Solutions to Attendance Quiz for Lecture 1

1. Use Gauss-Jordan Reduction to find all solutions of the system

$$x + y + z = 3 \quad , \quad 2x - y + z = 2 \quad , \quad 3x \quad + 2z = 5 \quad .$$

Sol. of 1: The **augmented matrix** is

$$\begin{bmatrix} 1 & 1 & 1 & | & 3 \\ 2 & -1 & 1 & | & 2 \\ 3 & 0 & 2 & | & 5 \end{bmatrix} \quad .$$

Doing $r_2 - 2r_1 \to r_2$ and $r_3 - 3r_1 \to r_3$ gives

$$\begin{bmatrix} 1 & 1 & 1 & | & 3 \\ 0 & -3 & -1 & | & -4 \\ 0 & -3 & -1 & | & -4 \end{bmatrix} \quad .$$

Doing $r_3 - r_2 \to r_3$ gives

$$\begin{bmatrix} 1 & 1 & 1 & | & 3 \\ 0 & -3 & -1 & | & -4 \\ 0 & 0 & 0 & | & 0 \end{bmatrix} \quad .$$

Doing $-\frac{1}{3}r_2 \to r_2$ gives

$$\begin{bmatrix} 1 & 1 & 1 & | & 3 \\ 0 & 1 & \frac{1}{3} & | & \frac{4}{3} \\ 0 & 0 & 0 & | & 0 \end{bmatrix} \quad .$$

Now the matrix is in **row-echelon form** but not yet in **reduced row-echelon form**. To get it to be in reduced-row echelon form we have to perform $r_1 - r_2 \to r_1$ getting

$$\begin{bmatrix} 1 & 0 & \frac{2}{3} & | & \frac{5}{3} \\ 0 & 1 & \frac{1}{3} & | & \frac{4}{3} \\ 0 & 0 & 0 & | & 0 \end{bmatrix} \quad .$$

x and y are **basic variables** while z is a **free variable**.

In **everyday notation** this is

$$x + \frac{2}{3}z = \frac{5}{3} \quad ,$$

$$y + \frac{1}{3}z = \frac{4}{3} \quad .$$

Getting

$$x = \frac{5}{3} - \frac{2}{3}z \quad ,$$

$$y = \frac{4}{3} - \frac{1}{3}z \quad .$$

$$z = free.$$

This is the correct answer in **scalar form**. Since I did not ask for the vector form, this would have given you full credit.

However, if I did ask for the **vector form**, you would do it as follows.

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \frac{5}{3} - \frac{2}{3}z \\ \frac{4}{3} - \frac{1}{3}z \\ z \end{bmatrix}$$

$$\begin{bmatrix} \frac{5}{3} \\ \frac{4}{3} \\ 0 \end{bmatrix} + z \begin{bmatrix} -\frac{2}{3} \\ -\frac{1}{3} \\ 1 \end{bmatrix} \quad , \quad (z \quad free) \quad .$$

This is the final answer in **vector form**.