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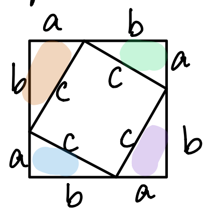
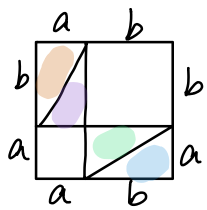
1. (1) First. we build a square, whose length of side is $(a+b)$ and another square of length c inserted into it. The area of the small square



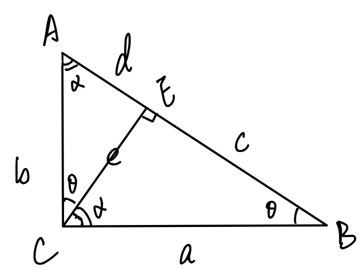
Then move and rearrange the four rectangular triangles, two a new square. whose length of side is also $(a+b)$.



We can see except the four triangles, there are two new square left, the lengths are a and b, the areas of them are a^2 an b^2.

So we know $a^2+b^2=c^2$

(2)



Proof. Since $\angle CDB = \angle ACB = 90°$,
$\angle CAB$ and $\angle CAE$ are the same angle,
$\angle ACE = \angle EBC$
Then $\triangle ABC \sim \triangle ACE$.
Since $\angle CBA$ and $\angle CBE$ are the same angle,
$\angle EBC = \angle CAB$
Then $\triangle ABC$ is similar to $\triangle CBE$.
Then $\triangle ACE$ is similar to $\triangle CBE$.

So $\frac{BC}{AB} = \frac{BE}{CB}$, $\frac{AC}{AB} = \frac{AE}{AC}$,

then we have $\frac{a}{d+c} = \frac{c}{a}$, $\frac{b}{d+c} = \frac{d}{b}$.

So $a^2 = c^2 + cd$, $b^2 = d^2 + dc$,
then $a^2+b^2 = c^2+2cd+d^2 = (c+d)^2$

2. Proof.

Assume $\sqrt[7]{3}$ is rational, then $\sqrt[7]{3} = \frac{m}{n}$, for $m, n \in \mathbb{Z}$

Now take the pair of (m, n) that give the smallest $m+n$, then (m, n) are coprime because if m, n share a factor $z > 1 \in \mathbb{N}$

Then $\frac{m}{n} = \frac{zm'}{zn'} = \frac{m'}{n'}$ and $m+n > m'+n'$.

So $\sqrt[7]{3} = \frac{m}{n}$ s.t. (m, n) are coprime.

Then $3 = \frac{m^7}{n^7}$, $3n^7 = m^7$. then 3 is a factor of m^7,

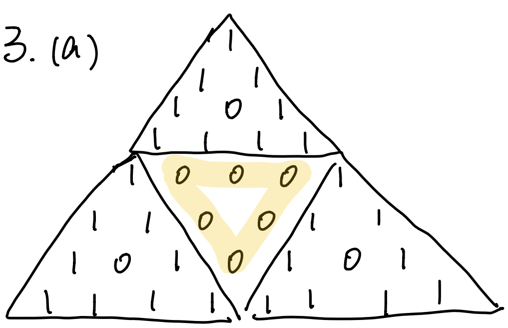
also 3 is a factor of m, so we can write $m = 3k$, $k \in \mathbb{Z}$.

Then $m^7 = 3^7 k^7 = 3n^7$, $3^6 k^7 = n^7$.

So 3^6 is a factor of n^7, so 3 is a factor of n.

it is contradicted with (m, n) are co-prime.

3. (a)



(b) When $3 < a < 3.449$, the period is 2.

(period length is the number of unique values of x_n, that it cycles). Let a_n be the bifurcatron parameter at which the period changes from 2^{n-1} to 2^n

And the Feigenbaum constant is

$$\lim_{n \to \infty} \frac{a_{n-1} - a_{n-2}}{a_n - a_{n-1}} \text{ which is equal to } 4.669\cdots$$

4. (a) Platonic Solid is a polyhedron where all the faces are identical perfect polygons.

(b) $V = \frac{2E}{a}$, $F = \frac{2E}{b}$

(c) $V = \frac{2E}{a}$, $\to 2E = Va$, then $F = \frac{Va}{b}$

We know that $V - E + F = 2$. plug in $V = \frac{2E}{a}$ and $F = \frac{2E}{b}$

Then $\frac{2E}{a} - E + \frac{2E}{b} = 2 \Rightarrow E\left(\frac{2}{a} + \frac{2}{b} - 1\right) = 2$, plug in $F = \frac{2E}{b}$,

$$F = \frac{\frac{4}{\frac{2}{a} + \frac{2}{b} - 1}}{b}, \text{ So } F = \frac{4}{\left(\frac{2}{a} + \frac{2}{b} - 1\right)b}$$

(d) when $a = 3$, $b = 3$, $F = \frac{4}{3\left(\frac{2}{3} + \frac{2}{3} - 1\right)} = 4$, it is a tetrahedro

When $a = 3$, $b = 4$, $F = \frac{4}{4\left(\frac{2}{3} + \frac{2}{4} - 1\right)} = 6$, it is a cube.

When $a = 4$, $b = 3$, $F = \frac{4}{3\left(\frac{2}{4} + \frac{2}{3} - 1\right)} = 8$ it is a octahedron.

When $a = 3$, $b = 5$, $F = \frac{4}{5\left(\frac{2}{3} + \frac{2}{5} - 1\right)} = 12$, it is a dodecahedron

When $a = 5$, $b = 3$, $F = \frac{4}{3\left(\frac{2}{5} + \frac{2}{3} - 1\right)} = 20$ it is an icosahedron.

When $a = 4$, $b = 5$, $F = \frac{4}{5\left(\frac{2}{4} + \frac{2}{5} - 1\right)} < 0$ X

When $a = 5$, $b = 5$, $F = \frac{4}{5\left(\frac{2}{5} + \frac{2}{5} - 1\right)} < 0$ X

When $a = 5$, $b = 4$, $F = \frac{4}{4\left(\frac{2}{5} + \frac{2}{4} - 1\right)} < 0$ X

They are exactly 5 Platonic solids.

5. Let $aH = \{ah \mid h \in H\}$ be the left coset of H including a.

Lemma: Every coset of H has the same numbers of elements as H.

Let $F: H \to aH$ s.t. $f(h) = ah$. If $f(h_i) = f(h_j)$ then $ah_i = ah_j$
So $h_i = h_j$.

For every element in H, known as h_k, $f(h_k) = ah_k$.

Since every element is unique, so every element in aH, subjection

So $f: H \to aH$ is bijection and $|aH| = |H|$

suppose G has n elements and H has m elements.

Let $H, a_1H, \dots, a_{k-1}H$ be the distinct cosets of H.

Each coset has the same cardinality as H and each element of G appears in exactly one coset so $n = mk$.

Then m divides n, $|H|$ divides $|G|$, so $|G|/|H|$ is always an integer.

6. "Cauchy-Riemann" equations;

The xy-plane conformally on the uv-plane and the existence of a function capable of transforming any simple connected region in one place into any simple connected region in one plane into any simply conception of the Riemann surface

7. William Rowan Hamilton; Dublin

8. $A = \sqrt{s(s-a)(s-b)(s-c)}$, where A is the area, s is the semi-perimeter, a, b, c are lengths of three sides;

1st century.

9. At Cambridge; Isaac Barrow; Barrow yielded the Lucasian professorship to his pupil - a remarkable academic event since Barrow acknowledged Newton to be his superior;

Then he accepted the position of warden, later of master, of the mint

10. Leipzig; Near the court of Hanover; George I..

11. (a) $\frac{2}{\pi} = \cos\frac{\pi}{4} \cos\frac{\pi}{8} \cos\frac{\pi}{16} \cos\frac{\pi}{32} \cdots$

(b) John Neper , then Henry Briggs

12. (a) A way of visiting all the edges of the graph, each exactly once starting at some vertex and ending at another vertex.

(b) Every vertex, except 2, has an even number of edges incident to it, and the path starts and ends at one of the vertices that have an odd number of edges incident to it.

(C). For Eulerian path, we start at a vertex and end at a different vertex. Each vertex need to go in and out, so it is even, except for the begining and the end.

The start point only need to be gone in, so it should be an odd degree vertex. The same as the end point.