Sarah Cabric Attendance Quiz 8 10/4/2021

AQ 1: Who proved that π is irrational? Johann Heinrich Lambert

AQ 2: Who proved that π is transcendental? Ferdinand von Lindemann

AQ 3: Why is the rational number $\frac{355}{113}$ important and famous?

It is the best rational approximation of π with a denominator of four digits or fewer, which is accurate to six decimal places.

1. $\frac{11}{4} = 2 + \frac{3}{4} = 2 + \frac{1}{4/3} = 2 + \frac{1}{1 + \frac{1}{3}}$ $[2, 1, 3]$

2. Assume to the contrary that $\sqrt{2}$ is rational so it can be written in the form $\frac{p}{q} = \sqrt{2}$ where p, q are positive integers. We are also assuming that p/q is in simplest form. Square both sides to get that $\frac{p^2}{q^2} = 2$ and multiply both sides by q^2 to get $p^2 = 2q^2$. So, p^2 must be even since it is equal to $2 \cdot q^2$, where q^2 is an integer. Since p^2 is even, p must be even. So $p = 2m$ for some integer m. Substituting that into the original: $(2m)^2 = 2q^2$ so $4m^2 = 2q^2$ so $q^2 = 2m^2$. So q^2 is even since it is equal to $2m^2$, so q must be even. However, we assumed that p and q was in simplest terms, but we have just shown that they are both even so they cannot be in simplest terms. So $\sqrt{2}$ must be irrational.