**NAME: (print!)** Vivian Choong

**E-Mail address:** vc387@scarletmail.rutgers.ed

**MATH 437 Exam II for Dr. Z.'s, Fall 2021, Dec. 6, 2021, 3:00-4:20pm, (on-line)**

**No Calculators! No Cheatsheets! YOU MAY USE YOUR HISTORY NOTEBOOK (But not your Math Notebook).**
**Show your work! An answer without showing your work will get you zero points.**

Do not write below this line (office use only)

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1. (out of 10)

2. (out of 10)

3. (out of 10)

4. (out of 10)

5. (out of 10)

6. (out of 10)

7. (out of 10)

8. (out of 10)

9. (out of 10)

10. (out of 10)

11. (out of 10)

12. (out of 10)

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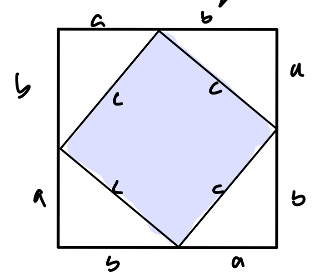
total: (out of 120)

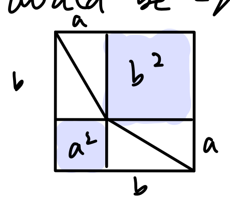
**1.** (10 points) Give **two** proofs of the Pythagorean theorem.

Proof #1 - Rearrangement of Triangles

Let us rearrange 4 triangles of the same sides, a, b, c. such that it forms a square measuring c on all sides

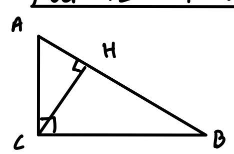



If we were to rearrange said triangles, we can see that the shaded area($c^2$) would be equal to $(a^2)+(b^2)$.




Thus $a^2+b^2=c^2$

Proof #2 - Proof using Similar Triangles




As we can see $\triangle AHC$ and $\triangle BHC$ are similar

Using the ratios of the similar triangles:

$\frac{BC}{AB}=\frac{BH}{BC}$ $\quad \frac{AC}{AB}=\frac{AH}{AC}$ After cross multiplying,

$BC^2=AB(BH)$, $AC^2=AH(AB)$

$(BC^2=AB(BH))+(AC^2=AH(AB))=AB\,(AH+BH)=AB^2=$

$BC^2+AC^2=AB^2$

$c^2=A^2+B^2$.

**2.** (10 points) Prove that $\sqrt[7]{3}$ is irrational.

Suppose $\sqrt[7]{3}$ can be expressed as a fraction $\frac{a}{b}$ s.t. $a,b\in\mathbb{Z}$, $b\neq 0$ and $\gcd(a,b)=1$.

This would mean $b^7(3)=a^7$. Let p be a prime number s.t. $p|3$. This implies $p|a^7$ and thus $p|a$. This can be rewritten as $p^7|a^7$. Since 3 is square free, $p^6|b^7$. Since $7\geq 2$ $p|b^7$, which means $p|b$. This implies $\gcd(a,b)=7$, this contradicts $\gcd(a,b)=1$ and $\sqrt[7]{3}$ is irrational

**3.** (10 points total)
(a) (5 points) Construct the Pascal triangle mod 2 Fractal using the first 8 rows (i.e, the row for $n = 0$ through row for $n = 7$). Highlight the middle 0 section, and show that the remaining part consists of three identical triangles with 4 rows,

| | | |
|---|---|---|
| 1 | | 1 |
| 1 1 | | 1 1 |
| 1 2 1 | | 1 0 1 |
| 1 3 3 1 | $\longrightarrow$ | 1 1 1 1 |
| 1 4 6 4 1 | | 1 0 0 0 1 |
| 1 5 10 10 5 1 | | 1 1 0 0 1 1 |
| 1 6 15 20 15 6 1 | | 1 0 1 0 1 0 1 |
| 1 7 21 35 35 21 7 1 | | 1 1 1 1 1 1 1 1 |
| 1 8 28 56 70 56 28 8 1 | | 1 0 0 0 0 0 0 0 1 |

It is a fractal because it has an inverted equalateral triangle

(b) (5 points) Define the Feigenbaum constant. Explain everything!

Feigenbaum noticed that $\lim_{k \to \infty} \dfrac{r_{k+1} - r_k}{r_k - r_{k-1}}$

in the logistic map $x_{n+1} = k x_n (1 - x_n)$ if $1 < k < 3$, the population will stabalize. If $k > 3$, then it will be the ultimate period of 2. Making $k$ bigger, there will be an ultimate period of 4. Thus the constant tends to 4.6692016.

**4.** (10 points altogether)
(a) (2 points) Define a **Platonic soild**
(b) (2 points) Let $a$ be the number of edges meeting each vertex, and let $b$ be the number of edges surrounding each face. Express $V$ (the number of vertices) and $F$ (the number of faces) in terms of $E$ (the number of edges), and $a$ and $b$.
(c) (2 points) Find an expressions for $F$, in terms of $a$ and $b$.
(d) (4 points) Obviously both $a$ and $b$ must be at least 3, and $F$ (and hence $V$ and $E$) must be positive. It is easy to see (you don't have to do it) that $a, b$ must be both between 3 and 5, leaving 9 potential scenarios. Find those values of $a$ and $b$ that make sense, and thereby prove that there are exactly 5 Platonic solids. For each of them, find $F$ (the number of faces) and give the name of the corresponding Platonic solid.

(a) A platonic solid is a polyhedron that have congruent faces that are regular polygons and the same number of faces meet at each vertex.

(b) $bF = 2E = aV$

(c) $F = \dfrac{4a}{4-(b-2)(a-2)}$

(d) Since $a, b$ must be between 3 and 5, we can have only 5 possibilities for $\{a, b\}$:
$\{3,3\}$, $\{4,3\}$, $\{3,4\}$, $\{5,3\}$, $\{3,5\}$. using Euler's formula that $\frac{2E}{b} - E + \frac{2E}{a} = 2 \Rightarrow \frac{1}{a} + \frac{1}{b} = \frac{1}{2} + \frac{1}{E} \Rightarrow \frac{1}{a} + \frac{1}{b} > \frac{1}{2}$.

Thus $a+b > 2$.

**5.** (10 points)
Prove Lagrange's theorem that if $H$ is any subgroup of a group $G$, and $|H|$ and $|G|$ are their number of elements, respectively, then $|G|/|H|$ is always an integer.

Let $H \leq G$, $|G| = n$ and $|H| = m$.

Since every coset (left and rights) of a subgroup has the same number of elements as $H$, we know $H$ has $m$ elements. Let $r$ be the number of cells in the partition of $G$ into left cosets of $H$.

Then $n = rm \Rightarrow |G| = |H|(G:H) \Rightarrow \frac{|G|}{|H|} = r$., $r \in \mathbb{N}$.

**6.** (10 points) What is the name of the following famous equation-pair?

$$u_x = v_y \quad , \quad u_y = -v_x \quad ,$$

or, in fuller notation

$$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y} \quad , \quad \frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x} \quad .$$

What is special about the function $u(x,y)+iv(x,y)$ where $u(x,y), v(x,y)$ satisfy the above system of two equations?

"Cauchy-Riemann" equations

Two smooth real functions $u(x,y)$ and $v(x,y)$ are the real and imaginary parts of the complex variable $z = x+iy$

**7.** (10 points) Who discovered the quaternions? What city did that person live in?

William Rowan Hamilton, Dublin

**8.** (10 points) What is Heron's formula, what century did Heron live in?

$A = \sqrt{s(s-a)(s-b)(s-c)}$

1st Century (lived from 10AD–70AD).

**9.** (10 points) Where did Isaac Newton study? Who was his teacher? What unusual action did that teacher do? What was Newton's position after he left Cambridge?

Isaac newton studied Astronomy under his teacher Isaac Barrow, who wrote Lectiones Opticae which was unusual.

Newton became warden and later master at the mint.

**10.** (10 points) In what city was Leibnitz born? Where did he spend most of his life? What King of England was once the employer of Leibnitz?.

Leipzig, Germany, spent most of his life near the court of Hanover. King George I.

**11.** (10 points total)
(a) (5 points) State Viète's infinite product for $\frac{2}{\pi}$.

$$\frac{2}{\pi} = \cos\frac{\pi}{4}\cos\frac{\pi}{8}\cos\frac{\pi}{16}\cos\frac{\pi}{32}\cdots$$

(b) ( 5 points) State the names of two people who initiated the use of logarithms

John Naper and Henry Briggs.

**12.** (10 points altogether) (a) (3 points) Define a *Eulerian path* in a graph.

A Eulerian path visits every edge of the graph exactly once

(b) (3 points) State the necessary condition for a graph to have a Eulerian path

A graph has a Eulerian path iff there are at most two vertices of odd degree.

(c) (4 points) Prove (or explain in your own words) why the condition in (b) is necessary.

If we add an edge between two - odd degree vertices, the graph would have a Eulerian circuit. If we remove the edge, we would then have a Eulerian path. Suppose we have a graph with vertices that contain an odd degree not be two and contains a Eulerian path. Let us add an edge at the two ends of the path. This would be a graph with an odd degree vertex and Euler circuit which is a contradiction. Thus a Euler path must have at most 2 vertices of odd degree.