

Getting to know you Quiz (does not count towards the grade)

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Email to DrZlinear@gmail.com when I tell you to

Subject: pre0

with an attachment: pre0FirstLast.pdf

1.: What are your career goals?

I want to use my knowledge in Computer Science and Math to become an expert Software Engineer. Also hopefully one day work at Google or Apple!

2.: What are your hobbies?

I really like to build computers for fun! Unfortunately because of the pandemic computer parts are super expensive so I haven't been able to indulge in my hobbies recently.

3. What is a rational number?

A rational number is a number that can be represented as a ratio of two integers.

4. Prove that the sum of two rational numbers is also a rational number,

Let  $r$  and  $s$  be a rational number. It follows that  $r = p/q$  and  $s = t/v$  for some integers  $p, q, t, v$ . Then  $r+s = p/q + t/v = (pv + tq)/qv$ . Since the multiplication and sum of two integers is an integer, we can conclude that  $(pv+tq)$  is an integer and  $qv$  is also an integer, thus  $r+s$  is a rational number.

5. Prove or disprove (by giving a counterexample) : "the sum of two irrational numbers is always also an irrational number"

We can disprove this statement by observing that  $\pi - \pi = 0$ .

6. Prove that there are infinitely many primes.

Didn't have time for this question!

7. Prove that  $\sqrt{5}$  is an irrational number.

Assume  $\sqrt{5}$  is rational. Then we can represent it as  $\sqrt{5} = p/q$  for some integers  $p, q$ . Further assume  $p$  and  $q$  have no common factors. Then it follows that  $5 = p^2 / q^2$ . Hence  $5q^2 = p^2$ . We can see that 5 divides  $p^2$ , therefore divides  $p$  since 5 is prime. It follows that  $p = 5s$  for some integer  $s$ . Furthermore  $5s^2 = p^2 = 5q^2$ . Similar to before we can conclude that 5 divides  $q$ . Since  $p$  and  $q$  can be further divided by 5, we have reached a contradiction, hence  $\sqrt{5}$  is irrational.