NAME: (print!) ____

E-Mail address: ____________________

MATH 354 (3), Dr. Z. , Exam 2, Mon. Nov. 27, 2023, 10:25-11:35am, TIL-246

FRAME YOUR FINAL ANSWER(S) TO EACH PROBLEM
No Calculators! No books! No Notes! To ensure maximum credit, organize your work neatly and be sure to show all your work.
Do not write below this line

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1. (out of 10)

2. (out of 10)

3. (out of 10)

4. (out of 10)

5. (out of 10)

6. (out of 10)

7. (out of 10)

8. (out of 10)

9. (out of 20)

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tot.: (out of 100)

Reminder from Linear Algebra:

The inverse of an $n \times n$ matrix $A = [a_{ij}]$ is

$$\frac{1}{\det(A)}[A_{ij}]^T \quad ,$$

where A_{ij} is $(-1)^{i+j}$ times the determinant of the (i,j) **minor**, which is the $(n-1)\times(n-1)$ matrix obtained by removing the i-th row and the j-th column.

Another way to find the inverse A^{-1} of an $n \times n$ matrix, A, is to stick the identity matrix I_n right after it, getting $[A|I_n]$, perform Gauss-Jordan elimination to get A to be the identity matrix, and whatever emerges to its right is the matrix A^{-1}.

1. (10 points) Consider the initial simplex tableau

BASIC	x_1	x_2	x_3	x_4	x_5	z	*RHS*
x_3	1	2	1	0	0	0	4
x_4	2	3	0	1	0	0	9
x_5	8	4	0	0	1	0	16
	-2	-3	0	0	0	1	0

.

Ans. entering variable: ; departing variable: ;
BASIC column of new tableau: [$]^T$;

Bottom line of next tableau:

(i) (1 point) What is the entering variable? Explain!

(ii) (2 points) What is the departing variable? Explain

(iii) (1 point): What is the BASIC column of the next tableau?

(iv) (6 points): What is the bottom (aka objective) line of the next tableau?

2. (10 points) If the initial simplex tableau was:

$BASIC$	x_1	x_2	x_3	x_4	x_5	z	RHS
x_3	1	2	1	0	0	0	4
x_4	2	3	0	1	0	0	9
x_5	8	4	0	0	1	0	16
	-2	-3	0	0	0	1	0

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and currently the BASIC column is

$$\begin{bmatrix} x_2 \\ x_5 \\ x_1 \end{bmatrix}$$

What is the x_3 column (inclduing the entry at the bottom line, i.e. what used to be 0) of the current tableau? (Explain!)

Ans.: The x_3 column of the current tableau is:

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3. (10 points) A certain linear programming problem with three variables x_1, x_2, x_3, and three constraints, has an optimal solution $x_1 = 0, x_2 = 1, x_3 = 10$, yielding the optimal value 101.

You are also told that its first constraint is **not tight**, i.e. if you plug-in the values of the optimal solution into the first constraint you get a **strict** inequality $<$ (they are **not** equal). Calling the dual variables corresponding to the first, second, and third constraints, w_1, w_2, w_3 respectively, you are also told that the dual constraints are

$$w_1 + w_2 + w_3 \geq 3 \quad , \quad w_1 + 2w_2 + 3w_3 \geq 13 \quad , \quad w_1 + 2w_2 + w_3 \geq 7 \quad .$$

Find (i) (8 points) the optimal solution of the dual problem. (ii) (2 points) the value of the goal function at that optimal solution of the dual problem

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Ans.: The optimal solution of the dual problem is

$$w_1 = \qquad\qquad w_2 = \qquad\qquad w_3 =$$

The value of the goal function at that optimal solution of the dual problem is:

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4. (10 points)
For the following transportation problem, find the initial basic feasible solution, M, using the **Minimal Cost Rule**. Also find the cost of that solution.

$$\mathbf{C} = \begin{bmatrix} 12 & 5 & 3 \\ 3 & 5 & 7 \\ 7 & 12 & 12 \end{bmatrix} \quad , \quad \mathbf{s} = \begin{bmatrix} 30 \\ 31 \\ 11 \end{bmatrix} \quad , \quad \mathbf{d} = \begin{bmatrix} 22 \\ 28 \\ 22 \end{bmatrix} \quad .$$

Ans.

$$\mathbf{M} = \begin{bmatrix} & & \\ & & \\ & & \end{bmatrix}$$

cost=

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5. (10 points)
For the following transportation problem, find the initial basic feasible solution, M, using **Vogel's Rule**. Also find the cost of that solution.

$$\mathbf{C} = \begin{bmatrix} 12 & 5 & 3 \\ 3 & 5 & 7 \\ 7 & 12 & 12 \end{bmatrix} \quad , \quad \mathbf{s} = \begin{bmatrix} 30 \\ 31 \\ 11 \end{bmatrix} \quad , \quad \mathbf{d} = \begin{bmatrix} 22 \\ 28 \\ 22 \end{bmatrix} \quad .$$

Ans.

$$\mathbf{M} = \begin{bmatrix} & & \\ & & \\ & & \end{bmatrix}$$

cost=

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6. (10 points) Find the permutation matrix, P, that solves the following assignment problem with four employees and four jobs, where C is the cost matrix whose (i, j) entry is the cost of assigning employee i to job j.

$$C = \begin{bmatrix} 6 & 4 & 1 & 5 \\ 2 & 7 & 4 & 8 \\ 2 & 2 & 2 & 2 \\ 13 & 17 & 10 & 18 \end{bmatrix}$$

Ans.:

$$P = \begin{bmatrix} \quad & \quad & \quad & \quad \\ \\ \\ \end{bmatrix}$$

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7. (10 points) In a certain transportation problem with four factories and four stores, the current basic feasible solution is

$$M = \begin{bmatrix} 0 & 21 & 0 & 11 \\ 0 & 0 & 34 & 0 \\ 25 & 12 & 13 & 0 \\ 0 & 4 & 0 & 0 \end{bmatrix} .$$

It was found out that the **entering variable** is x_{34}. find a cheaper solution, M', by performing the relevant step in the Transportation Problem algorithm.

Ans:.

$$M' = \begin{bmatrix} & & & \\ & & & \\ & & & \\ & & & \end{bmatrix} .$$

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8. (10 points) In the course of solving an assignment problem with four employees and four jobs, the following partial solution was arrived at:

$$\begin{bmatrix} 0^* & 1 & 2 & 0 \\ 3 & 0 & 4 & 0^* \\ 0 & 5 & 0^* & 4 \\ 2 & 1 & 0 & 3 \end{bmatrix}$$

find the permutation matrix that is the final solution.

Ans.

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9. (20 points altogether) For the following transportation problem.

$$\mathbf{C} = \begin{bmatrix} 6 & 5 & 4 \\ 3 & 3 & 3 \\ 5 & 6 & 6 \end{bmatrix} \quad , \quad \mathbf{s} = \begin{bmatrix} 14 \\ 16 \\ 12 \end{bmatrix} \quad , \quad \mathbf{d} = \begin{bmatrix} 15 \\ 16 \\ 11 \end{bmatrix} \quad .$$

(i) (2 points) Explain why the following solution

$$\begin{bmatrix} 0 & 3 & 11 \\ 15 & 1 & 0 \\ 0 & 12 & 0 \end{bmatrix}$$

is a **basic feasible solution**. Also find its cost.

(ii) (18 points) Starting with the above basic feasible solution as the initial basic feasible solution, find the optimal solution, and the **minimal cost**.

Ans.:

$$\begin{bmatrix} & & \\ & & \\ & & \end{bmatrix} \qquad \qquad minimal \quad cost =$$
