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Lecture 5 Attendance 09/22/2021

Part I

1. look up a joke based on the Pythagorean theorem?

The squaw on the Hippopotamus is equal to the sum of the squaws on the other two sides.

2. Proofe $(m^2-n^2,\ 2mn,\ m^2+n^2)$ are Pythagorean triplets for any m, n that are integers.

Proof: $(m^2-n^2)^2+(2mn)^2=(m^2+n^2)^2$

$n^4+m^4-2m^2n^2+4m^2n^2=m^4+2m^2n^2+n^4$

$m^4+2m^2n^2+n^4 \overset{\checkmark}{=} m^4+2m^2n^2+n^4$

Therefore, it obeys $a^2+b^2=c^2$.

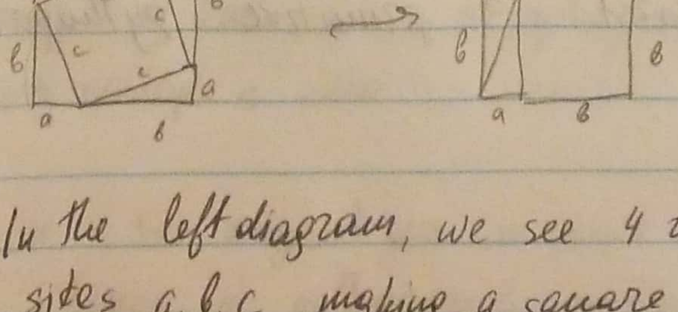
Part II

1. Pythagorean theorem:

$\Rightarrow$ In a right triangle: $a^2+b^2=c^2$, or the sum of the squares of two sides; equals the square of the hypotenuse

Proof:

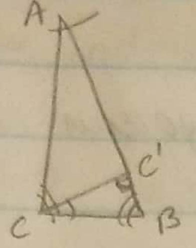
I. Geometric:



In the left diagram, we see 4 right triangles with sides a, b, c making a square with sides c and $A=c^2$

Rearranging, the triangles remain, with the $A=a^2+b^2$

II Similar triangle proof



The triangles ΔABC, $\Delta AC'C$ and $\Delta C'CB$ are similar. Since they're similar $\tan\theta$ will be same constant and: the ratios between sides will also be the same:

$$\frac{BC}{AB} = \frac{BC'}{BC} \quad \text{and} \quad \frac{AC}{AB} = \frac{AC'}{AC}.$$

Therefore: $BC^2 = BC' \cdot AB$ and $AC^2 = AC' \cdot AB$

and $BC^2 + AC^2 = BC' \cdot AB + AC' \cdot AB$

$= AB\,(BC' + AC') = AB \cdot AB$

$= AB^2$.

2. First 3 smallest primitive Pythagorean triples

triples $= (m^2 - n^2,\ 2mn,\ m^2 + n^2)$

$m=2,\ n=1 \Rightarrow (3,\ 4,\ 5)$

$m=3,\ n=1 \Rightarrow (8,\ 6,\ 10)$ $\times 2$ $\Rightarrow$ not primitive

$m=4,\ n=1 \Rightarrow (15,\ 8,\ 17)$

$m=3,\ n=2 \Rightarrow (5,\ 12,\ 13)$

Therefore: $(3,4,5)$, $(15,8,17)$ and $(5,12,13)$ are the first 3 primitive pythagorean triples.