

1.  $\frac{d^2 V}{dx^2} + \frac{d^2 V}{dy^2} + \frac{d^2 V}{dz^2} = 0$ ; Euler
2. Brunsvick, Germany; son of a day laborer
3. Yes. Gauss.
4. a) It has an identity element 0 such that any element of  $G$  "multiplied" by 0 gives you the element again. Any two elements "multiplied" together gives an element of the group. For example  $4 * 3 = 1$ . There is an inverse element for every element, that is for each element it has an inverse such that when "multiplied" by the inverse you get the identity element. For example,  $2 * 4 = 0$ . So 2 is the inverse of 4 and 4 is the inverse of 2. It is associative because addition is associative.
 

b) It is not a subgroup because it does not contain the identity element 0.

c)  $H = \{0, 2, 4\}$   
 $0 * 2 = 2 * 0 = 2$ ,  $0 * 4 = 4 * 0 = 4$ ,  $2 * 4 = 4 * 2 = 0$   
 so when you multiply any of the elements in the set, you get an element of the set. It is associative because addition is associative. It also contains the identity element of  $G$ .

d)  $1H = \{1 * 0, 1 * 2, 1 * 4\} = \{1, 3, 5\}$   
 so  $G = 1H \cup H$