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Attendance 8 10/04/2021

1. Who proved that π was irrational
   Johann Heinrich Lambert in 1760s

2. Who proved that pi is transcendental
   Ferdinand von Lindemann

3. Use this style of argument to prove that $\sqrt{3}$ is irrational
   $\sqrt{3} = \frac{a}{b}$ then $3 = \frac{a^2}{b^2}$ take (a, b) to be the smallest values for $(a+b)$. Then:
   $a^2 = 3b^2$. It follows that a^2 and a must be divisible by 3. so let $a = 3n$. Then $a^2 = (3n)^2 = 3b^2$.
   Now we have $b^2 = 3k^2$. So b is also divisible by 3.
   let $b = 3m$ and $b^2 = (3m)^2 = 3k^2 \Rightarrow k^2 = 3m^2$ so (m, k) are smallest pair.
   Contradiction

4. Why is $\frac{355}{113}$ number important and famous.
   $\Rightarrow$ best rational approximation of π.

Part II.

1. $\frac{11}{4} = 2 + \frac{3}{4} = 2 + \frac{1}{\frac{4}{3}}$
   $= 2 + \frac{1}{1 + \frac{1}{3}}$

2. If $\sqrt{2}$ is rational then we can write
   $\sqrt{2} = \frac{a}{b}$. Take a, b to be smallest $(a+b)$.
   Then: $2 = \frac{a^2}{b^2}$ and $a^2 = 2b^2$. From this it follows that a^2 is even, so a must be even as well. Write $a = 2n$ and plug that in:
   $(2n)^2 = 4n^2 = 2b^2$ so $b^2 = 2n^2$

By the same logic as for a, b must be even. So write $b = 2m$ and plug in.

$$(2m)^2 = 4m^2 = 2n^2 \quad \text{and} \quad n^2 = 2m^2.$$

It follows then that $2 = \frac{n^2}{m^2}$ and

$\sqrt{2} = \frac{n}{m}$. Moreover $m + n = \frac{a+b}{2} < a + b$,

which is a contradiction, since we took (a, b) to be such that they give the smallest sum.