Solutions to Attendance Quiz for Lecture 7

1. Consider the linear programming problem

Maximize $z = 3x + 2y$ subject to

$$2x - y \leq 6 \quad , \quad 2x + y \leq 10 \quad , \quad x \geq 0 \quad , \quad y \geq 0 \quad .$$

(a) Transform this problem to a problem in **canonical form**.

(b) Find all **basic solutions** and label them according to whether there are *feasible* (f) or *not faesible* (n)

(c) Find the optimal solution (or solutions in case there is more than one) and the optimal value.

Sol. to 1a: Introducing the *slack variables*, u and v we have that the above linear programming problem in **canonical form** is

Maximize $z = 3x + 2y$ subject to

$$2x - y + u = 6 \quad , \quad 2x + y + v = 10 \quad , \quad x \geq 0 \quad , \quad y \geq 0 \quad , \quad u \geq 0 \quad , \quad v \geq 0 \quad .$$

Sol. to 1b. There are 4 variables and 2 equations, so we have to look at all $\binom{4}{2} = (4 \cdot 3)/2$ ways of picking basic variables.

• Non Basic variables $\{x, y\}$; Basic variables $\{u, v\}$. Plugging-in $x = 0, y = 0$ we have the system

$$u = 6 \quad , \quad v = 10 \quad ,$$

and hence $(x, y, u, v) = (0, 0, 6, 10)$ is a basic solution. Since none of the variables are negative, it is a *feasible* basic solution aka as *extreme point*.

• Non Basic variables $\{x, u\}$; Basic variables $\{y, v\}$. Plugging-in $x = 0, u = 0$ we have the system

$$-y = 6 \quad , \quad y + v = 10 \quad ,$$

getting $y = -6$ and $v = 16$, and hence $(x, y, u, v) = (0, -6, 0, 16)$ is a basic solution. Since $y = -6$ is **negative**, this is **not** a feasible solution hence and it is **not** an extreme point.

• Non Basic variables $\{y, u\}$; Basic variables $\{x, v\}$. Plugging-in $y = 0, u = 0$ we have the system

$$2x = 6 \quad , \quad 2x + v = 10 \quad ,$$

getting $x = 3$ and hence $v = 10 - 2 \cdot 3 = 10 - 6 = 4$, and hence $(x, y, u, v) = (3, 0, 0, 4)$ is a basic solution. Since none of the variables are negative, it is a *feasible* basic solution aka as *extreme point*.

• Non Basic variables $\{y, v\}$; Basic variables $\{x, u\}$. Plugging-in $y = 0, v = 0$ we have the system

$$2x + u = 6 \quad , \quad 2x = 10 \quad ,$$

getting $x = 5$ and $u = 6 - 2 \cdot 5 = -4$, and hence $(x, y, u, v) = (5, 0, -4, 0)$ is a basic solution. Since $u = -6$ is **negative**, this is **not** feasible and hence **not** an extreme point.

• Non Basic variables $\{u, v\}$; Basic variables $\{x, y\}$. Plugging-in $u = 0, v = 0$ we have the system

$$2x - y = 6 \quad , \quad 2x + y = 10 \quad ,$$

Adding them gives $4x = 16$ hence $x = 4$, and $y = 10 - 2 \cdot 4 = 2$ and hence $(x, y, u, v) = (4, 2, 0, 0)$ is a basic solution. Since none of the variables are negative, it is a *feasible* basic solution aka as *extreme point*.

Summarizing we have the following

x	y	u	v	*Type*	z	*Truncated*
0	0	6	10	f	0	$(0, 0)$
0	-6	0	16	n	$-$	$-$
0	10	16	0	f	20	$(0, 10)$
3	0	0	4	f	9	$(3, 0)$
5	0	-4	0	n	$-$	$-$
4	2	0	0	f	16	$(4, 2)$

.

Since the **largest** value is 20 when $(x, y, u, v) = (0, 10, 16, 0)$, whose **truncated version** is $(x, y) = (0, 10)$ we get

Ans. to 1c: The optimal solution is $x = 0, y = 10$ and the **optimal value** is 20.