

Quiz 6

10/20/2021

1. 'Discorsi' was written by Galileo Galilei in 1638.
2. Algebra and geometry were unified by Réne Descartes in 1637 when he published 'Géométrie'.
3. Differential calculus was first discovered by Isaac Newton, but Gottfried Leibniz was the first to publish it.

4. $x^3 + 6x - 7 = 0$ Cardano's method to solve for x :

1. Set $x = u + v$. Then:

$$(u+v)^3 + 6(u+v) - 7 = 0$$

$$u^3 + v^3 + 3u^2v + 3uv^2 + 6(u+v) - 7 = 0$$

$$u^3 + v^3 + 3uv(u+v) + 6(u+v) - 7 = 0$$

$$u^3 + v^3 + (3uv + 6)(u+v) - 7 = 0.$$

2. Find u, v such that the term $(3uv + 6)(u+v) = 0$.

$$3uv + 6 = 0$$

$$uv = -2, \text{ and then } (uv)^3 = u^3v^3 = (-2)^3 = -8.$$

3. Now the cubic equation from 1. becomes:

$$u^3 + v^3 - 7 = 0$$

$$u^3 + v^3 = 7.$$

4. Since $u^3v^3 = -8$ and $u^3 + v^3 = 7$, u^3 and v^3 will be the solutions to the following quadratic equation:

$$x^2 - 7x - 8 = 0.$$

5. Find x s.t:

$$x^2 - 7x - 8 = 0$$

$$(x - 8x) + (x - 8) = 0$$

$$(x - 8)(x + 1) = 0$$

$$x = 8 \text{ and } x = -1.$$

6. Now we have that:

$$u^3 = 8 \quad \text{and} \quad v^3 = -1$$

$$u = 2$$

$$v = -1$$

(since $x = u^3$ or $x = v^3$)

7. There are 3 solutions to $x^3 + 6x - 7$ based on u and v that we found:

$$1. \quad x = u + v = 2 + (-1) = \boxed{1}$$

$$2. \quad x = \frac{-1 + i\sqrt{3}}{2} \cdot u + \frac{-1 - i\sqrt{3}}{2} \cdot v$$

$$= \frac{-1 + i\sqrt{3}}{2} \cdot 2 + \frac{-1 - i\sqrt{3}}{2} \cdot (-1) = \boxed{\frac{-1 + 3i\sqrt{3}}{2}}$$

$$3. \quad x = \frac{-1 + i\sqrt{3}}{2} \cdot v + \frac{-1 - i\sqrt{3}}{2} \cdot u$$

$$= \frac{-1 + i\sqrt{3}}{2} \cdot (-1) + \frac{-1 - i\sqrt{3}}{2} \cdot 2 = \boxed{\frac{-1 - 3i\sqrt{3}}{2}}$$

Answer: The 3 solutions to $x^3 + 6x - 7 = 0$ are

$$x = 1, \quad x = \frac{-1 + 3i\sqrt{3}}{2}, \quad x = \frac{-1 - 3i\sqrt{3}}{2}.$$