Quiz 6

1. Galileo Galilei, 1638
2. Descartes, 1637
3. Isaac Newton and Gottfried Leibniz
   - Newton discovered
   - Leibniz published

(4) $x^3 - 6x - 7 = 0$

$x = u + v \rightarrow (u+v)^3 - 6(u+v) - 7 = 0$

$= u^3 + 3u^2v + 3uv^2 + v^3 - 6(u+v) - 7 = 0$

$= u^3 + v^3 + [3uv(u+v) - 6(u+v)] - 7 = 0$

$= u^3 + v^3 + (u+v)(3uv - 6) - 7 = 0$

Set $3uv - 6 = 0$

$uv = 2$

$u^3 + v^3 = 7$

$u^3 v^3 = 8$

$X^2 - 7X + 8 = 0$

$\rightarrow \frac{7 \pm \sqrt{49 - 32}}{2} = \frac{7 \pm \sqrt{17}}{2}$

$u^3 = \frac{7 + \sqrt{17}}{2}$ $\quad v^3 = \frac{7 - \sqrt{17}}{2}$

$u = \sqrt[3]{\frac{7+\sqrt{17}}{2}}$ $\quad v = \sqrt[3]{\frac{7-\sqrt{17}}{2}}$

$x_1 = u + v = \sqrt[3]{\frac{7+\sqrt{17}}{2}} + \sqrt[3]{\frac{7-\sqrt{17}}{2}}$

$x_2 = \frac{-1 + \sqrt{3}i}{2} \cdot \sqrt[3]{\frac{7+\sqrt{17}}{2}} + \frac{-1 - \sqrt{3}i}{2} \cdot \sqrt[3]{\frac{7-\sqrt{17}}{2}}$

$x_3 = \frac{-1 - \sqrt{3}i}{2} \cdot \sqrt[3]{\frac{7+\sqrt{17}}{2}} + \frac{-1 + \sqrt{3}i}{2} \cdot \sqrt[3]{\frac{7-\sqrt{17}}{2}}$