

### Solutions to Attendance Quiz for Lecture 3

1. Set up a linear programming model of the situation described. Determine whether it is in standard form. If not make it standard.

A restaurant chef is planning a meal consisting of two foods, A, and B.

- Each kg of A contains 3 units of fat and 6 units of protein
- Each kg of B contains 1 unit of fat and 3 units of protein

The chef wants the meal to consist of at least 18 units of protein and at most 6 units of fat.

If the profit that he makes is 3 dollars per kg for food A and 5 dollars for food B, how many kilograms of each food should be served so as to **maximize** his profit?

#### Solution to 1.

Let  $x_1$  be the amount in kilograms that the chef makes of food A, and let  $x_2$  be the amount in kilograms that the chef makes of food B.

- The total amount of fat is  $3x_1 + x_2$  units.
- The total amount of protein is  $6x_1 + 3x_2$  units.

Since the total amount of fat in the two foods is  $3x_1 + x_2$  units, and it may not exceed 6, the first constraint is  $3x_1 + x_2 \leq 6$ .

Since the total amount of protein in the two foods is  $6x_1 + 3x_2$  units, and it must be at least 18 units, the second constraint is  $6x_1 + 3x_2 \geq 18$ .

The **profit** is  $3x_1 + 5x_2$ .

The **Mathematical model** is

Maximize  $z = 3x_1 + 5x_2$

subject to the constraints (or restrictions)

$$3x_1 + x_2 \leq 6 \quad ,$$

$$6x_1 + 3x_2 \geq 18 \quad ,$$

$$x_1 \geq 0 \quad , \quad x_2 \geq 0 \quad .$$

This is **not** in standard form, since the second constraint has a “ $\geq$ ” rather than a “ $\leq$ ”. To remedy it, we multiply both sides by  $-1$ , and of course change the “ $\geq$ ” to “ $\leq$ ”.

The **standard** form of the same linear programming problem is

$$\text{Maximize } z = 3x_1 + 5x_2$$

subject to the constraints (or restrictions)

$$3x_1 + x_2 \leq 6 \quad ,$$

$$-6x_1 - 3x_2 \leq -18 \quad ,$$

$$x_1 \geq 0 \quad , \quad x_2 \geq 0 \quad .$$