

## Solutions to Attendance Quiz for Lecture 8

1. Set up an initial simplex tableau to the following linear programming problem.

Maximize  $z = x_1 + 3x_2 + 5x_3 - 4x_4$  subject to the restrictions

$$\begin{aligned} 2x_1 - 3x_2 + 4x_3 - x_4 &\leq 5 \quad , \\ 3x_1 \quad \quad + x_4 &\leq 6 \quad , \\ 2x_3 + 5x_4 &\leq 7 \quad , \\ x_1 \geq 0 \quad , \quad x_2 \geq 0 \quad , \quad x_3 \geq 0 \quad , \quad x_4 \geq 0 \quad . \end{aligned}$$

**Sol. of 1:** We must first convert the problem to **canonical form**. Since there are **three** inequalities, we need three **slack variables**, let's call them  $x_5, x_6, x_7$ . The canonical form is

Maximize  $z = x_1 + 3x_2 + 5x_3 - 4x_4 + 0 \cdot x_5 + 0 \cdot x_6 + 0 \cdot x_7$  subject to the restrictions

$$\begin{aligned} 2x_1 - 3x_2 + 4x_3 - x_4 + x_5 &= 5 \quad , \\ 3x_1 \quad \quad + x_4 + x_6 &= 6 \quad , \\ 2x_3 + 5x_4 + x_7 &= 7 \quad , \\ x_1 \geq 0 \quad , \quad x_2 \geq 0 \quad , \quad x_3 \geq 0 \quad , \quad x_4 \geq 0 \quad , \quad x_5 \geq 0 \quad , \quad x_6 \geq 0 \quad , \quad x_7 \geq 0 \quad . \end{aligned}$$

We first rewrite the goal equation as

$$-x_1 - 3x_2 - 5x_3 + 4x_4 + 0 \cdot x_5 + 0 \cdot x_6 + 0 \cdot x_7 + z = 0 \quad .$$

Now we transcribe the set of constraints in the form of a **simplex** tableau

$$\left[ \begin{array}{c|cccccccc|c|c} & x_1 & x_2 & x_3 & x_4 & x_5 & x_6 & x_7 & z & \\ \hline - & - & - & - & - & - & - & - & - & - \\ x_5 & 2 & -3 & 4 & -1 & 1 & 0 & 0 & 0 & 5 \\ x_6 & 3 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 6 \\ x_7 & 0 & 0 & 2 & 5 & 0 & 0 & 1 & 0 & 7 \\ \hline - & - & - & - & - & - & - & - & - & - \\ & -1 & -3 & -5 & 4 & 0 & 0 & 0 & 1 & 0 \end{array} \right] .$$

Note that the leftmost column is that of the **basic variables**. At the initial tableau these are always the slack variables. For example the basic variable of the first row is  $x_5$  since,  $x_5$  has coefficient 1 in the first row, but does not show up (i.e. has coefficients 0) in the other rows.

Similarly, the basic variable of the second row is  $x_6$  since the coefficient of  $x_6$  in the second row is 1 but it is 0 in the first and third rows. Finally, the basic variable of the third row is  $x_7$  since the coefficient of  $x_7$  in the third row is 1 but it is 0 in the first and second rows.

This is the **initial tableau**. Later on we will construct 'better' and 'better' tableaux and the set of basic-variables will keep changing.