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Attendance Quiz for Lecture 8

Part I:

① Who proved that π is irrational? Johann Heinrich Lambert

② Who proved that π is transcendental? Ferdinand von Lindemann

③ Use this style of argument to prove that $\sqrt{3}$ is irrational.

Suppose that there are two integers a and b such that

$$\sqrt{3} = \frac{a}{b}$$

Thus $3 = \frac{a^2}{b^2}$, so $a^2 = 3b^2$. Then there also exist a pair (a,b) with the smallest value of $a+b$. WLOG, the above pair has $a+b$ as small numbers.

Intermediate fact: If n can not be divided by 3, then n^2 can not be divided by 3.

Proof: Suppose that $n = 3m+1$. Then $n^2 = (3m+1)^2 = 9m^2 + 6m + 1 = 3(3m^2+2m)+1$ so 3 does not divide in evenly.

Next we use the corollary that if n^2 can be divided by 3, then n can be divided by 3. Thus $a = 3n$ for some integer n, so $n = \frac{a}{3}$. Then we can say

$$(3n)^2 = 3b^2$$
$$9n^2 = 3b^2$$
$$3n^2 = b^2$$

Hence b^2 can be divided by 3, so b must be able to be divided by 3 as well. We let $b=3m$ for some integer m, so $m=\frac{b}{3}$. Then we write

$$(3m)^2=3n^2$$
$$9m^2=3n^2$$
$$3m^2=n^2$$

So (m,n) is also a possibility. But $m+n=\frac{b}{3}+\frac{a}{3}=\frac{a+b}{3}<a+b$. Thus we arrive at a contradiction.

4. The rational number $\frac{355}{113}$ is important and famous because it is a good approximation for the value of π. It comes within 0.000009% of the actual value of π.

Part II:

1. $\frac{11}{4}=2+\frac{3}{4}=2+\frac{1}{\frac{4}{3}}=2+\frac{1}{1+\frac{1}{3}}$

2. Using the geometrical representation, we see that

$$(2b-a)^2=2(a-b)^2$$

Simplifying, we obtain

$$(2b-a)^2-2(a-b)^2=-(a^2-2b^2)$$

Suppose that there exist integers a,b such that $a^2-2b^2=0$. Let (a,b) be the pair with the smallest sum $(a+b)$. Let $c=2b-a$ and $d=a-b$. Then $c^2-2d^2=0$. But $c+d=2b-a+a-b=b<a+b$. We arrive at a contradiction.