

Sarah Magno

Real Quiz 7

- ① Guido Grandi. In his "proof," a father leaves a gem to his two sons, who may each keep it for one year in alternation. This way, it belongs to each son for $\frac{1}{2}$.
- ② Mme. Du Châtelet; Voltaire
- ③ Joseph Louis Lagrange
- ④ The cycles are $(1\ 9)(2\ 8)(3\ 7)(4\ 5\ 6)$, which is

$(1\ 9)(2\ 8)(3\ 7)(4\ 5\ 6)$ in cycle notation. The lengths of the cycles are 2, 2, 2, and 3, so the $\text{lcm}(2, 2, 2, 3) = 6$, thus 6 is the smallest such i that Π^i is the identity permutation.

- ⑤ Let 9 = the blank spot in the puzzle. We know that the invariant S is
$$S = \text{number of inversions} + \text{taxicab distance from } 9$$

where taxicab distance = row number + column number - 2 of the blank spot (9). It measures the distance to the top left hand corner of the puzzle

By the lemma stated, in any legal move (horizontal or vertical), the number of inversions changes by an odd integer, and the taxicab distance changes by ± 1 (since either the row or column number changed by 1). Thus the parity never changes, and so if it starts odd, it stays odd, and if it starts even it stays even.

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no inversions in initial state

Initial State: $S = 0 + (3 + 3 - 2) = 4$

Final State: $S = 3 + (3 + 3 - 2) = 7$

↑
because it took 3 inversions from
1 2 3 4 5 6 7 8 9 to 3 2 1 4 5 6 7 8 9

Since 4 is even and 7 is odd, they have different parities, so it is impossible by sliding to get to the final state.