Sarah Magno

Real Quiz 6

① Galileo Galilei was the author. It was published in 1638.

② René Descartes unified Algebra and Geometry. The book was published in 1637.

③ Isaac Newton and Gottfried Leibniz were the two people involved. Newton was the first to discover it, and Leibniz was the first to publish it.

④ Let $x = u+v$. Plugging in for x, we obtain

$$(u+v)^3 + 6(u+v) - 7 = 0$$

Expanding,

$$u^3 + 3u^2v + 3uv^2 + v^3 + 6(u+v) - 7 = 0$$

Rewrite this as

$$u^3 + v^3 + (3u^2v + 3uv^2 + 6(u+v)) - 7 = 0$$

Factoring,

$$u^3 + v^3 + 3uv(u+v) + 6(u+v) - 7 = 0$$

$$u^3 + v^3 + (u+v)(3uv+6) - 7 = 0$$

We demand that $3uv + 6 = 0$. This means that $uv = -2$. Thus $u^3v^3 = -8$.

Returning to the original equation, we see that

$$u^3+v^3=7$$

thus u^3 and v^3 have a sum of 7 and a product of -8. We use the formula

$$x^2-(\text{sum})(x)+\text{product}$$

to obtain

$$x^2-7x-8$$

Setting equal to 0 and factoring,

$$x=8,\ x=-1$$

So $u^3=8$ and $v^3=-1$, thus $u=2$ and $v=-1$.

So one solution is $x=u+v=2+(-1)=1$

We find the other two roots by using $w=\frac{-1+\sqrt{3}i}{2}$ and $w^2=\frac{-1-\sqrt{3}i}{2}$

$$wu+w^2v=\left(\frac{-1+\sqrt{3}i}{2}\right)(2)+\left(\frac{-1-\sqrt{3}i}{2}\right)(-1)=\frac{-1+3\sqrt{3}i}{2}$$

$$w^2u+wv=\left(\frac{-1-\sqrt{3}i}{2}\right)(2)+\left(\frac{-1+\sqrt{3}i}{2}\right)(-1)=\frac{-1-3\sqrt{3}i}{2}$$

So the three roots of the equation are 1, $\frac{-1+3\sqrt{3}i}{2}$, and $\frac{-1-3\sqrt{3}i}{2}$