Solutions to the Intended Attendance Quiz for Lecture 6

This is the attendance quiz meant for Lecture 6, but Mr. Spahn gave another one.

1. Find the extreme points of the set of feasible solutions for the given linear programming problem and (b) find the optimal solution(s).

Mimimize $z = 2x + y$ subject to the restrictions

$$x + 2y \leq 6 \quad , \quad 2x + y \leq 6 \quad , \quad x + y \geq 2 \quad , \quad x \geq 0 \quad , \quad y \geq 0 \quad .$$

Sol. of 1: The **feasible region**, is the subset of the positive quadrant that is

• **below** the line $2x + y = 6$,

• **below** the line $x + 2y = 6$,

• **above** the line $x + y = 2$.

It is a **pentagon** with the set of **extreme points** (i.e. vertices)

$$\{(2,0)\,,\,(3,0)\,,\,(2,2)\,,\,(0,3)\,,\,(0,2)\} \quad .$$

(Note that the point $(2,2)$ is the intersection of the lines $x + 2y = 6$ and $2x + y = 6$ (subtracting we get $x - y = 0$ hence $x = y$, hence $3x = 6$ hence $x = 2$ and hence $y = 2$)

For the **final contest** we have

We have, since $z(x,y) = 2x + y$,

$$z(2,0) = 2 \cdot 2 + 0 = 4 \quad , \quad z(3,0) = 2 \cdot 3 + 0 = 6 \quad , \quad z(2,2) = 2 \cdot 2 + 2 = 6 \quad ,$$

$$z(0,3) = 2 \cdot 0 + 3 = 3 \quad , \quad z(0,2) = 2 \cdot 0 + 2 = 2 \quad .$$

The **minimal value** is 2 at the point $x = 0, y = 2$.

Ans. to 1a.: The set of extreme points is $\{(2,0)\,,\,(3,0)\,,\,(2,2)\,,\,(0,3)\,,\,(0,2)\}$.

Ans. to 1b.: The optimal solution is $x = 0$, $y = 2$ and the value at the optimal solution is 2.