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Real Quiz 8

① Laplace's partial differential equation is $\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = 0$

Euler derived it before Laplace.

② Gauss was born in Brunswick. His father was a day laborer.

③ Yes they can. Gauss proved this.

④a We must check the four properties

(i.) It is closed under "multiplication" because any two integers added together mod 6 "stays in the family."

(ii.) Addition is associative when done mod 6

(iii.) Each element has an inverse. 0 and 3 are inverses of themselves, and they are both in the set, 1 and 5 are inverses of each other and they are both in the set, and 2 and 4 are inverse of each other and they are both in the set.

(iv.) The identity element in this case would be 0, and 0 is in the set.

④b {1, 3, 5} is not a subgroup because it does not contain the identity element, which is 0.

(4c) H is a subgroup because

(i.) It contains the identity element, which is 0.

(ii.) It is closed under "multiplication" because

$0 * 2 = 2$ which is in the set

$0 * 4 = 4$ which is in the set

$2 * 4 = 0$ which is in the set

$2 * 0 = 2$ which is in the set

$4 * 0 = 4$ which is in the set

$4 * 2 = 0$ which is in the set

(iii.) 0 is its own inverse, and that is in the set, and 2 and 4 are inverses of each other and they are both in the set.

(4d) One coset is eH, so first we kick out the members of H from G

Let $a = \{1, 3, 5\}$

$1 * 0 = 1$
$3 * 2 = 5$
$5 * 4 = 3$

So another coset is $\{1, 3, 5\}$. Thus the coset decomposition is

$G = eH + aH = \{0, 2, 4\} + \{1, 3, 5\}$