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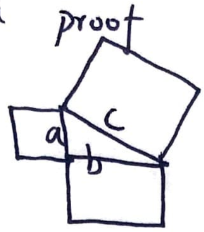
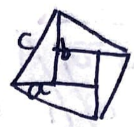
1、first proof



a^2 is one square b^2 is also square

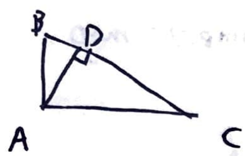
if we have four triangles like this



$$c^2 = a \cdot b \cdot \frac{1}{2} \cdot 4 + (b-a)^2$$
$$= 2ab + b^2 - 2ab + a^2$$
$$= a^2 + b^2$$

so $c^2 = a^2 + b^2$ is proved

second proof



from this triangle ABC, DBA and DAC we have similar triangles

so $\frac{AB}{BC} = \frac{BD}{AB}$ and $\frac{AC}{BC} = \frac{DC}{AC}$

$AB \cdot AB = BC \cdot BD$ $\quad AC \cdot AC = BC \cdot DC$

$$AB \cdot AB + AC \cdot AC = BC \cdot BD + BC \cdot DC$$
$$= BC \cdot BC$$

So $a^2 + b^2 = c^2$ is proved.

2、

$$\sqrt[7]{3} = \frac{a}{b}$$

$3 = \frac{a^7}{b^7}$ $\quad 3b^7 = a^7$

but this is impossible becaus 3 appears b^7 times to be a^7

Assume a^7 is divisible by 3 so $3b^7 = (3k)^7$ and $(3k)^7 = 3^7 k^7$

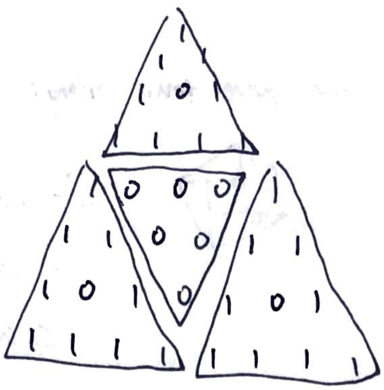
$b^7 = 3^6 k^7$ which is a contradiction

So $\sqrt[7]{3}$ is irrational

12 a)

3. a)

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
1 6 15 20 15 6 1
1 7 21 35 35 21 7 1



b) The first Feigenbaum constant δ is the limiting ratio of each bifurcation interval to the next between period doubling, of a one parameter map

$$\delta = \lim_{n\to\infty} \frac{a_{n-1}-a_{n-2}}{a_n - a_{n-1}} = 4.669201609$$

the second Feigenbaum constant $\alpha = 2.502907875095 89$ is the ratio between the width of a tine and width of one of its two subtines. A negative sign is applied to α when the ratio between the lower subtine and the width of the tine is measured.

$\frac{13}{11} \times \frac{17}{11} \times \frac{37}{21} = \frac{817}{3267}$

4. a) Platonis solid is a regular polyhedron that the faces ar congruent regular polygons, and the same numbers of faces meet each vertex.

b) $V - E + F = 2$

$V = \frac{2E}{a}$ $F = \frac{2E}{b}$

c) $F = \frac{2E}{b}$ $\frac{2E}{a} - E + \frac{2E}{b} = 2$ $E = \frac{2ab}{2a+2b-ab}$

d) $a=3$ $b=3$ $E=6$ $F=4$ is tetrahedron

$a=3$ $b=4$ $E=12$ $F=6$ is ~~[illegible]~~ hexahedron

$a=4$ $b=3$ $E=12$ $F=8$ is octahedron

5\. Assume $a_1H, a_2H \cdots a_nH$ denote left cosets of H in G

for each $a \in G$ $\quad aH = a_nH$ for n

H is a subgroup of G $\quad a \in aH$

so each element of G belong to the cosets a_nH

so $\quad G = a_1H \cup a_2H \cup \cdots \cup a_nH$

$|G| = |a_1H \cup \cdots \cup a_nH|$

$\Rightarrow |G| = |a_1H| + \cdots + |a_nH|$

$|a_nH| = |H|$ for $1 \leq n \leq n$

so $|G| = \underbrace{|H| + \cdots + |H|}_{n \text{ times}}$

$|G| = n|H|$

$\frac{|G|}{|H|} = n$

therefor $\frac{|G|}{|H|}$ is always integer.

imar triangles lik

$(1-a)^2$

6. $u_x = v_y$ $u_y = -v_x$

if $u(x,y) + iv(x,y) = f(z)$ be any function
then condition for $f(z)$ to be ~~[illegible]~~ possible is
that it must satisfy Cauchy-Riemann equations

7. Sir William Rowan Hamilton discovered quaternions.
He lived in ~~[illegible]~~ Dublin, Ireland.

8. Heron's formula is Area $= \sqrt{s(s-a)(s-b)(s-c)}$

where s is half the perimeter or $\frac{a+b+c}{2}$

century is c. 10 AD to c. 70 AD

9. Issac Newton study at The King's School, Grantham
his teacher is Isaac Barrow
his teacher first ~~developed a method of determining~~ recognized the processes of integration and differentiation in calculus

are inverse operations
Newton's position is that he appointed his teacher's place and hold the Lucasian chair.

10. Leibnitz born in Leipzig.
He spent most of his life in Hanover.
George I of Great Britain

11. a) $\frac{2}{\pi} = \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{2+\sqrt{2}}}{2} + \frac{\sqrt{2+\sqrt{2+\sqrt{2}}}}{2} \cdots$

b) Henry Briggs and Leonard Euler

12. a)

Euler path is E D C A B C

Euler path is a path that uses every edges of a graph

b) condition is all vertices in the graph have an even edges

c) Assume the graph has Euler path but not a circuit

every time the path passes through a vertex, it needs two to the degree of vertex. and the first and last vertices will have odd degree, other vertices have even degree.

if assume two vertices, u, v have odd degree

if we connect these two vertices. then every vertex will have even degree. so we have an Euler circuit. in this graph. then if we remove u and v. we get an Euler path for original path.