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1. I hope to go to medical school for an MD/PhD. and then become a clinical researcher.

2. Hobbies: writing, reading, growing plants, training my dog, mathematics.

3. A rational number is any number that isn't irrational. Such numbers can be written in the form p/q where p, q are integers and $q \neq 0$.

4. Sum of two rational numbers is rational.

Proof:

Suppose $M = \frac{a}{b}$ and $N = \frac{c}{d}$ where $a, b, c, d \in \mathbb{Z}$ and $b, d \neq 0$. Then M and N are rational numbers.

Now take $S = M + N = \frac{a}{b} + \frac{c}{d} = \frac{a \cdot d + c \cdot b}{bd}$

Since $a, b, c, d \in \mathbb{Z}$ then $ad + bc$ is also an integer.

Assign $p = ad + cb$.

Since $b, d \in \mathbb{Z}$ and $b, d \neq 0$ then $bd \in \mathbb{Z}$ and $bd \neq 0$.

Assign $q = bd$.

We have that $S = \frac{p}{q}$

which perfectly fits the definition of a rational number.

5. Sum of two irrational numbers is always irrational

Disproof.

Take $\sqrt{2}$ and $-\sqrt{2}$. Both are irrational.

But $\sqrt{2} + (-\sqrt{2}) = 0$ which is rational. //

6. There are infinitely many prime numbers.

Proof:

Assume there are only finitely many prime numbers:

$p_1, p_2, p_3 \dots p_n$.

Now take some number $M = p_1 \cdot p_2 \cdot \dots \cdot p_n + 1$.

Assuming we do have $p_1, p_2 \dots p_n$ prime numbers then M is bigger than all of them since it's the product of them plus one. (The list of prime numbers is nonempty, because at least 2 is a prime number).

When divided by any prime number from our list M has a remainder of 1. The two addends thus don't have a common divisor other than 1, therefore M doesn't have a multiple other than 1 and M.