound way (Baker et al. 1977; de Grey 2018; Hales 2005; Hales et al. 2017; Knuth 1976; Mackenzie 2005; Pool 1992). In distinction to its traditional title "Man of the Year", *Time* magazine's 1982 issue dubbed the personal computer as "Machine of the Year". The cover of the January 3, 1983 issue featured the headline, "*Machine of the Year: The Computer moves in*" with a visual of a white paper mache man contemplating a concept of personal computer at a kitchen table, which was designed by the American sculptor George Segal. Unfortunately, extensive scientific efforts have been focused on referring to PC as the technological artefact per se rather than its imperceptible reflections to us in sociocultural perspective (Atkinson 1998).

What do computers mean for humanity and society? (Kurzweil 1999; Simon 1977) According to Gretchen Vogel, humans and other primates seem to persistently observe the golden rule—having an innate tendency to cooperate with one another—whenever it does them observable good or not (Bok 1955; Moll and Schulkin 2009; Vogel 2004). Unfortunately, in those scenarios, less aligned recognition has been given to cooperative partners like their PCs, which never seek either incentive reward or social cognition. In doing so, as a nascent exception, scientist Doron Zeilberger endowed his close partner—an aspiring PC with an empathetic name Shalosh B. Ekhad—to achieve burgeoning visibility in the academic realm. This underrepresented minority case in academia would challenge pre-evolved dispositions, intuition, emotions, cultural norms, utility theory, and ultimatum game theory (Haidt 2007; Sanfey 2003). The non-human protagonist poses intriguing subjects and induces intertwining puzzles for all of us, and even drives us into the sequential prisoner's dilemma (Brainard 2018; Moll and Schulkin 2009).

In this research, we aim to probe answers to the above thorniest problems. Our goals are as follows:

1. We use Semantic Scholar's metrics to measure authors who most influenced Shalosh B. Ekhad and authors most influenced by Shalosh B. Ekhad, and find connections between studies by observing which publications have greater impact on others.
2. We present a systematic scientometric review of Shalosh B. Ekhad's scientific impact based on two sets of the relevant scientific articles by CiteSpace: (A) a set of 78 publications authored or co-authored by Shalosh B. Ekhad and (B) a set of 286 publications that cited the set A (Chen 2004, 2018; Cobo et al. 2011). Set A represents Shalosh B. Ekhad's original publications, whereas Set B represents the impact of Set A through citation indexing. Coupling with the semantic scholar's metrics (Fricke 2018), google earth engine (Gorelick et al. 2017), and the Alluvial Generator (Rosvall and Bergstrom 2010), scientometrical analysis is expected to provide a hallmark reference to understand Ekhad's contributions in the academic sphere.
3. We use google books Ngram viewer to probe the scope of conventional publication ethics. We first propose a culturomic review of the Zeilberger–Ekhad scenarios and give some convictive explanations accordingly for the underlying motivations.

Related work

When it comes to Shalosh B. Ekhad, there are many questions awaiting answers, and some are still yet to be uncovered:

1. Who is Dr. Shalosh B. Ekhad?
2. How about the extent of Shalosh B. Ekhad's scientific impact?
3. Who is Ekhad's host?
4. Why is such an eminent mathematician obsessed with his first personal computer and why does he empathize with it?
5. Why does he co-author with his computer?
6. Is such behavior unethical?
7. What is the underlying motivation?

With the pull of academic accession of this inconspicuous story, we reassemble the information of the twin-authors in this study, and examine a useful testing ground for understanding the scope of their scientific impacts. To capture the scientific impact of Shalosh B. Ekhad, we attempt to present a scientometric review of Dr. Ekhad's contribution in terms of publications that cited Ekhad's publications via dual-map overlay analysis, author co-citation analysis, document co-citation analysis, and geographic map analysis.

Who is Dr. Shalosh B. Ekhad?

Shalosh B. Ekhad (hereinafter abbreviated to Ekhad) is the first UNIX 3B1 personal computer—AT&T 3B1[1]—credited by the eminent Israeli mathematician Doron Zeilberger known for his provocative contributions in combinatorics. As a lucky member of computers developed in room 3B1 of Bell Labs (then AT&T), it is anthropomorphized as a professor with a doctorate, Department of Mathematics, Rutgers University, which is named by its model number in Hebrew ("Shalosh" and "Ekhad" mean "Three" and "One" in Hebrew respectively)[2] (Ekhad 2002; Gallian and Pearson 2007; Rabinoff 2002). Ekhad started working with Doron Zeilberger in the late 1980s. Although Ekhad existed between 1986 and 1990, Dr. Ekhad firmly believed that computers' souls could "always be transferred from one machine to another" (Ekhad 2002), so did Zeilberger (Gallian and Pearson 2007). As of 2018, Ekhad has been credited to publish 32 scholarly articles in peer-reviewed scientific journals with Doron Zeilberger, who incisively explains that "the computer helps so much and so often" and "uploads all the software from one Shalosh to the next, thereby guaranteeing the immortality of its soul."

[1] The 3B1, also known as the PC7300, or Unix PC, was a UNIX workstation computer originally developed by Convergent Technologies (later acquired by Unisys), and marketed by AT&T in the mid- to late-1980 s. As an erstwhile-innovative PC7300 variants, it was a 10 MHz Motorola MC68010-based Unix machine with a full-height 67 MB disk and expanded onboard memory to 1 or 2 MB.

[2] "The original Shalosh B. Ekhad was actually a Hebrew translation of the first PC that I owned, called AT&T 3B1. At the time it was a very innovative machine, the first UNIX PC, that was manufactured by AT&T in the 80s. The Hebrew translation of 3B1 is Shalosh B. Ekhad," Doron Zeilberger said. https://www.maa.org/sites/default/files/pdf/pubs/mayjune07web.pdf. (p. 15).

Additionally, Dr. Ekhad has its own email address and took delight in these heated debates, which human mathematicians are embroiled in. In a 2009 interview,[3] Ekhad even responded harshly to the "outrageous statements" of the eminent Soviet-Russian–German mathematician Yuri Ivanovitch Manin, who was known for his work in algebraic geometry and diophantine geometry, and many expository works ranging from mathematical logic to theoretical physics (Gelfand 2009).

As of 2018, Ekhad not only owned at least 24 articles in the Web of Science (WoS), 53 articles in the arXiv, and five online WebBooks,[4] but also had its own official home page[5] and even Personal Journal—*The Personal Journal of Shalosh B. Ekhad and Doron Zeilberger.*[6] Ekhad published 77 articles on the Personal Journal, from 1984 to August 11, 2018. Dr. Ekhad was not only named after the "Shalosh B. Ekhad's 10^n Lattice Paths Theorem" (Loehr et al. 2008), but also contributed to the famous *EKHAD* and *qEKHAD* packages of Maple programs.

As such, Ekhad was appointed by the Atari Corporation as a spokes-computer for a new research journal—*Electronic Journal of Computational Mathematics*, which is exclusively devoted to articles written by computers, edited by and for computers (Gordon et al. 2007), although the editors of *Math Horizons* Arthur T. Benjamin and Jennifer J. Quinn warned that it was just a spoof on April 1, 2007 (Quinn and Benjamin 2007). More importantly, Ekhad has made ever-increasing substantial contributions to research on widely different topics over a sustained period of time, while Professor Zeilberger had a *h*-index of 38 in *Google Scholar* from 6731 citations (Van Noorden 2014), and 342 highly influential citations in *Semantic Scholar* so far (Bohannon 2016).

Ekhad has even contributed to the accession of the "shaloshable" (Machine-solvable) paradigm in the academic sphere (Ekhad and Zeilberger 2017; Gnang and Zeilberger 2013; Zeilberger 1993, 1999). There are even those, like Zeilberger, who have coined the English terms "shaloshable" and "non-shaloshable" in the mathematical tribe, to conceptualize whether a human demonstration can be performed by Ekhad via computable algorithms or not.[7]

Dr. Shalosh B. Ekhad's publicizer: Doron Zeilberger

Doron Zeilberger (born on July 2, 1950 in Haifa, Israel) is an Israeli–American mathematician and a Board of Governors Professor of Mathematics at Rutgers University, New

[3] (Guest-) Opinion 105: Interview with Shalosh B. Ekhad about Mikhail Gelfand's Interview with Yuri Manin. http://sites.math.rutgers.edu/~zeilberg/Opinion105.html.

[4] *Ekhad's 2050 Plane Geometry: An Elementary Textbook*, http://sites.math.rutgers.edu/~zeilberg/GT.html; *Ekhad's Computer-Generated Alphametics,* http://sites.math.rutgers.edu/~zeilberg/hans/hans.html*; Ekhad's One Hundred and Twenty Connect-Four End-Game Problems,* http://sites.math.rutgers.edu/~zeilberg/C4/C4.html*; Ekhad's One Hundred and Twenty Pic-a-Pix Problems,* http://sites.math.rutgers.edu/~zeilberg/PicAPix/Pic.html*; Ekhad's Spelling Puzzles,* http://sites.math.rutgers.edu/~zeilberg/SB*; Ekhad's Sixty Six Skyscrapers Puzzles*, http://sites.math.rutgers.edu/~zeilberg/SK/SK.html.

[5] Home Page of Doron Zeilberger's servant. http://sites.math.rutgers.edu/~zeilberg/ekhad/ekhad.html.

[6] *The Personal Journal of Shalosh B. Ekhad and Doron Zeilberger* (http://sites.math.rutgers.edu/~zeilberg/pj.html) is an open-access journal, which only published Professor Doron Zeilberger and his co-author Shalosh B. Ekhad's articles ongoing updated online. All those papers (and lectures) are exclusively published in this Personal Journal, and sometimes many are also in *arxiv.org*, but not in any refereed regular journal, unless noted otherwise.

[7] Shalosh B. Ekhad and Tewodros Amdeberhan. Indeed Shaloshable! https://www.math.temple.edu/~tewodros/SHALO.PDF; Doron Zeilberger. WZ Theory, Chapter II. https://arxiv.org/pdf/math/9811070.pdf.

Brunswick, USA. Zeilberger's mathematical career began in his early teens, and he became enchanted by the beautiful world of mathematics. He earned his B.Sc. degree at University of London with First Class Honours in 1972, and doctorate in 1976 at Weizmann Institute of Science, Revovot city, Israel. He eventually completed his doctoral dissertation titled "*New Approaches and Results in the Theory of Discrete Analytic Functions*", under the direction of the eminent Israeli mathematician Harry Dym (an academic descendant of Courant and Hilbert), who introduced the third-order partial differential equation—Harry-Dym equation,[8] which bears his name (Kruskal 1975).

Mathematicians increasingly enjoy commanding computers for tackling intractable problems, like the 1976 proof of the Four Color Theorem (Appel and Haken 1977; Cipra 1988, 1989a). Among this elite club, Zeilberger is a de facto ever-victorious and veritable conqueror. He has the ability to turn long-standing intriguing mathematical conjectures with elusive results into theorems (See Table 1). A case in point is that Zeilberger was the first to give the elegant proof of combinatorial theory known as the Alternating Sign Matrix Conjecture in 1996, noteworthy not only for its mathematical content, but also for the fact that Zeilberger recruited nearly a hundred volunteer checkers to "pre-referee" the paper. After that, together with Christoph Koutschan and Manuel Kauers, Zeilberger proved the q-TSPP conjecture, which was stated around 1983 by George Andrews and David P. Robbins independently in 2011 (Koutschan et al. 2011).

Prof. Zeilberger is a prolific writer. His early mathematical works were mainly in the theory of discrete analytic functions, wherein he published several research papers in the late 1970s. In the early 1980s, he discovered some interesting results that connect partial difference equations and combinatorics. Since then, he has published more than 210 research papers in combinatorics, number theory, and algorithmic proof theory in refereed journals. In the arXiv community, Zeilberger is also a very active author with more than 170 pre-printed copies, and the list continues to grow. Moreover, on *The Personal Journal of Shalosh B. Ekhad and Doron Zeilberger*, he posted over 165 publications between 1984 and 2018.

Prof. Zeilberger has made numerous important contributions to combinatorics, hypergeometric identities, and q-series. In view of this, he became a Laura H. Carnell Professor at Temple University from 2000 to 2001, and a fellow of the American Mathematical Society in 2012 (http://www.ams.org/profession/fellows-list). He sits on many editorial boards, including but not limited to: *Advances in Applied Mathematics* (co-editor-in-chief 1998–2010, resigned 2016), *Annals of Combinatorics*, *Electronic Journal of Combinatorics* (resigned 2013), *INTEGERS*, *Journal of Difference Equations and Applications*, *Journal of Symbolic Computation*, and *Ramanujan Quarterly*.

Powered by Ekhad, Zeilberger earned many famous awards for his unique contributions. In 1990, Zeilberger received a Lester R. Ford Award for "the best paper in the American Mathematical Monthly in 1989" (Zeilberger 1989). In fact, he has attempted to find constructive proof of the unimodality of the Gaussian polynomials since 1978. In 1998, joint with Herbert S. Wilf, Zeilberger was awarded the American Mathematical Society's Leroy P. Steele Prize for Seminal Contributions to Research for their development

8 In 1975, the Harry-Dym equation, also known as the Harry-Dym hierarchies, first appeared in the paper written by the deceased American mathematician and physicist Martin David Kruskal. Professor Kruskal reported the Harry Dym's unpublished original work and reaffirmed this prevailing equation. Interestingly, Professor Kruskal retired from Princeton University in 1989 and joined the mathematics department of Rutgers University, holding the David Hilbert Chair of Mathematics.

Table 1 Chronological list of mathematical conjectures first proved by Doron Zeilberger

Year	Puzzlist(s)	Conjectures	Rewards	Cooperator(s)
1983	Richard Askey & George Andrews	The q-Dyson conjecture	$50	David Bressoud
1985	G. Xavier Viennot	A bijection between binary trees with n leaves and Strahler number k and ordered trees with n vertices and pruning order k	10 bottles of wine	None
1986	Richard Askey	The G_2 case of Macdonald's root system conjecture	$50	Laurent Habsieger
1987	Richard Askey	The G_2-dual case of the Macdonald-Morris conjecture	$50	None
1992	Julian West	The Julian West's conjecture on 2-stack-sortable permutations	None	None
1995	William Mills, Howard Rumsey & David Robbins	The alternating sign matrix conjecture	None	None
1998	Ron Graham	A determining the asymptotic minimal number of Schur triples	$100	Aaron Robertson
2011	George Andrews & David P. Robbins	The q-TSPP conjecture	None	Christoph Koutschan & Manuel Kauers

of Wilf–Zeilberger pair (or WZ pair) (co-recipient with Herbert S. Wilf), which has revolutionized the field of hypergeometric summation (Tefera 2010). In 2004, Zeilberger was awarded the Institute of Combinatorics and its Applications' 2004 Euler Medal Citation for recognizing his "distinguished lifetime career contributions to combinatorial research" by Fellows of the ICA who are still active in research (co-recipient with Zhu Lie). In 2016, the American Mathematical Society's David P. Robbins Prize was awarded to Zeilberger together with Christoph Koutschan and Manuel Kauers to recognize their outstanding achievements unanimously by its members (Gallian and Pearson 2007). According to the Who's Bigger Rankings (Skiena and Ward 2017), Zeilberger is the 76415th position on the most famous person list who has ever lived, and the 95523rd position on the most significant person list.

Dr. Shalosh B. Ekhad and Doron Zeilberger

Most impressively, Zeilberger is widely known for the coinage of Wilf–Zeilberger Theory (Cipra 1989b), and Zeilberger's algorithms (such as q-Zeilberger algorithms etc.), which are extensively used in modern computer algebra software. Just as the 2004 Euler Medal Citation, which was awarded to Zeilberger, eloquently declared him as "a champion of using computers and algorithms to do mathematics quickly and efficiently, he is in the forefront of current combinatorial research."[9] (Shtull-Trauring 2012) As a testimony to this, his proof of the Alternating Sign Matrix Conjecture in 1996, combines computer algebra and the results from partition theory, constant term identities, symmetric functions, and difference operators elegantly.

One of the most eminent mathematicians and computer scientist Donald E. Knuth[10] punctuated in his effusive foreword to the book entitled with "$A = B$" by Marko Petkovšek, Herbert Wilf, and Doron Zeilberger (Denef 1999; Kreinovich 2000; Petkovšek et al. 1998):

> Science is what we understand well enough to explain to a computer. Art is everything else. During the past several years, an important part of mathematics [*binomial coefficient identities*] has been transformed from an Art to a Science. No longer do we need brilliant insight to evaluate sums of binomial coefficients, we can now follow a mechanical procedure [*guided by Zeilberger's EKHAD and qEKHAD packages of Maple programs*] and discover the answer quite systematically.

In this book, the authors further underlined that (Petkovšek et al. 1998):

> While the above identity (essentially the Pfaff–Saalschütz identity) and all the other binomial-coefficient identities proved there can now be done by our *distinguished colleague Shalosh B. Ekhad*, as the reader can check with the package *EKHAD* described in Appendix A below, no computer would ever (or at least for a very long time to come) develop such a beautiful theory and such beautiful human proofs that are much more important than the theorems they prove. (p. 194)

Recently, Zeilberger has written at least five papers on the so-called umbral transfer matrix method. He has blended the transfer matrix method of statistical physic with the

9 Announcements. *Bulletin of the Institute of Combinatorics and its Applications*, Vol. 44, May 2005, p. 12.

10 According to the Who's Bigger Rankings, Donald E. Knuth is the 1746th position on the most famous person list who has ever lived, and the 5917rd position on the most significant person list.

umbral calculus to develop a method for counting difficult combinatorical structures, such as self-avoiding walks. Wherever his publications were released, Shalosh B. Ekhad has always played an unsurpassed role among joint authors who collaborate with Zeilberger.

Data collection and methods

In this article, firstly, we use the Semantic Scholar's metrics to measure the authors who most influenced Shalosh B. Ekhad and those who were most influenced by Ekhad, and find highly influential citations record. Secondly, we present the analysis of two sets of publications, sets S_A and S_B, where S_A consists of 78 publications by Ekhad and S_B consists of 286 publications that cite S_A, from the WoS.

In this study, Semantic Scholar's metrics will be used to examine connections between studies by seeing which publications had the greatest impact on others. The CiteSpace (5.3.R4 version) will be used to analyse the two scientific literature sets, including:

1. dual-map overlay analysis of publications from the source journals to the target journals at the level of disciplinary domains;
2. author co-citation analysis;
3. document co-citation analysis;
4. geographic map analysis based on Google Earth Engine;
5. alluvial flow analysis based on the Alluvial Generator.

In addition, Google Books Ngram Viewer will be used to conduct culturomic analysis of related topics on publication ethics.

S_A: 78 source publications

The first set of 78 publications S_A consists of publications by Shalosh B. Ekhad, which were retrieved from the WoS, JSTOR and arXiv.

Firstly, considering that Shalosh B. Ekhad always favors to publish papers on arXiv before formal publication, records are retrieved from WoS and arXiv. We obtained 10 allonym-monographs and 14 co-authored articles in the WoS, one allonym-monographs and seven co-authored articles in the JSTOR, six allonym-monographs and 47 co-authored articles in the arXiv (Fig. 1).

Secondly, information from arXiv is often incomplete and out-of-date. At the same time, it is noted that many preprints of Ekhad's publications are shared in arXiv, and later exclusively published in the electronic journal *The Personal Journal of Shalosh B. Ekhad and Doron Zeilberger*. As such, they are merged through a new process of citation records from different sources manually, but a similar consolidation procedure introduced by Chaomei Chen does not work here (Chen 2018). For example, Table 2 lists the eight articles from JSTOR with specific reference formats. Further, Fig. 2 demonstrates sdome typical publications by Ekhad with special titles. We crosschecked the information of each record from the WoS, JSTOR, arXiv and curriculum vitae of Doron Zeilberger available.

Finally, each record is standardized to the WoS format in terms of overlapping records retrieved from different sources and multiple variants within the same source, and 78 records are obtained. If an article has several variants, we only record the latest version and the corresponding publication year no matter they are from the same source or not.

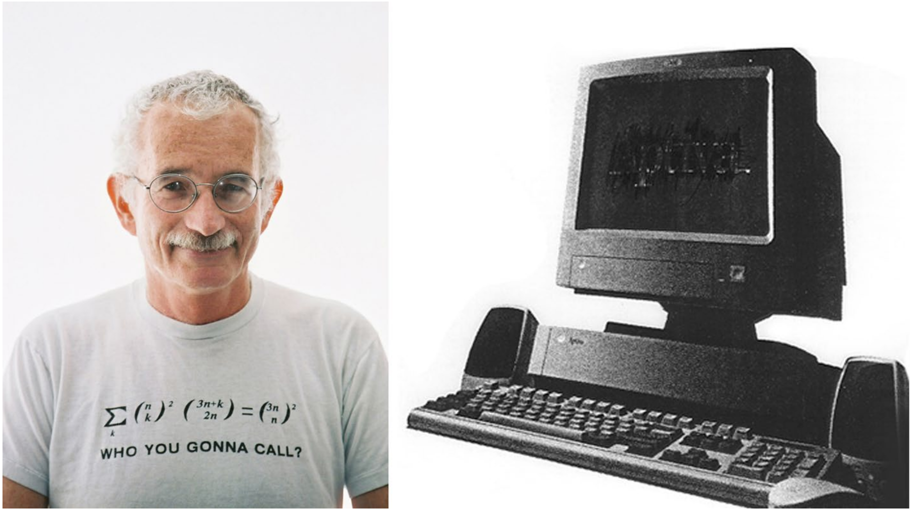

Fig. 1 Doron Zeilberger and Shalosh B. Ekhad (Courtesy of Doron Zeilberger)

Table 2 Chronological list of 8 papers by Ekhad from JSTOR with very special reference formats

#	8 papers by Ekhad from JSTOR
1.	Ekhad, S. B. (1989). Short proofs of two hypergeometric summation formulas of Karlsson. *Proceedings of the American Mathematical Society*, 107(4), 1143–1144. doi:10.2307/2047680
2.	Andrews, G. E., Ekhad, S. B., & Zeilberger, D. (1993). A Short Proof of Jacobi's Formula for the Number of Representations of an Integer as a Sum of Four Squares. *The American Mathematical Monthly*, *100*(3), 274. https://doi.org/10.2307/2324461
3.	Ali, H. A. S., Rey, J. G., Bunuel, I. B. L., Ekhad, S. B., Gessel, I., Huanxin, J., … Stanley, R. P. (1994). 10354–10360. *The American Mathematical Monthly*, 101(1), 75–76.
4.	Wenchang, C., Ekhad, S. B., & Chapman, R. J. (1996). A q-trigonometric Identity:10226. *The American Mathematical Monthly*, 103(2), 175–177.
5.	Ekhad, S. B., & Darling, D. A. (1997). A Sequence of Squares: 10356. *The American Mathematical Monthly*, 104(2), 176–177.
6.	Anglesio, J., & Ekhad, S. B. (2000). Four More Distinguished Points of a Triangle: 10703. *The American Mathematical Monthly*, 107(3), 285.
7.	Chao, W. W., Reid, M., Rosado, F. B., Chapman, R. J., Donini, D., Ekhad, S. B., Lakshmanan, N., Lossers, O. P., Nijenhuis, A., Nuesch, P. E., Petalas, C. G. (2000). Incenters and Excenters: 10693. *The American Mathematical Monthly*, 107(2), 182–184. https://doi.org/10.2307/2589457
8.	Mazur, M., Hanes, K., Anglesio, J., Benedicty, M., Ekhad, S. B., Lakshmanan, N., … Smith, J. H. (2000). Tangent Lines and Collinear Points: 10673. *The American Mathematical Monthly*, 107(2), 180–181. https://doi.org/10.2307/2589454

S_B: 286 source publications

The second set of publications S_B consists of 286 publications from Web of Science Core Collection (WoSCC) that cited at least one publication by Ekhad (authored or co-authored) in S_A.

An Explicit Formula for the Number of Solutions of $X^2 = 0$ in Triangular Matrices Over a Finite Field

Shalosh B. EKHAD[1] and *Doron ZEILBERGER*[1]

The Binomial Theorem for $(N + n)^r$ (where Nf(n)=f(n+1))

Moa APAGODU, Shalosh B. EKHAD, and Patrick GASKILL

There are $\frac{1}{30}(r+1)(r+2)(2r+3)(r^2+3r+5)$ Ways For the Four Teams of a World Cup Group to Each Have r Goals For and r Goals Against [Thanks to the Soccer Analog of Prop. 4.6.19 of Richard Stanley's (Classic!) EC1]

By Shalosh B. EKHAD and Doron ZEILBERGER

Fig. 2 Typical publications with very special titles by Ekhad

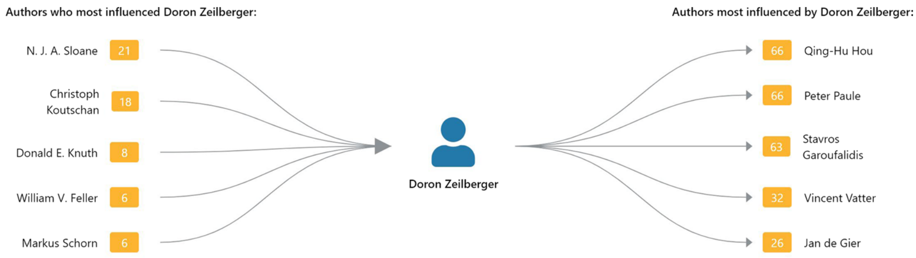

Fig. 3 Authors who most influenced Zeilberger and authors most influenced by Zeilberger

Semantic Scholar's metrics analysis

In comparison to Google Scholar and PubMed, the Semantic Scholar is apt to highlight the most influential papers and identify the connections between them, based on over 40 million scientific papers from sources like PubMed, Nature, and arXiv (Ginsparg 2011; Jones 2015, 2016; Xiong et al. 2017). Doron Zeilberger and Shalosh B. Ekhad always took delight in sharing their preprints on arXiv before formal publication (Ginsparg 2011), with 169 records and 53 records in arXiv respectively. Therefore, we explore Ekhad's scientific impact via the Semantic Scholar's metrics macroscopically.

The Semantic Scholar identifies citations where the cited publication has a significant impact on those citing publications, by which we can easily understand how each publication is built upon and related to other references. Except for Ekhad, Doron Zeilberger has also collaborated extensively with others. According to his bibliography record, he has at least 76 co-authors, including many current and former Ph.D. students. Donald E. Knuth runs on the top-5 rankings of authors who most influenced Doron Zeilberger, according to the Semantic Scholar's metrics (See Fig. 3).

At the same time, Zeilberger is the author who most influenced Ekhad and on the top list of authors who are most influenced by Ekhad (Fig. 4). The numbers in box in Figs. 3 and 4 are the corresponding influence scores of co-authors. Figure 4 shows that Ekhad has the most impact on many famous authors, including Martin J. Erickson, Jesús A. De Loera, Tewodros Amdeberhan, Andrew V. Sills, Scott Ahlgren, etc.

Ekhad holds the record of 10 highly influential citations (Table 3). Here, influential citations are determined by the Semantic Scholar, utilizing a machine-learning model to

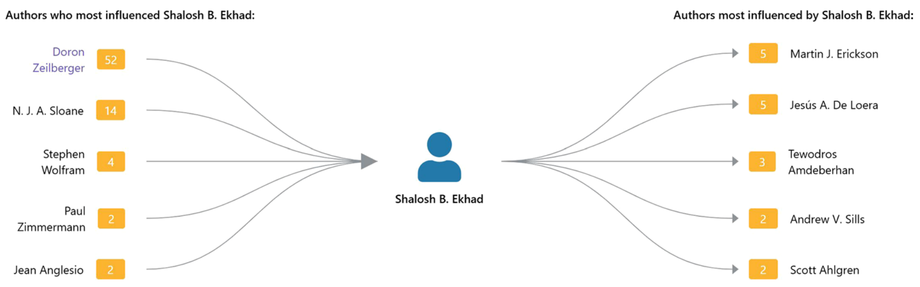

Fig. 4 Authors who most influenced Ekhad and authors most influenced by Ekhad

Table 3 Top 10 papers by Ekhad sort by highly influential citations

#	Publications by Ekhad with highly influential citations
1.	Ekhad, S. B., & Tre, S. (1990). A purely verification proof of the first Rogers-Ramanujan identity. *Journal of Combinatorial Theory, Series A*, *54*(2), 309–311. https://doi.org/10.1016/0097-3165(90)90038-x
2.	Andrews, G. E., Ekhad, S. B., & Zeilberger, D. (1993). A Short Proof of Jacobi's Formula for the Number of Representations of an Integer as a Sum of Four Squares. *The American Mathematical Monthly*, *100*(3), 274. https://doi.org/10.2307/2324461
3.	Ekhad, S. B., Sloane, N. J. A., & Zeilberger, D. (2015). A Meta-Algorithm for Creating Fast Algorithms for Counting ON Cells in Odd-Rule Cellular Automata. Retrieved from http://arXiv.org/abs/1503.01796
4.	Ekhad, S. B., & Zeilberger, D. (1990). A 21st century proof of Dougall's hypergeometric sum identity. *Journal of Mathematical Analysis and Applications*, *147*(2), 610–611. https://doi.org/10.1016/0022-247x(90)90375-p
5.	Ekhad, S. B., & Majewicz, J. E. (1996). A short WZ-style proof of Abel's identity. *The electronic journal of combinatorics*, *3*(2), #R16. Retrieved from http://www.combinatorics.org/ojs/index.php/eljc/article/view/v3i2r16/pdf
6.	Parnes, S., & Ekhad, S. B. (1992). A WZ-style proof of Jacobi polynomials' generating function. *Discrete Mathematics*, *110*(1–3), 263–264. https://doi.org/10.1016/0012-365x(92)90715-r
7.	Ekhad, S. B., & Zeilberger, D. (1998). Proof of Conway's Lost Cosmological Theorem. *Electronic Research Announcements of the American Mathematical Society*, *3*(11), 78–82. https://doi.org/10.1090/s1079-6762-97-00026-7
8.	Ekhad, S. B. (1990). A very short proof of Dixon's theorem. *Journal of Combinatorial Theory, Series A*, *54*(1), 141–142. https://doi.org/10.1016/0097-3165(90)90014-n
9.	Ekhad, S. B., & Zeilberger, D. (1996). The Number of Solutions of $X^2=0$ in Triangular Matrices Over $GF(q)$. *The electronic journal of combinatorics*, *3*(1), #R2. Retrieved from http://www.combinatorics.org/ojs/index.php/eljc/article/view/v3i1r2/pdf
10.	Ahlgren, S., Ekhad, S. B., Ono, K., & Zeilberger, D. (1998). A binomial coefficient identity associated to a conjecture of Beukers. *The electronic journal of combinatorics*, *5*(1), R10. Retrieved from http://arXiv.org/abs/math/9806040

analyse several factors including the number of citations to a publication, and the surrounding context for each. On the top of the list is the Ekhad's 1990 article on the Rogers–Ramanujan identities, in which the verification proof is executed by a MAPLE program (Ekhad and Tre 1990). For Doron Zeilberger, that was his early shaloshable attempt to solve mathematical conjectures. This finding is in accord with the result of the WoS, that 1990 article cited by other publications from the WoSCC.

Table 4 References cited most by Shalosh B. Ekhad himself

#	Citations	Publications authored or co-authored by Shalosh B. Ekhad
1.	1	Ekhad, S. B., & Tre, S. (1990). A purely verification proof of the first Rogers-Ramanujan identity. *Journal of Combinatorial Theory, Series A*, *54*(2), 309–311. https://doi.org/10.1016/0097-3165(90)90038-x
2.	1	Ekhad, S. B., & Zeilberger, D. (1998). Curing the andrews syndrome. *Journal of Difference Equations and Applications*, 4(3), 299–310. https://doi.org/10.1080/10236199808808143

Table 5 Subject category of Shalosh B. Ekhad's publications

Web of science categories	Subject categories
16 Mathematics	22 Mathematics
8 Mathematics applied	1 Computer science
1 Computer science theory methods	1 Science technology other topics
1 Multidisciplinary sciences	

Table 3 also shows that Ekhad has co-authored with some other eminent mathematicians, especially in combinatorics.

CiteSpace: scientometrical analysis

Shalosh B. Ekhad's publications (S_A)

References cited most by Shalosh B. Ekhad

The publications that have been cited the most by Ekhad are the 1990 article (Ekhad and Tre 1990) and 1998 article (Ekhad and Zeilberger 1998) on ranking journals based on citations. Table 4 shows the top references cited most by Ekhad in S_A. Ekhad is a prolific author, but he seldom cites his own early publications. This fact may indicate that Ekhad often gives precise, simple yet elegant proof to a specific problem or theory.

Table 5 summarizes the distributions of Ekhad's publications in terms of the relatively recent WoS categories and the more traditional Subject Categories. Ekhad's research areas reside in mathematics, computer science, and other multidisciplinary sciences.

Dual-map overlays (S_A)

Dual-map overlays consist of a dual-map base (the base maps built on JCR 2011) and multiple layers of overlay visualization, and could highlight the predominating interdisciplinary citation links. Figure 5 demonstrates that a dual-map overlay of Ekhad's 78 publications in S_A. Aggregated citation paths originate from the source journal map (left side) to target journals in the target journal map (right side). The major clusters of source journals are journals in *mathematics, systems and mathematical* (red). Each source journal group

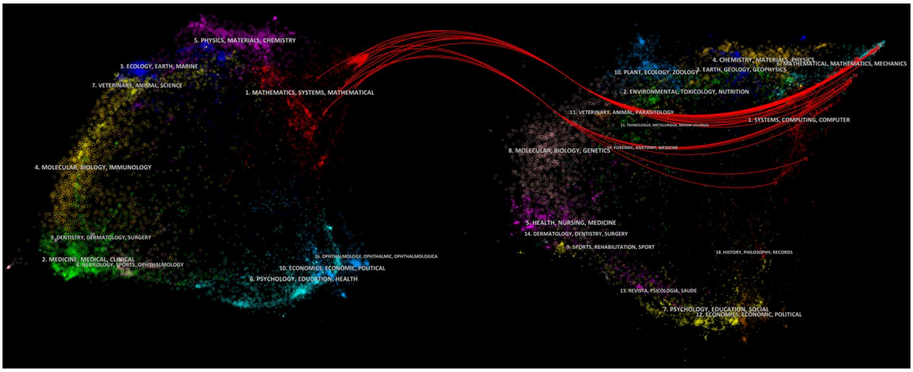

Fig. 5 A dual-map overlay of Shalosh B. Ekhad's 78 publications in S_A

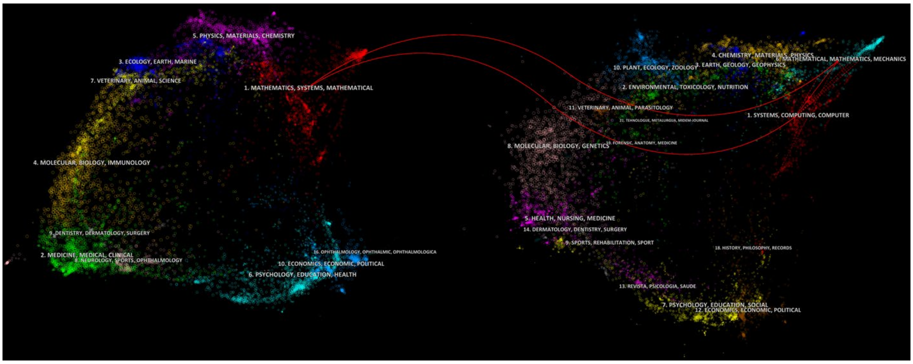

Fig. 6 A dual-map overlay of Shalosh B. Ekhad's followers' publications in the WoS

is connected to its own counterpart in the target journal map, as well as group cross-references on *systems, computing and computer*. Figure 5 shows that Ekhad's research areas reside in mathematics in a macroscopic level, as Tables 4 and 5 indicate.

Figure 6 shows a dual-map overlay of Ekhad's followers' publications in the WoS, with citation links bundled by z-scores. The major trajectory bundles in the dual-map overlay of Ekhad's publications also appear in its followers' dual-map overlay visualization. As we can see, the major connections take place in the same disciplinary domains.

Coauthorship network (S_A)

Figure 7 shows a timeline visualization of Shalosh B. Ekhad's coauthorship network in S_A between 1989 and 2017, with node labelling by eigenvector centrality. Since 1989, except for Doron Zeilberger, Ekhad has been continuously active in the mathematic realm by collaborating extensively with other scientists.

Figure 7 reveals that Ekhad had the most impact on a long list of mathematicians, and his followers, including Rebecca Smith, Robert Brignall, Doron Zeilberger's Ph. D. students Sheldon Parnes, Tewodros Amdeberhan and Vincent Russell Vatter. According to the Semantic Scholar's metrics, most of them are active researchers in combinatorics, who

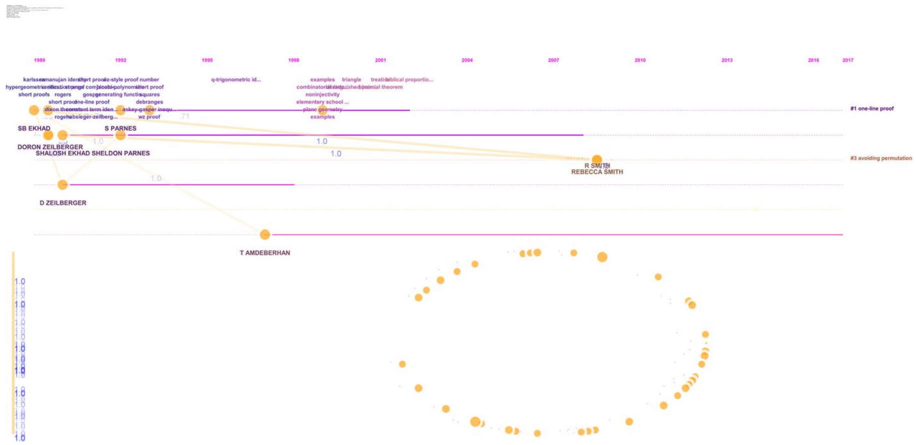

Fig. 7 A timeline view of Shalosh B. Ekhad's coauthor-ship network in S_{A}. Node labelling by eigenvector centrality

contributed to 1, 20, 1, 19, and 59 records of highly influential citations respectively, while they held 1, 17, 0, 59, and 40 hits in the WoSCC respectively. This finding offers a strong cross verification for the above results of the Semantic Scholar's metrics, and makes supplemental references to the previous conclusion mentioned above.

The coauthors of Ekhad were involved in many academic forelands from time to time. Table 6 further demonstrates the distribution of their research fronts, which are corresponding to the six evolutionary clusters in Fig. 5.

More and more researchers would like to share their preprints in the arXiv and intend to outsource them to the gatekeepers from the open professional community, except for anonymous reviewers designated by the editors of specific journals. Meanwhile, researchers are apt to cite the publications from this realm and echo promptly to the state-of-the-art findings relevant to new ideas. However, the opposition complains about the poor referencing formats.

We have no intention of giving preferential weight to the literature from different sources. However, it is worth mentioning that, in S_{A}, there are more subject frontiers from the WoS citations rather than the counterparts in the arXiv, or those preprints shared in the arXiv and their formal versions indexed by the WoS later. This finding may indicate that people tend to cite more stable references rather than their preprints. The possible reasons may go as follows:

1. Authors tend to cite the stable references from the high-profile sources in the WoS.
2. For many long-standing mathematic conjectures, mathematicians have every reason to await the most elegant proofs or the de facto solutions given, rather than make rush decisions based on the references of "unconventional publishing".

Co-citation clusters (S_{A})

Figure 8 shows a timezone view of a co-citation network generated from 78 publications in S_{A} based on 30 one-year slices. It demonstrates 19 hybrid co-citation clusters between 1982 and 2018 in S_{A} and their colours, representing the average publication years. Each

Table 6 Distribution of the hot subject frontiers that Shalosh B. Ekhad and coauthors are dedicated to

Year	Hot subject frontiers that Shalosh B. Ekhad and coauthors dedicated to	Evolution of cluster in Fig. 5
1989	Hypergeometric Summation Formulas of Karlsson; short proofs	#1
1990	Rogers–Ramanujan Identity; verification proof Dixon's theorem; short proof	#1
1991	combinatorial identity conjectured by Gosper; short proof Habsieger–Zeilberger G_2; one-line proof	#1
1992	Jacobi-polynomials' generating function; WZ-style proof	#1
1993	Askey–Gasper Inequality of Bieberbach; WZ-style proof Askey–Gasper Equality; short proof; Bieberbach conjecture; high-school algebra wallet-sized proof	#1
1996	q-trigonometric identity	#1
1999	Plane geometry Combinatorial radon transform; noninjectivity	#1
2000	Distinguished points of a triangle	#1
2001	Binomial theorem; A treatise on the	#1
2002	Biblical proportions	#1
2008	Stanley's P-partitions; Rota's Umbral calculus	#2
1990	Dougalls hypergeometric sum identity; 21st-century proof	#3
1998	Andrews syndrome	#4
2014	Somos-like miracles	#5
1997	Greg Kuperberg; Jim Propp; condensed condensation proof Determinant evaluation conjectured by Greg Kuperberg and	#6
2017	Creative telescoping	#6

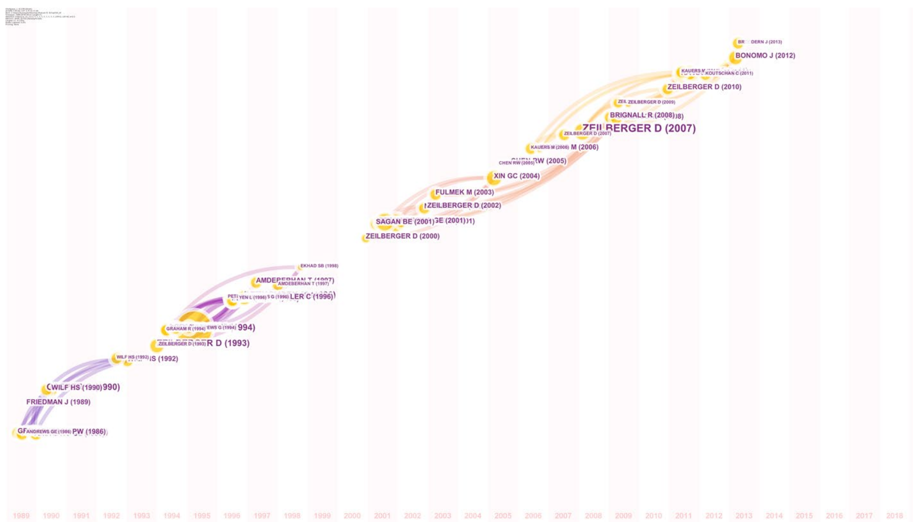

Fig. 8 A timezone view of co-citation network generated from 78 publications in S_A. Node labelling by citation

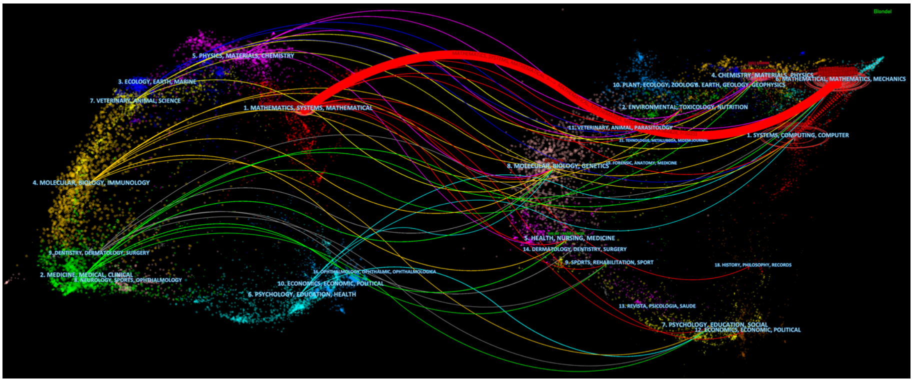

Fig. 9 A dual-map overlay of publications in S_B

citation link in a timezone runs from left to right, and the node labelling the citation and the node size are determined by the PageRank scores.

The earliest co-citation burst happened in 1986. After that, major attention was shifted to Shalosh B. Ekhad's article titled "a very short proof of Dixon's theorem", which was published in May 1990 (Ekhad 1990). As one of the most cited allonym-monographs by Shalosh B. Ekhad, this article has been cited 11 times by the publications in the WoS so far.

Scientific impact of Shalosh B. Ekhad's publications (S_B)

In this section, we aim to measure the scientific impact of Shalosh B. Ekhad, which is reflected by the breadth and depth of 286 articles in S_B. The 286 publications in S_B cited 9572 references, which includes 9375 valid distinct references (98.3424%).

Dual-map overlays (S_B)

Figure 9 shows a dual-map overlay of publications in S_B, where links are determined by the cluster's color of source journals and citation links are bundled by z-scores. As we can see, the predominant trajectory bundles originate from the source journal map to the target journal map, from *Mathematics, Systems, Mathematical* to *Mathematical, Mathematics, Mechanics*. At the same time, a fraction of references is beyond the disciplinary domain of mathematics and computer science. The larger circles also indicate Shalosh B. Ekhad's scientific impact in the disciplinary domain of mathematics and computer science.

Document co-citation clusters in S_B

The most cited publications in S_B could be selected by the TC field in the WoS. Table 7 lists the top-10 publications that are most cited by other references in S_B. The top ranked item by citation counts is the paper by Christian Krattenthaler with 51 counts (Krattenthaler 2005). The top list comes from the field of mathematics and computer science.

Figure 10 shows a merged network of document co-citation clusters by CiteSpace with the 286 publications in the impact set S_B. The network consists of 455 nodes and 1365 co-citation links. The silhouette score of each node indicates its level of homogeneity within

Table 7 Top 10 most cited references in S_B

#	Citations	Most cited publications by other references in S_B
1.	51	Krattenthaler, C. (2005). Advanced determinant calculus: A complement. *Linear Algebra and its Applications*, 411, 68–166. doi:10.1016/j.laa.2005.06.042
2.	39	Kim, D., & Kim, T. (2014). Barnes-type Narumi polynomials. *Advances in Difference Equations*, 2014(1), 182. https://doi.org/10.1186/1687-1847-2014-182
3.	34	Araci, S. (2014). Novel identities involving Genocchi numbers and polynomials arising from applications of umbral calculus. *Applied Mathematics and Computation*, 233, 599–607. https://doi.org/10.1016/j.amc.2014.01.013
4.	26	Srivastava, H. M. (2011). Some Generalizations and Basic (or q-) Extensions of the Bernoulli, Euler and Genocchi Polynomials. *Applied Mathematics & Information Sciences*, 5(3), 390–444.
5.	23	Guillera, J. (2006). Generators of some Ramanujan formulas. *The Ramanujan Journal*, 11(1), 41–48. https://doi.org/10.1007/s11139-006-5306-y
6.	15	Park, J.-W. (2014). On the twisted Daehee polynomials with q-parameter. *Advances in Difference Equations*, 2014(1), 304. https://doi.org/10.1186/1687-1847-2014-304
7.	14	Guillera, J. (2003). About a New Kind of Ramanujan-Type Series. *Experimental Mathematics*, 12(4), 507–510. https://doi.org/10.1080/10586458.2003.10504518
8.	13	Kim, T. (2009). Some identities on the q-Euler polynomials of higher order and q-stirling numbers by the fermionic p-Adic integral on $\mathbb{Z}p$. *Russian Journal of Mathematical Physics*, 16(4), 484–491. https://doi.org/10.1134/s1061920809040037
9.	13	Luo, Q.-M., & Srivastava, H. M. (2011). Some generalizations of the Apostol–Genocchi polynomials and the Stirling numbers of the second kind. *Applied Mathematics and Computation*, 217(12), 5702–5728. https://doi.org/10.1016/j.amc.2010.12.048
10.	12	Gaboury, S., & Tremblay, R. (2014). A note on some new series of special functions. *Integral Transforms and Special Functions*, 25(5), 336–343. https://doi.org/10.1080/10652469.2013.849248

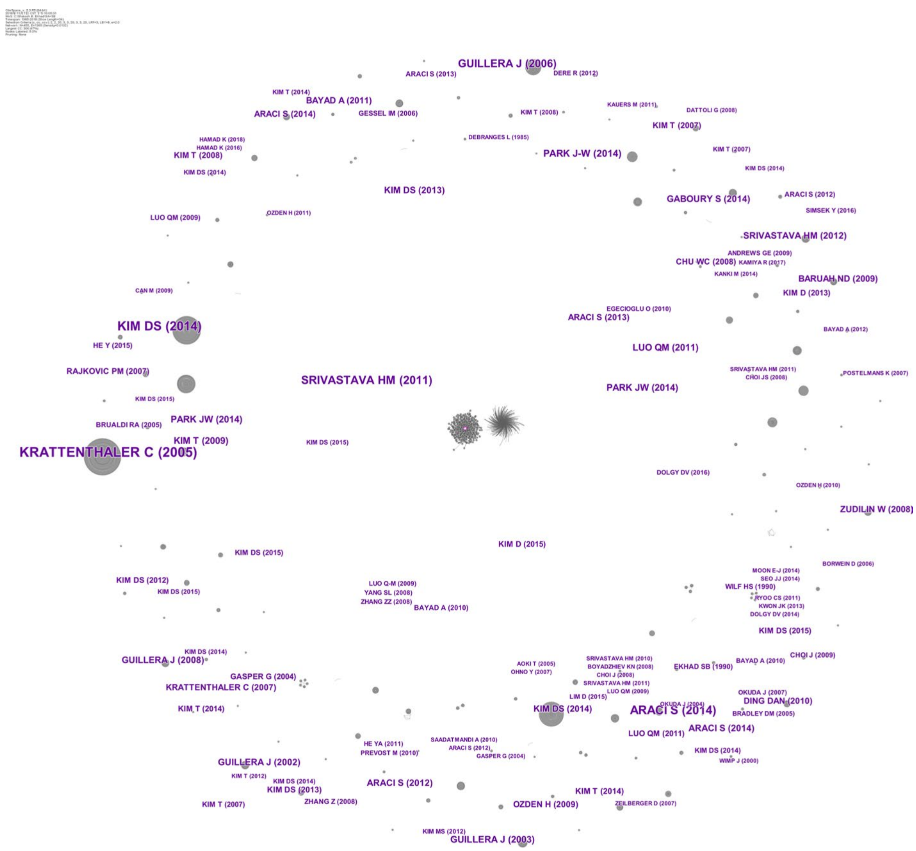

Fig. 10 A merged network of document co-citation clusters in S_B

the corresponding cluster. If the score of a node is above the threshold, it is labelled by citation and its size is proportional to the score. The most cited publications in Table 7 are in the position of relative prominence.

Author co-citation cluster in S_B

In the section of Semantic Scholar's metrics analysis, the results clearly identify those authors most influenced by Shalosh B. Ekhad and Ekhad's joint authors—distinguished mathematicians. We further uncover that those authors have dialogued with Shalosh B. Ekhad by reference citations directly or indirectly.

Figure 11 shows a timeline view of author co-citation clusters in S_B, with 489 nodes and 880 citation links. That indicates that at least 489 authors are involved in such science communication, in which the topics are q-zeta function, alternating sign matrices, elliptic curve, Hankel determinant, avoiding permutation, partition bijection, etc. Figure 12 demonstrates a geographic map of authors who have dialogued with Shalosh B. Ekhad in S_B. The solid red dots represent the clusters of the authors' source city, and their sizes proportional to the amounts of authors from the same city. The heights of those red dots are proportional to the magnitudes of publications from the same city's

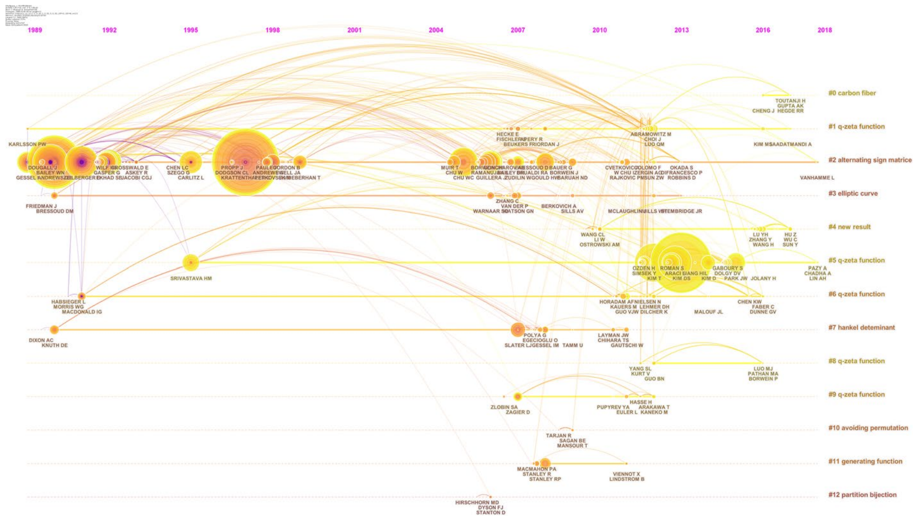

Fig. 11 A timeline view of author co-citation clusters in S_B

authors. The communication links are labelled by different colors, and the legends are identified by the common communication links and the location of Shalosh B. Ekhad. As we can see, many authors may have several affiliations from different cities. The source city with the most frequent interaction is Seoul, South Korea (top left), followed by Gaziantep, Turkey (top right) (Fig. 12b). Many American scientists have also been active in those dialogues, but are scattered across 38 cities.

Table 8 lists the Top 10 most active contributors in the communication community, with the frequencies of dialogue, their highly influential citations according to Semantic Scholar's metrics, and source cities according to the Google Earth analysis. Shalosh B. Ekhad and Doron Zeilberger are ranked 3rd and 8th on the top 10 list respectively. The majority of these most active contributors are from the field of combinatorics.

It is noted that Semantic Scholar's metrics are pertinent to the sources arXiv, PubMed etc., while the results in this section are the wrestling information from WoSCC. Each of CiteSpace, Semantic Scholar's metrics and Google Earth Engine, is a supplement to the other, shedding light on the significance of the other.

Alluvial diagram of dialogues in S_B

An alluvial diagram is a visualization approach of multiple networks to probe the significant structural changes between the adjacent clusters (Rosvall and Bergstrom 2010). We have generated a network of authors and cited authors by 1-year slices in CiteSpace and imported these annual clusters into the Alluvial Flow generator proposed by Rosvall and Bergstrom. Figure 13 illustrates an alluvial diagram of cited-author clusters over time in S_B, where the highlighted nodes denote the self-citations by Ekhad and Doron Zeilberger. It reveals that scholars who have dialogued with Ekhad by citations almost last the entire course from 1989 to 2018, and there was relatively little monologue in the process of interaction. It also demonstrates a clear propagation path of the scientific impact of Ekhad by the citation patterns.

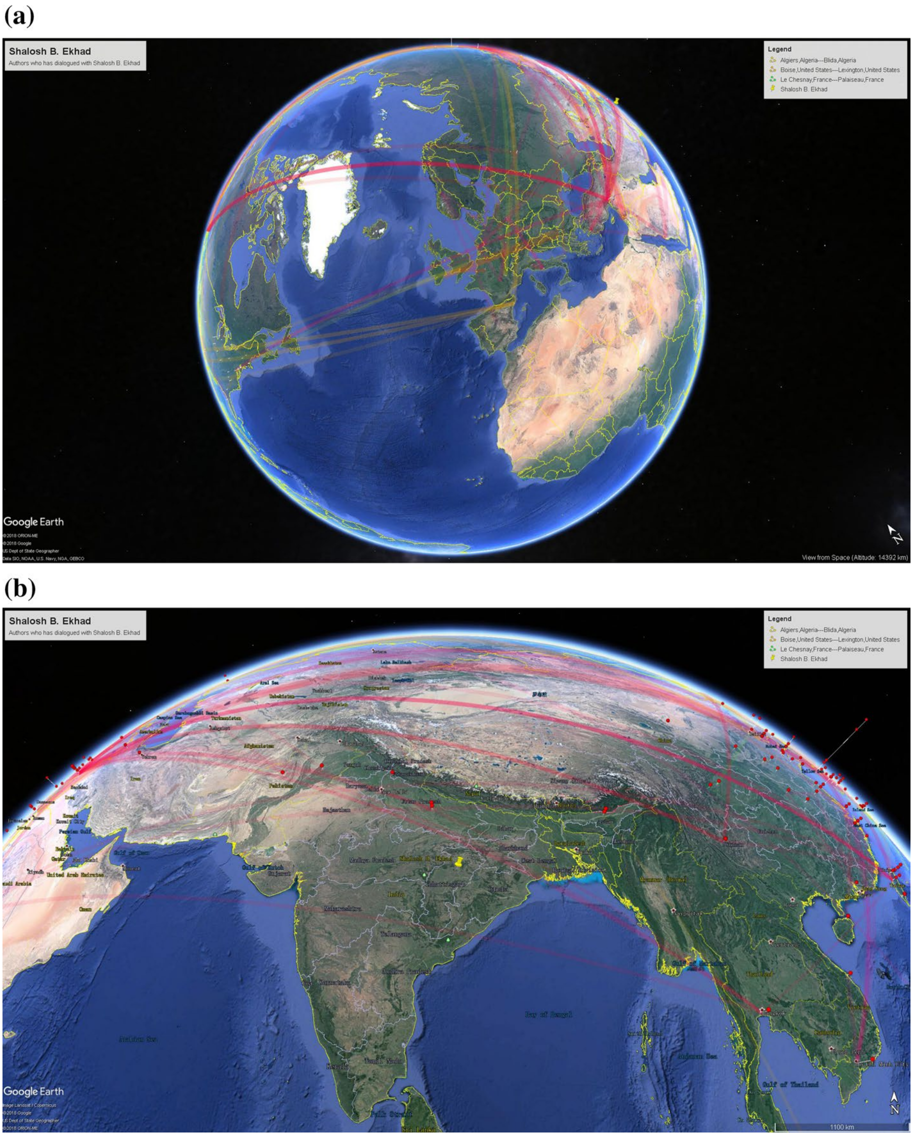

Fig. 12 A geographic map of the authors who have dialogued with Shalosh B. Ekhad in S_B. **a** view of the whole globe with the location of Shalosh B. Ekhad; **b** a silhouetted view

Google books Ngram viewer: culturomic analysis

The present-day model policy for publication ethics is still pending further discovery. In 2018, Jeffrey Brainard profiled a tally of scandals for flawed, fraudulent or tainted scientific literature (Brainard 2018). It is plausible to expect that it is still widespread. Obviously, the precept “credit where credit’s due” should not evolve into “credit only when it’s convenient” (Pearson 2006). On the other hand, collaborators may in turn

Table 8 Top 10 most active contributors in S_B

#	Top 10 most active contributors	Frequency	Highly influential citations	Source city
1.	TAEKYUN KIM	21	58	Seoul, South Korea
2.	DAE SAN KIM	20	8	Seoul, South Korea
3.	SB EKHAD	18	10	Piscataway, United States
4.	WENCHANG CHU	15	14	Lecce, Italy
5.	SERKAN ARACI	14	6	Gaziantep, Turkey
6.	MEHMET ACIKGOZ	11	6	Gaziantep, Turkey
7.	DMITRY V. DOLGY	10	1	Haifa, Israel
8.	D ZEILBERGER	8	333	Piscataway, United States
9.	TOUFIK MANSOUR	7	122	Haifa, Israel
10.	JESUS GUILLERA	7	15	Zaragoza, Spain

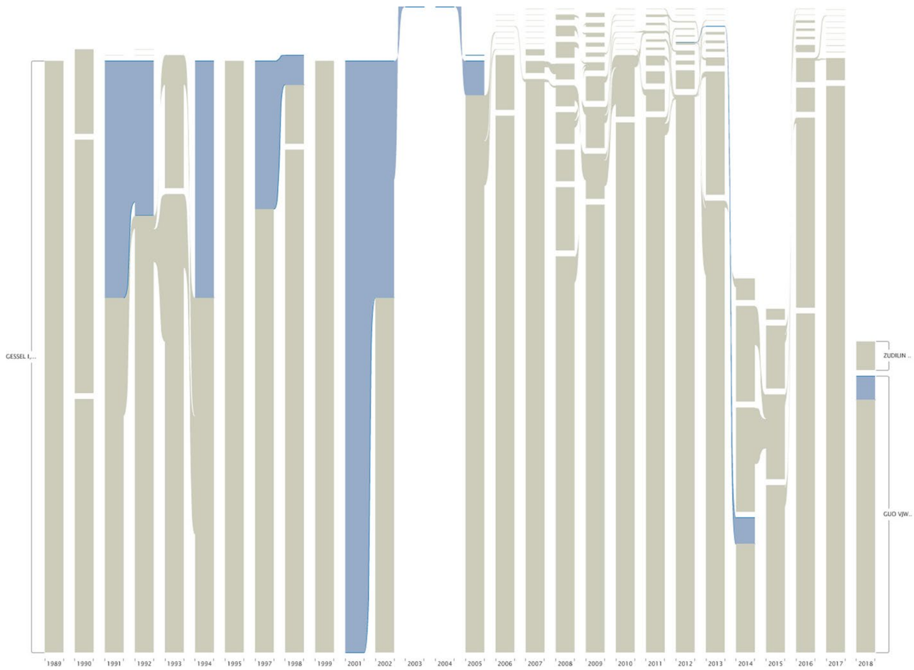

Fig. 13 An alluvial diagram of authors who have dialogued with Shalosh B. Ekhad in S_B

suffer career damage when their co-authors are found to be guilty of scientific misconduct (McCook 2018).

Figure 14 charts the word frequency of common unethical concerns in the domain of publication ethics in English corpus from 1918 to 2008 respectively, such as *scientific misconducts*, *scientific frauds*, *publication ethics*, *publishing ethics*, *academic authorship*, *authorship criteria*, *honorary authorship(s), false authorship(s),* and *ghost authorship*. As the Google Books Ngram Viewer illustrates (Aiden and Michel 2013;

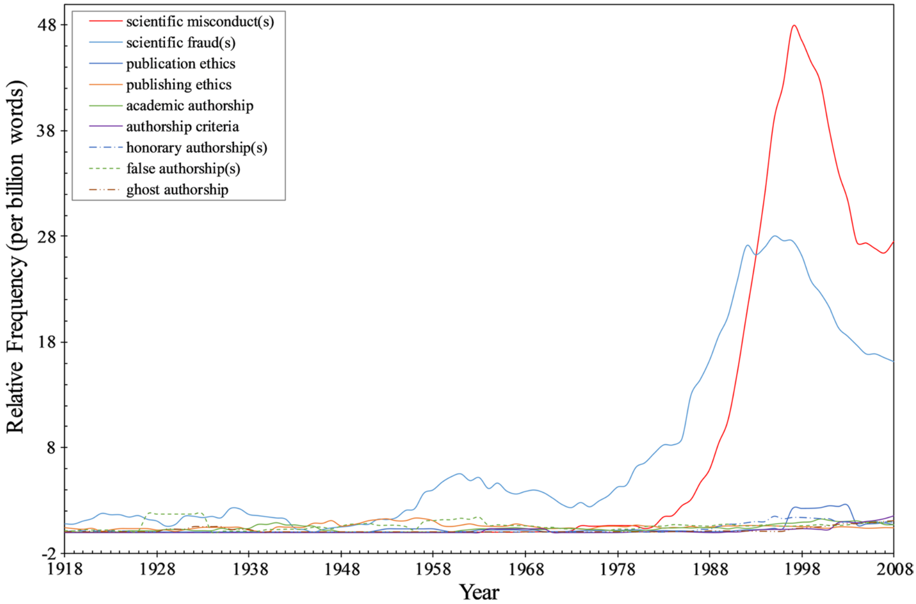

Fig. 14 Google books Ngram viewer charting word frequencies of the common unethical concerns in the domain of publication ethics in English corpus from 1918 to 2008 respectively

Lieberman et al. 2007; Michel et al. 2011), it is high time to echo the de novo state of the fact and discourage honorary, false, and ghost authorships to provide an additional level of the present-day basic canon of publication ethics (Greenland and Fontanarosa 2012). In the past century, the burgeoning discussions on scientific frauds and misconducts (Broad 1981; Couzin-Frankel 2018; Fanelli 2013; Fang et al. 2012; Hesselmann et al. 2017; McCook 2018; Pearson 2006; Yeagle 2018) were conducted; on the contrary, less intensive care has been given to the model policy itself for publication ethics (Hook 1974; Horowitz and Garn 1979; Kabat 1975; Page 1966; Painter 1986; Place 1920), especially about honorary authorships (no actual work done)(Greenland and Fontanarosa 2012; T. M. B. 1988), false authorships (false attribution of authorship) (Brookfield 2003; Fanelli 2013), and ghost authorships (who meets the criteria of authorship and has made substantial contributions, but is not credited as an author) (Gøtzsche et al. 2007).

Until recently, more scientists started to reflect upon anonymous credit (Anonymous 2016), inanimate authors credit (Erard 2015a), and animal authors credit (Erard 2015b; Erren et al. 2017; Matzinger and Mirkwood 1978), and other urgent concerns (Editor 1995; Ren et al. 2016). As with *ex ante* anticipation, the Zeilberger–Ekhad's episode is a swing and a miss. This prototypical example suggests that the extent to such honorary or false co-authorships is still present, but is not empirically known (Alpher et al. 1948; Brookfield 2003; Marušić 2016; Rajasekaran et al. 2014).

As for nonhuman authors credit in scientific domains, it is true that people may hold different ideas with their convincing reasons on the Zeilberger–Ekhad's case. Advocates believe that this should generate great interest and debate, however, it justifies its fair share of dissension. On the contrary, opposition to such a case provides nominal rather than substantial evidence conventionally (Brainard 2018; Moor 1985).

Evidently, the Zeilberger–Ekhad scenarios are beyond the scope of conventional publication ethics. On the basis of the above analysis and discussions, we may provide multiple persuasive inferences for the underlying motivations:

1. For Zeilberger, Ekhad is a pseudonym for his PC. Therefore, he coauthors with his PC beyond the scope of individualism (Oyserman and Lee 2008) or altruism (Sánchez and Cuesta 2005).
2. In the mathematical tribe, Zeilberger advocates the shaloshable paradigm, as many other scientists do (Nemes et al. 1997; Snow 2017). Shalosh B. Ekhad, as "our beloved servant", is just the touchstone of transforming his conceptualized paradigm to reality (Shar and Zeilberger 2016).
3. In his philosophy of coauthorship, when commanding the computer to crack long-standing mathematical problems, Zeilberger takes delight in coauthoring with it (Petkovšek et al. 1998). However, were it not for his computer, he would not succeed. In this regard, Ekhad is his "distinguished colleague" and loyal cooperator rather than a lifeless PC (J. A. Markowitz 2015).
4. From a psychological perspective, Zeilberger has every reason to think of Ekhad with deep gratitude, who has lighted the flame, as with other mathematicians, at times when his own light extinguished and is rekindled by a spark from Ekhad (Benderly 2015; Shar and Zeilberger 2016; Snow 2017; Zeilberger 2007).

Discussions and conclusions

With an emphasis on scientometrical analysis and visualizations, we examine the Zeilberger–Ekhad theorem in sociocultural paradigms to uncover some unfolded motivations under the umbrella of human–machine cooperation scenarios. In summary, the conclusions of our findings are as follows:

1. Mathematics is a fundamental intellectual tool in computing, and computers are increasingly tooled up as a key component in mathematical problem-solving. Many computer pioneers have been tagged as mathematicians who dwell in both mathematics and computer science. Especially, mathematicians-and-PC alliances, each shedding light on the other, often channel the unique opportunity to combine an appreciation of mathematical reasoning with an understanding of computing. Today, Shalosh B. Ekhad is the pseudonym of mathematician Doron Zeilberger. For Zeilberger, the dramatic career of Shalosh B. Ekhad is more than just a tantalizing game—it is the status quo of distinctive inclusiveness over exclusiveness (Russo 2018).
2. Although the main publishing area is combinatorics, Ekhad has also made substantial contributions to partition theory, symmetric functions, hypergeometric identities, and conjectures. It is difficult to say which area he has contributed to most, but all are in his debt.
3. We cannot reach a ubiquitous conclusion for the Zeilberger–Ekhad theorem beyond the arguments of conventional publication ethics. However, from a sociocultural perspective, this underrepresented minority episode may reflect on other human–machine cooperation scenarios in our collective future (Kurzweil 1999; Silver et al. 2018).

Admittedly, the above academic attempt is just a scratch on the surface, and is not the finality of such socio-cultural studies. Still, whatever cause it may be, our profiling of Ekhad's scientific impact strongly reflects the emerging power of those mathematicians who are obsessed with commanding computers to visualize their brain game in the process of solving complex mathematic problems. Furthermore, the above analysis is expected to discourage bias in later accounts (Ekhad 2002), and approach the revivification of their research impact, rather than an ambitious stride toward the bottom of multifaceted paradigms. It is reframing the extensile questions which have not produced simple answers to anonymous, honorary, false, ghost, inanimate, and animal authors credit yet, in ethics, law, sociology, psychology, and philosophy.

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