Quiz 9

① Fermat

② Victor Poncelet wrote "Traité des propiétées projectives des figures" which contained a new form of geometry

③ mayor of a small town

④ Balzac

⑤ a) $$\int_{-\infty}^{\infty} e^{-\frac{x^2}{2}}\,dx = \sqrt{2\pi}$$

b) Proof: x is a dummy variable which we can replace with y, so

$$C = \int_{-\infty}^{\infty} e^{\frac{-x^2}{2}}\,dx = \int_{-\infty}^{\infty} e^{\frac{-y^2}{2}}\,dy$$

$$\text{so } C^2 = \left(\int_{-\infty}^{\infty} e^{\frac{-x^2}{2}}\,dx\right)\left(\int_{-\infty}^{\infty} e^{\frac{-y^2}{2}}\,dy\right)$$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{-(x^2+y^2)/2}\,dx\,dy$$

Convert to Polar:

$$C^2 = \int_0^{\infty}\int_0^{2\pi} re^{-r^2/2}\,d\theta\,dr = 2\pi\int_0^{\infty} re^{-r^2/2}\,dr$$

$$\frac{d}{dr}\left(-e^{-r^2/2}\right) = re^{-r^2/2} \quad \text{so} \quad \int re^{-r^2/2} = -e^{-r^2/2} + C$$

So $\int_0^{\infty} r e^{-r^2/2}\, d\theta\, dr = -e^{-r^2/2}\Big|_0^{\infty} = -e^{-\infty} - -e^{0} = 1$

$c^2 = 2\pi$

so $c = \sqrt{2\pi}$