P5

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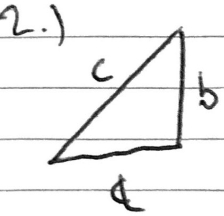
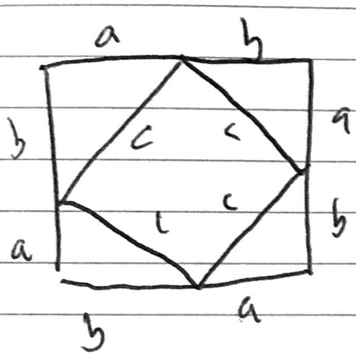
1.) JOKE: Pytagoras walks into a bar muttering "If a right angled triangle has a short side X, long side Y, and hypotenuse Z, then the square of Z must equal the sum of the square of X and of, ...um...

[The Barman says "Y, the long face"]

2.)



$a^2 + b^2 = c^2$



$(a+b)^2 = a^2 + 2ab + b^2 = \text{Area}$ ★

$\text{Area} = 4\Delta + \square$

$= 4\left(\frac{ab}{2}\right) + c^2$ area of square

$= 2ab + c^2$ (B)

Set ★ = to (B)

$a^2 + \cancel{2ab} + b^2 = \cancel{2ab} + c^2$

$a^2 + b^2 = c^2$

3.) Finish proof of formula for Pythagorean triples

Note: $(m^a)^b = m^{(ab)}$

$(m^2 - n^2)^2 = (m^2)^2 - 2(m^2)(n^2) + (n^2)^2$

$A^2 = m^4 - 2m^2n^2 + n^4$

$B = 2mn \quad B^2 = 4m^2n^2$

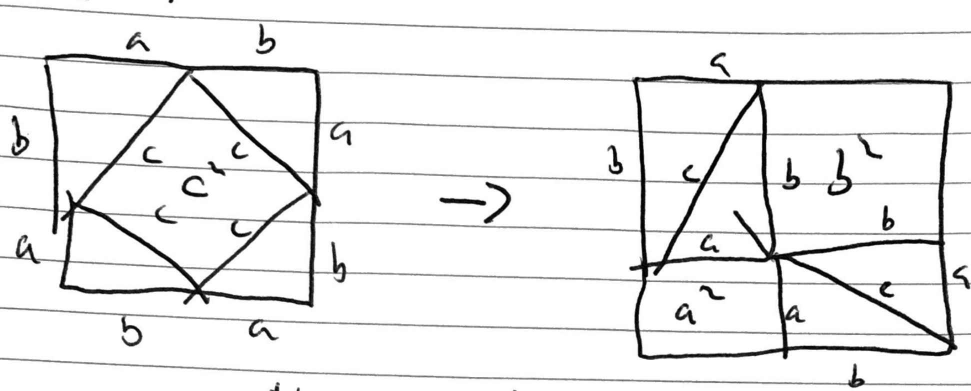
$c = m^2 + n^2 \quad c^2 = m^4 + 2m^2n^2 + n^4$

$A^2 + B^2 = (m^4 - 2m^2n^2 + n^4) + 4m^2n^2 = m^4 + 2m^2n^2 + n^4 = c^2$

Part 2

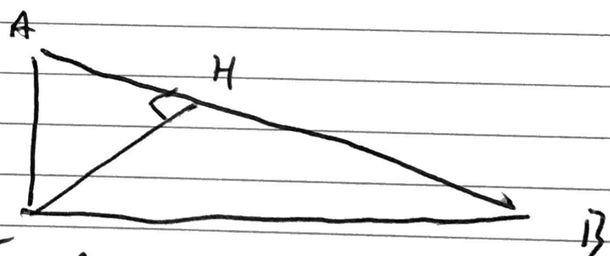
1.) $a^2+b^2=c^2$

a.)



~~[scribble]~~ The two large squares shown in each figure contain 4 identical triangles, and ~~only~~ the difference between the 2 large squares is that the triangles are arranged differently, therefore, the white space within each of the 2 squares must have equal area. Equating the area of the white space yields the Pythagorean theorem

b.) ~~[scribble]~~



$$\frac{BC}{AB}=\frac{BH}{BC} \quad \text{and} \quad \frac{AC}{AB}=\frac{AH}{AC}$$

The first result equals the cosines of angle θ, whereas the second result equals their sines

~~these Ratios can be written as~~

$$BC^2 = AB \times BH \quad \text{and} \quad AC^2 = AB \times AH$$

Summing these 2 equalities results in

$$BC^2 + AC^2 = AB \times BH + AB \times AH = AB \times (AH + BH) = AB^2$$

which, after simplification, expresses the Pythagorean theorem

$$BC^2 + AC^2 = AB^2$$

2.) (5, 12, 13), (21, 20, 29), (15, 8, 17)