Real Quiz # 7 for Dr. Z.'s MathHistory

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Email DrZlinear@gmail.com as soon as I tell you (around 3:15pm)

Subject: q7

with an attachment called

q7FirstLast.pdf (e.g. q7PaulErdos.pdf)

1. (2 points) Who "proved" that

$$1 - 1 + 1 - 1 + 1 - 1 + 1 - 1 + \ldots$$

equals $\frac{1}{2}$? Briefly describe his "proof".

Guido Drandi

He thaght of if 2 siblings share a inhertance switching every yer than each sibling wold have it 1/2 the time

2. (2 points) Who translated Newton's *Principia* into French, and who wrote *Lettres sur les Anglais*?

Marquise du Chatelet

Voltaire

3. (1 point) Who proved that every integer is a sum of four or less squares?

Lagrange

4. (2 point) Express the permutation

$$\begin{pmatrix} \cancel{1} & \cancel{2} & \cancel{3} & \cancel{4} & \cancel{5} & \cancel{6} & \cancel{7} & \cancel{8} & \cancel{9} \\ 9 & 8 & 7 & 5 & 6 & 4 & 3 & 2 & 1 \end{pmatrix},$$

as a product of disjoint cycles. What is the smallest i such that π^i is the identity permutation?

$$\begin{pmatrix} 1 & 9 \\ 9 & 1 \end{pmatrix} \begin{pmatrix} 2 & 8 \\ 8 & 2 \end{pmatrix} \begin{pmatrix} 3 & 7 \\ 7 & 3 \end{pmatrix} \begin{pmatrix} 4 & 5 & 6 \\ 5 & 6 & 4 \end{pmatrix}$$

$$i = \mathrm{lcm}(2,3) = 6$$

5. (3 points) Prove that in the nine puzzle, if you start with

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & \end{pmatrix} \quad \textit{it is impossible to get to} \quad \begin{pmatrix} 3 & 2 & 1 \\ 4 & 5 & 6 \\ 7 & 8 & \end{pmatrix} = B$$

by sliding.

Note: You can use (without proving) the lemma that whenever two elements of a permutation trade places, and all the other elements stay where they are, the number of inversions changes by an odd integer (i.e. is $\pm 1, \pm 3, \pm 5, \pm 7, \ldots$ of what it used to be.

The invariant is defined as the sum of number of inversions and the taxi cab distance of the blank from the top ~~right~~ left corner.

From the LEMMA the ~~number~~ change in the number of inversions will always be odd for every move

The taxicab distance will always change by ± 1 for every move

From this, at any given move the invariant will change by an even number because an odd number ± 1 will always be even.

If the invariant always changes by an even number then the parity of Invariant will never change if all moves made are legal moves

	Inversions		Taxi cab	
Invariant (A) =	0	+	4	= 4 ⇒ Even
Invariant (B) =	3	+	4	= 7 ⇒ Odd

Because the Invariant's ~~changes~~ parity changes from even to odd when it moves from A to B, this is impossible to occur by the legal moves.