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MATH 437 Exam II for Dr. Z.'s, Fall 2021, Dec. 6, 2021, 3:00-4:20pm, (on-line)

No Calculators! No Cheatsheets! YOU MAY USE YOUR HISTORY NOTEBOOK (But not your Math Notebook).
Show your work! An answer without showing your work will get you zero points.

Do not write below this line (office use only)

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1. (out of 10)
2. (out of 10)
3. (out of 10)
4. (out of 10)
5. (out of 10)
6. (out of 10)
7. (out of 10)
8. (out of 10)
9. (out of 10)
10. (out of 10)
11. (out of 10)
12. (out of 10)

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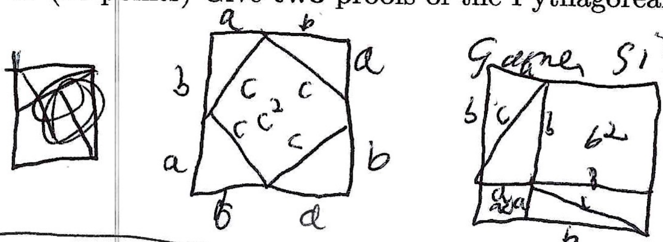
total: (out of 120)

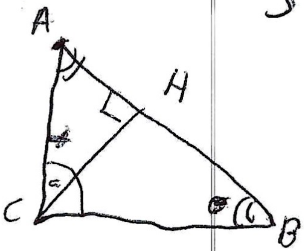
1. (10 points) Give **two** proofs of the Pythagorean theorem.



Same size box

so same number of triangle of same volume in same size box but open represent two different way still has same space so $a^2+b^2=c^2$

proof 2



similar triangle

new triangle are similar since share con 90 angle degree

90° angle and hi is create two so that means $HBC\angle = ACH\angle$ and $HCB\angle = HAC$,

so can state that $\frac{BC}{AB} = \frac{BH}{BC}$ $\frac{AC}{AB} = \frac{AH}{AC}$

$BC^2 = AB + DK +$ $AC^2 = AB \times AH$

$(BC)^2 + (AC)^2 = AB^2$

2. (10 points) Prove that $\sqrt[7]{3}$ is irrational.

$\sqrt[7]{3} = a/b$ / a and b are postive intergers and relative primer

$3 = a^7/b^7$

$3b^7 = a^7$

3 / a then

$3 / a^7$, $(3c)^7 = a^7$ & hence follow lemma must have sameunder to equalence

let $3c = a$,

~~$3b^7 = (2k+1)^7$~~

$3b^7 = (3c)^7$

$b^7 = 3^6c^7$

hence b^7 is multiple of 3 and by lemma also b is divisiblue by 3

so both a and b are divilible by 3

this contradiction the assumption a are not both divisble by 3, this means $\sqrt[7]{3}$ is irrational

$V = \frac{2E}{a}$, $F = 2E/b$

4. (10 points altogether)
(a) (2 points) Define a **Platonic soild**
(b) (2 points) Let a be the number of edges meeting each vertex, and let b be the number of edges surrounding each face. Express V (the number of vertices) and F (the number of faces) in terms of E (the number of edges), and a and b.
(c) (2 points) Find an expressions for F, in terms of a and b.
(d) (4 points) Obviously both a and b must be at least 3, and F (and hence V and E) must be positive. It is easy to see (you don't have to do it) that a, b must be both between 3 and 5, leaving 9 potential scenarios. Find those values of a and b that make sense, and thereby prove that there are exactly 5 Platonic solids. For each of them, find F (the number of faces) and give the name of the corresponding Platonic solid.

(a) faces are all identical, regular polygons meeting at same three angle,

(b) $V + F - 2 = E$, $\frac{2E}{a} + \frac{2E}{b} = 2 = E$ | $E(\frac{2}{a} + \frac{2}{b} - 1) = 2$

$E = \frac{2}{(\frac{2}{a} + \frac{2}{b} - 1)}$

$E = \frac{2ab}{2a + 2b - ab}$

(c) $\frac{2E}{b} = F$, $F = \frac{4a}{4 - (a-2)(b-2)}$

(d)

	Tetra	Cube	Octo	~~Icosa~~ dode	icosa 20 someth.
E =	6	12	12	30	30
F	4	6	8	12	20
a	3	3	4		
b	4	3	4		

5. (10 points)
Prove Lagrange's theorem that if H is any subgroup of a group G, and $|H|$ and $|G|$ are their number of elements, respectively, then $|G|/|H|$ is always an integer.

There explicitly constructed a left coset decomposition of a $|G|$ ~~previous group~~ into cosets of H, each with same number of elements, no overlap. Hence pr~~oven~~

$|G| / |H|$ is must be st integer

6. (10 points) What is the name of the following famous equation-pair?

$$u_x = v_y \quad , \quad u_y = -v_x \quad ,$$

or, in fuller notation

$$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y} \quad , \quad \frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x} \quad .$$

L a place's partial d.q uation

? What is special about the function $u(x,y)+iv(x,y)$ where $u(x,y), v(x,y)$ satisfy the above system of two equations?

power for the solution of difference equation inverse probabilites.

7. (10 points) Who discovered the quaternions? What city did that person live in?

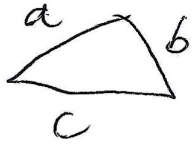
~~Rodrigues~~ William Rowan Hamilton, Dublin, Ireland

8. (10 points) What is Heron's formula, what century did Heron live in?

gives area of a triangle with side lenghts.

$$A = \sqrt{s(s-a)(s-b)(s-c)}$$

$$s = \frac{a+b+c}{2}$$



? **9.** (10 points) Where did Isaac Newton study? Who was his teacher? What unusual action did that teacher do? What was Newton's position after he left Cambridge?

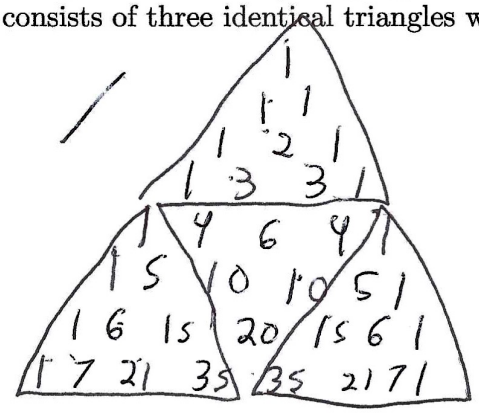
Cambridge, Isaac Barrow, yielded the Lucasian professorship to his pupil, Warden later master of the mint.

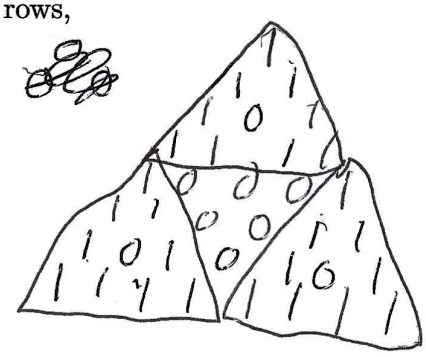
10. (10 points) In what city was Leibnitz born? Where did he spend most of his life? What King of England was once the employer of Leibnitz?.

born in Leipzig, work king George I, most life need court hanover

3. (10 points total)

(a) (5 points) Construct the Pascal triangle mod 2 Fractal using the first 8 rows (i.e, the row for $n = 0$ through row for $n = 7$). Highlight the middle 0 section, and show that the remaining part consists of three identical triangles with 4 rows,





I just go it is the same, since middle is all even so equal = 0 and the out triag are odd besides the third row middle is even)

(b) (5 points) Define the Feigenbaum constant. Explain everything!

ratio between the diameters of successive circles on the real axis,

11. (10 points total)
(a) (5 points) State Viète's infinite product for $\frac{2}{\pi}$.

simplified

$\frac{2}{\pi} = \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{2+\sqrt{2}}}{2} \cdot \frac{\sqrt{2+\sqrt{2+\sqrt{2}}}}{2} \cdots$ ✓

$\frac{2}{\pi} = \cos\frac{\pi}{4}\left(\cos\frac{\pi}{8}\right)\cos\frac{\pi}{16}\cos\left(\frac{\pi}{32}\right)$

(b) (5 points) State the names of two people who initiated the use of logarithms

Ezechiel de Decker, Vlacq

12. (10 points altogether) (a) (3 points) Define a *Eulerian path* in a graph.

A path that visits every edge exactly one, starting and ending at different vertices

(b) (3 points) State the necessary condition for a graph to have a Eulerian path

If the degree of even vertex is even beside two can be odd.

(c) (4 points) Prove (or explain in your own words) why the condition in (b) is necessary.

~~It is necessary since~~

since going in and out each vertex needs a even number beside first and last vertex which can odd since going another point from them ✓