

**Part I.**

1. Joke involving Pythagorean Theorem

Once upon a time there were three ladies of the First Peoples of America sitting around the campfire.

On a reindeer skin sat a lady who was the mother of a fine young warrior who weighed 140 pounds.

On a buffalo skin sat a lady who was the mother of a fine young warrior who weighed 160 pounds.

The third lady, as well she might, was sitting on the skin of a hippopotamus, as she herself weighed a mighty 300 pounds.

As you can see:

*The squaw on the hippopotamus is equal to the sons of the squaws on the other two hides.*

2. Proof that values always give Pythagorean Triplets

Let  $m$  and  $n$  be arbitrary integers.

Define  $a = m^2 - n^2$ ,  $b = 2mn$

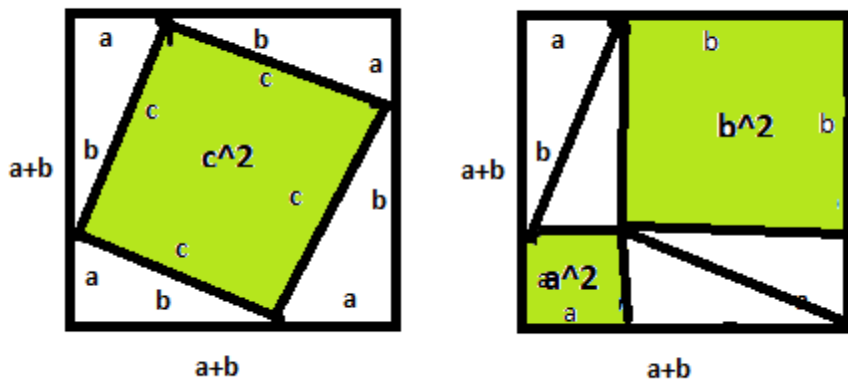
$$\begin{aligned} \text{Then } a^2 + b^2 &= (m^2 - n^2)^2 + (2mn)^2 \\ &= m^4 - 2m^2n^2 + n^4 + 4m^2n^2 \\ &= m^4 + 2m^2n^2 + n^4 \\ &= (m^2 + n^2)^2 \end{aligned}$$

Define  $c = m^2 + n^2$ . Because linear combinations of integers are integers,  $a$ ,  $b$ , and  $c$  are integers.

Thus  $a^2 + b^2 = c^2$ , making  $a, b, c$  a Pythagorean triplet.

**Part II**

1.  $a^2 + b^2 = c^2$ , where  $a$ ,  $b$  are sides of a right triangle and  $c$  is the hypotenuse



- II. Unsure
2. (3,4,5)

(5,12,13)

(8,15,17)