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Real Quiz 9

① Adrien Marie Legendre. He wrote Elements de géométrie.

② Victor Poncelet. He wrote Traité des propriétés projectives des figures. It contains the new form of geometry, such as cross ratio, perspectivity, projectivity, involution, and circular points at infinity.

③ His father was a small town mayor.

④ Balzac

⑤ a.) $\sqrt{2\pi}$

b.) Let $a = \int_{-\infty}^{\infty} e^{-x^2/2}\,dx$

dummy variable, so we can replace with y

Then $a^2 = \left(\int_{-\infty}^{\infty} e^{-x^2/2}\,dx\right)\left(\int_{-\infty}^{\infty} e^{-y^2/2}\,dy\right) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{-(x^2+y^2)/2}\,dx\,dy$

Thus in polar coordinates, we have

$$a^2 = \int_0^{\infty}\int_0^{2\pi} re^{-r^2/2}\,d\theta\,dr$$

$$a^2 = \int_0^{\infty} re^{-r^2/2}\left(\int_0^{2\pi} d\theta\right) dr$$

$$a^2 = \int_0^{\infty} re^{-r^2/2}(2\pi)\,dr$$

$$a^2 = 2\pi \int_0^{\infty} re^{-r^2/2}\,dr$$

We use u-substitution where $u = \frac{-r^2}{2}$. Then $du = -r$, so we have

$$-\int e^u \, du = -e^{-r^2/2} + C \quad \text{since } \int e^u = e^u$$

Thus

$$\left[-e^{-r^2/2}\right]_0^\infty = -\frac{1}{e^\infty} - (-e^0) = 0 - (-1) = 1$$

Thus $a^2 = 2\pi(1)$, so $a^2 = 2\pi$ and hence $a = \sqrt{2\pi}$.