Solutions to Dr. Z.'s Math 354 REAL Quiz #4

1. (8 pts.) (a) Find the extreme points of the set of feasible solutions for the following linear programming problem (b) Find the optimal solution(s)

Minimize $z = 5x - 3y$ subject to

$$x + 2y \leq 4 \quad , \quad x + y \geq 3 \quad , \quad x \geq 0 \quad , \quad y \geq 0 \quad .$$

Sol. of 1(a)

The **feasible region**, in the **postive quadrant** is the region **below** the line $x+2y = 4$ and **above** the line $x + y = 3$. It is a **triangle** whose **vertices** are

- $(3, 0)$ where the line $x + y = 3$ meets the line $y = 0$ (aka as the x axis)

- $(4, 0)$ where the line $x + 2y = 4$ meets the line $y = 0$ (aka as the x axis)

- $(2, 1)$ where the lines $x + 2y = 4$ and $x + y = 3$ meet each other.

[This is gotten by solving the sytem of two equations $\{x + 2y = 4, x + y = 3\}$ with the set of two unknowns $\{x, y\}$. Subtractiong the first equation from the second gives $y = 1$ and plugging into the first gives $x = 4-2\cdot 1 = 2$, (or if you wish, plugging into the second, also getting $x = 3-1 = 2$].

These vertices of the feasible regions are the **extreme points**.

Answer to 1(a): The extreme points of the set of feasible solutions for the following linear programming problem are $(3, 0)$, $(4, 0)$ and $(2, 1)$.

Solution to 1(b): Now comes the **final contest**. We plug the above **finalists** into the **goal function**, $z(x, y) = 5x - 3y$ and find who gives the **mininal value**.

- : For $(x, y) = (3, 0)$, we have $z(3, 0) = 5 \cdot 3 - 3 \cdot 0 = 15 \quad ,$

- : For $(x, y) = (4, 0)$, we have $z(4, 0) = 5 \cdot 4 - 3 \cdot 0 = 20 \quad ,$

- : For $(x, y) = (2, 1)$, we have $z(2, 1) = 5 \cdot 2 - 3 \cdot 1 = 7 \quad .$

Since the smallest value is 7 the lucky winner is $(x, y) = (2, 1)$, and the **value** of the optimal solution is 7.

Ans. to 1(b): The optimal solution is $x = 2$, $y = 1$ with the optimal value being 7.