

1. Johann Heinrich Lambert
2. Ferdinand von Lindemann
3.  $\sqrt{3}$  is irrational
4.  $\frac{355}{113} \approx \pi$

$$1. \frac{11}{4} = 2\frac{3}{4} = 2 + \frac{1}{\frac{4}{3}} = 2 + \frac{1}{1 + \frac{1}{3}}$$

$$4 \sqrt{\frac{11}{4}}$$

$$3 \sqrt{\frac{11}{3}} = 1\frac{1}{3}$$

2. Assume  $\sqrt{2}$  is rational such that  $\sqrt{2} = \frac{a}{b}$ , where  $\gcd(a, b) = 1$  and  $b \neq 0$

then  $2 = \frac{a^2}{b^2} \rightarrow 2b^2 = a^2$  since  $2b^2$  is

even then  $a^2$  is even implying that  $a$  is even so  $a = 2k$ ,  $k \in \mathbb{Z}$

Now we have  $2b^2 = (2k)^2 \rightarrow 2b^2 = 4k^2$   
 $b^2 = 2k^2$  and now we see  $b$  is even.

We have that  $a, b$  are both even

meaning that they can be divided by 2

but we already assumed  $\gcd(a, b) = 1$

So contradiction. Thus  $\sqrt{2}$  is irrational.