Sarah Magno 11/1/21

# Attendance Quiz for Lecture 14

Part I.

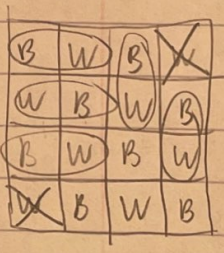
① Who were the two geniuses who proved the impossibility of a formula for solving a quintic?

Abel and Ruffini

② Find a way to place 31 domino pieces and cover completely (no overlap, tiling) an 8x8 square (like a chessboard) where two opposite corners have been removed.

A domino covers one black square and one white square. An 8x8 square can be reduced to a 4x4 square and then 2 opposite corners are removed without

Thus there is no way to cover the board with dominoes.




③ What ages did the above geniuses die?

Abel died at age 26. Ruffini died at age 56.

④ What university did the most in classifying so-called simple groups? What math department has the most number of faculty members (dead or alive) with groups named after them?

University of Chicago did the most in classifying simple groups.

University of Göttingen has the most number of faculty members with groups named after them.

Part II.

① 1 goes to 4, 4 goes to 6
2 goes to 5, 5 goes to 7
3 goes to 7, 7 goes to 5
4 goes to 6, 6 goes to 4
5 goes to 1, 1 goes to 3
6 goes to 2, 2 goes to 1
7 goes to 3, 3 goes to 2

Answer: $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 6 & 7 & 5 & 4 & 3 & 1 & 2 \end{pmatrix}$

② $\pi = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 4 & 1 & 2 & 3 \end{pmatrix}$

1 goes to 4, 4 goes to 3
2 goes to 1, 1 goes to 4
3 goes to 2, 2 goes to 1
4 goes to 3, 3 goes to 2

$\pi^2 = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 4 & 1 & 2 \end{pmatrix}$

1 goes to 3, 3 goes to 2
2 goes to 4, 4 goes to 3
3 goes to 1, 1 goes to 4
4 goes to 2, 2 goes to 1

$\pi^3 = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 4 & 1 \end{pmatrix}$

1 goes to 2, 2 goes to 1
2 goes to 3, 3 goes to 2
3 goes to 4, 4 goes to 3
4 goes to 1, 1 goes to 4

$\pi^4 = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 1 & 2 & 3 & 4 \end{pmatrix} = I$

③ As a cycle, it can be written as

$\begin{pmatrix} 1 & 3 & 2 \\ 3 & 2 & 1 \end{pmatrix}\begin{pmatrix} 4 & 5 \\ 5 & 4 \end{pmatrix} = (132)(45)$

③ 1 goes to 3, 3 goes to 2
2 goes to 1, 1 goes to 3
3 goes to 2, 2 goes to 1
4 goes to 5, 5 goes to 4
5 goes to 4, 4 goes to 5

$\pi^2 = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 3 & 1 & 4 & 5 \end{pmatrix}$

1 goes to 2, 2 goes to 1
2 goes to 3, 3 goes to 2
3 goes to 1, 1 goes to 3
4 goes to 4, 4 goes to 5
5 goes to 5, 5 goes to 4

$\pi^3 = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 2 & 3 & 5 & 4 \end{pmatrix}$

1 goes to 1, 1 goes to 3
2 goes to 2, 2 goes to 1
3 goes to 3, 3 goes to 2
4 goes to 5, 5 goes to 4
5 goes to 4, 4 goes to 5

$\pi^4 = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 3 & 1 & 2 & 4 & 5 \end{pmatrix}$

1 goes to 3, 3 goes to 2
2 goes to 1, 1 goes to 3
3 goes to 2, 2 goes to 1
4 goes to 4, 4 goes to 5
5 goes to 5, 5 goes to 4

$\pi^5 = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 3 & 1 & 5 & 4 \end{pmatrix}$

1 goes to 2, 2 goes to 1
2 goes to 3, 3 goes to 2
3 goes to 1, 1 goes to 3
4 goes to 5, 5 goes to 4
5 goes to 4, 4 goes to 5

$\pi^6 = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 2 & 3 & 4 & 5 \end{pmatrix} = I$

6 is the smallest such $i$ that $\pi^i$ is the identity permutation.

④ Step 1: Flip the two rows

$$\begin{pmatrix} 3 & 1 & 2 & 5 & 4 \\ 1 & 2 & 3 & 4 & 5 \end{pmatrix}$$

Step 2: Reorganize them so top row follows convention of 12345

$$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 3 & 1 & 5 & 4 \end{pmatrix} = \pi^{-1}$$