

## Quiz 8

11/17/2021

1. The following is Laplace's equation:

$$\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = 0$$

2. Gauss was born in Brunswick, Germany.  
His father was a day laborer.

3. Yes, proven by Fermat.

4.  $G = \{0, 1, 2, 3, 4, 5\}$

- a)  $G$  is a group because:

1. it has an identity member: 0 s.t.  
 $0 * n = n$  for  $n \in G$ .

2.  $G$  contains all of the possible products of its members, since it contains numbers 0-5, and the modulo of 6 of any number falls within  $\{0, 5\}$ .

3. By linear algebra,  $G$  is also associative.

- b) No  $\{1, 3, 5\}$  is not a subgroup of  $G$  because

$$1 * 3 = 4, \text{ and } 4 \text{ is not in } \{1, 3, 5\}$$

so this set doesn't include it's numbers.

d)  $H = \{0, 2, 4\}$  is a subgroup.

1. It has the identity element 0.
2. It includes the products of its members:

$$0 * 2 = 2 \in H \quad 0 * 4 = 4 \in H$$

$$2 * 4 = 0 \in H$$

d) Take  $0 * H$  to be the first coset.

then take

$$1 * H = \{1 * 0, 1 * 2, 1 * 4\} = \{1, 3, 5\}$$

Therefore the left coset decomposition of  $G$  with respect to  $H$  is

$$G = 0 * H \cup 1 * H$$