**Getting to know you Quiz (does not count towards the grade)**

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Subject: pre0

with an attachment: pre0FirstLast.pdf

**1.**: What are your career goals?

I would like to become a highschool mathematics teacher.

**2.**: What are your hobbies?

I enjoy knitting, baking, making custom keyboards.

**3.** What is a rational number?

A rational number is a number that can be written in the form $\frac{a}{b}$ such that $a, b \in \mathbb{Z}$.

**4.** Prove that the sum of two rational numbers is also a rational number,

Let $a, b, c, d \in \mathbb{Z}$. Two rational numbers can be written as $\frac{a}{b}, \frac{c}{d}$.

$$\frac{a}{b} + \frac{c}{d} = \left(\frac{a}{b}\right)d + \left(\frac{c}{d}\right)b = \frac{ad}{bd} + \frac{cb}{db} = \frac{ad+cb}{bd}.$$

The sum of $ad$ and $cb$ is an integer, likewise $bd \in \mathbb{Z}$.

$\therefore$ sum of two rational numbers is also a rational number.

**5.** Prove or disprove (by giving a counterexample) : "the sum of two irrational numbers is always also an irrational number"

The sum of two irrational numbers is always irrational is false.

$-\sqrt{2}$ is irrational and $\sqrt{2}$ is irrational

The sum of $-\sqrt{2}$ and $\sqrt{2}$ is $0$, which is is not an irrational number.

**6.** Prove that there are infintely many primes.

Suppose there is a finite list of primes called P.

If we were to take the list of primes, called $q$, and add 1, we can look at $q+1$ to see if it is prime. If $q+1$ is prime, it would be outside the bounds of P, disproving our statement of a finite list of primes. Suppose $q+1$ is not prime, then it would be divisible by some prime $r$ within our list $p$. This would not be possible as our $r$ would need to divide $q+1-q$, which means $r \mid 1$ $\times$.

$\therefore$ There are infinitely many primes.

**7.** Prove that $\sqrt{5}$ is an irrational number.

Suppose $\sqrt{5}$ is rational. This means that $\sqrt{5}$ can be represented as a distinct ratio between $\exists p, q \in \mathbb{Z}$, $\frac{p}{q}$.

$$\left(\sqrt{5} = \frac{p}{q}\right)^2$$

$$5 = \frac{p^2}{q^2}$$

$$p^2 = 5q^2$$

This implies that $5 \mid p^2$, which means $5 \mid p$

Since $5 \mid p$, we can say $5r = p$ for some $r \in \mathbb{Z}$.

The same can be said to $q$, which means 5 is a common factor. $\times$.

$\sqrt{5}$ is not a rational number, therefore is irrational