Getting to know you Quiz (does not count towards the grade)

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Email to DrZlinear@gmail.com when I tell you to

Subject: pre0

with an attachment: pre0FirstLast.pdf

1.: What are your career goals?

My career goal is to become a high school math teacher.

2.: What are your hobbies?

My hobbies include photography, playing the piano, and calligraphy.

3. What is a rational number?

A rational number is a fraction in the form $\frac{p}{q}$, where p and q are integers and q is not 0.

4. Prove that the sum of two rational numbers is also a rational number,

Let a and b be two rational numbers. Thus $a = \frac{p}{q}$ and $b = \frac{m}{n}$ for some $p, q, m, n \in \mathbb{Z}$ and $q, n \neq 0$. Thus $a+b = \frac{pn+mq}{qn}$. By the closure property of addition and multiplication for integers, $(pn+mq)$ and $(qn) \in \mathbb{Z}$ and $qn \neq 0$, so $a+b$ is a rational number.

5. Prove or disprove (by giving a counterexample) : "the sum of two irrational numbers is always also an irrational number"

This is false. A counterexample would be $\sqrt{2}$ and $-\sqrt{2}$, which added together give 0, which is rational.

6. Prove that there are infintely many primes.

For the sake of contradiction, suppose that the number of primes is finite. Let $S = \{p_1, p_2, \dots, p_k\}$ be the finite list of prime numbers. Let $N = p_1 p_2 \cdots p_k + 1$. Thus $N \in \mathbb{N}$, so N is either prime or composite. If N is prime, then N is a different prime from the primes in S, contradicting that S is finite. If N is composite, then N has a prime divisor, call it q. (continued on next page)

7. Prove that $\sqrt{5}$ is an irrational number.

For the sake of contradiction, suppose that $\sqrt{5}$ is rational, and it can be written as $\sqrt{5} = \frac{a}{b}$, where $a, b \in \mathbb{Z}$ and $b \neq 0$. Squaring both sides gives $5 = \frac{a^2}{b^2}$, thus $a^2 = 5b^2$.

⑥ Continued. None of the elements in S can equal q, since none of the elements in S can divide N; there will be a remainder of 1. Therefore q is a prime that is not in the finite list of primes, so both cases show that there can not be a finite list of primes.