Getting to know you Quiz (does not count towards the grade)

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Subject: pre0
with an attachment: preOFirstLast.pdf
1.: What are your career goals?

I want to be a high school math teacher.
2.: What are your hobbies?

I like to draw flowers and nature scenes. I like to cook vegetarian foods. I also enjoy watching Star Wars and Disney, specifically Pixar related movies and shows.
3. What is a rational number?

A rational number is a number of the form $\mathrm{p} / \mathrm{q}$, where p and q are integers, in simplest form.
4. Prove that the sum of two rational numbers is also a rational number,

Let $\mathrm{p} / \mathrm{q}$ be a rational number and $\mathrm{m} / \mathrm{n}$ be a rational number, where $\mathrm{p}, \mathrm{q}, \mathrm{m}, \mathrm{n}$ are integers. The sum of $p / q$ and $m / n$ would be $(p n+m q) / q n$. Since $p, q, m, n$ are all integers, then the product of any two would yield an integer. Hence, qn, pn, and mq are all integers. Also, the sum of two integers is an integer, and since we have established that pn and mq are integers, their sum would be an integer. Therefore, the sum of two rational numbers is a rational number.
5. Prove or disprove (by giving a counterexample) : "the sum of two irrational numbers is always also an irrational number"

Let $\mathrm{a}=$ square root of 3 be an irrational number. And let $\mathrm{b}=-$ square root of 3 be an irrational number. $a+b=$ square root of $3-$ square root of $3=0$, and 0 is a rational number. This counter example disproves the above statement.
6. Prove that there are infinitely many primes.

I remember doing this proof in Math 300, but I do not know how to approach it.
7. Prove that 5 is an irrational number.

Assume to the contrary that square root of 5 is a rational number. So it must be of the form $\mathrm{p} / \mathrm{q}$
with $p, q$ integers and in lowest terms. So let $p / q=$ square root of 5 , and square both sides. We get that $5=p^{\wedge} 2 / q^{\wedge} 2$, so $p^{\wedge} 2=5 q^{\wedge} 2$. If $q$ is even, then $p$ must be even, which would mean that it is not in simplest form. If $q$ is odd, then it would be of the form $2 b+1$, where $b$ is an integer. It would also imply that $p$ is odd, so $p=2 a+1$ So, let $q=2 b+1$. $p^{\wedge} 2=(2 a+1)^{\wedge} 2=4 a^{\wedge} 2+4 a+1$ $q^{\wedge} 2=5(2 b+1)^{\wedge} 2=5\left(4 b^{\wedge} 2+4 b+1\right)=20 b^{\wedge} 2+20 b+5$.
This simplifies to $2\left(2 a^{\wedge} 2+2 a\right)=2\left(10 b^{\wedge} 2+10 b+2\right)=>$ one side is even and the other is odd

