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MATH 437 Exam II for Dr. Z.'s, Fall 2021, Dec. 6, 2021, 3:00-4:20pm, (on-line)

No Calculators! No Cheatsheets! YOU MAY USE YOUR HISTORY NOTEBOOK (But not your Math Notebook).
Show your work! An answer without showing your work will get you zero points.

Do not write below this line (office use only)

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1. (out of 10)

2. (out of 10)

3. (out of 10)

4. (out of 10)

5. (out of 10)

6. (out of 10)

7. (out of 10)

8. (out of 10)

9. (out of 10)

10. (out of 10)

11. (out of 10)

12. (out of 10)

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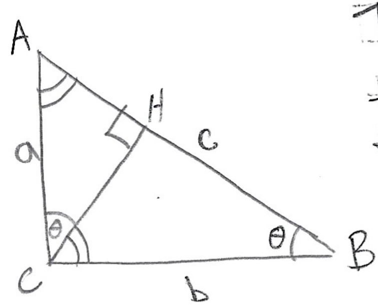
total: (out of 120)

1. (10 points) Give **two** proofs of the Pythagorean theorem.

Because these two squares cover the same area: $a^2+b^2=c^2$



Triang A, B, and C are similiar triangles each triangles Arear is proportional to the square of its hypoten use. because of the similiar triangles there constant of Proportionality are the same, γ;

$$\gamma a^2+\gamma b^2=\gamma c^2$$

$$a^2+b^2=c^2$$

2. (10 points) Prove that $\sqrt[7]{3}$ is irrational.

Suppose: $\sqrt[7]{3}=\frac{m}{n}$

$$3=\frac{m^7}{n^7}$$

$$3n^7=m^7 \quad \text{eq } 1$$

By the fundemental thm of Arithmetic

$$N=P_1^{a_1}\times P_2^{a_2}\times\ldots\times P_n^{a_n}$$

$$N^7=P_1^{7a_1}\times P_2^{7a_2}\times\ldots\times P_4^{7a_n}$$

LEMMA: For any integer N, the exponents of all primes in the prime decomposition of N^7 are factors of 7

The exponent of 3 on the Left hand side of eq 1 is of the form $7b+1$; and therefore not a factor of 7.

This cannot be true and gives a contradiction because the LHS and RHS prime decomposition cannot be equivent. $\rightarrow\leftarrow$

3. (10 points total)

(a) (5 points) Construct the Pascal triangle mod 2 Fractal using the first 8 rows (i.e, the row for $n = 0$ through row for $n = 7$). Highlight the middle 0 section, and show that the remaining part consists of three identical triangles with 4 rows,

1
1 1
1 0 1
1 1 1 1
1 0 0 0 1
1 1 0 0 1 1
1 0 1 0 1 0 1
1 1 1 1 1 1 1 1

All three remain triangles are of the form

1
1 1
1 0 1
1 1 1 1

There is self similiarity amongst these of the form

$$A(n) = \begin{matrix} & A(n-1) & \\ A(n-1) & 0 & A(n-1) \end{matrix}$$

(b) (5 points) Define the Feigenbaum constant. Explain everything!

The Feiganbaum constant is

$$\lim_{k \to \infty} \frac{r_{k+1} - r_k}{r_k - r_{k-1}} = 4.6692016$$

Where r_k are the the values k where in the logistic map where $X_{n+1} = kX_n(1 - X_n)$ where k is the reproductivity constant, $k > 1$. If $1 < k < 3$ the long run population will stabilize. at $k > 3$ there will be cycle 2 in the long run. Eventually ther will be a k which has a cycle of 4, this pattern continues. Thus r_k will be the transition from period 2^{k-1} to 2^k. When k approaches ∞ the above ratio approaches Feigan baums constant of 4.692016.-

4. (10 points altogether)
(a) (2 points) Define a **Platonic soild**
(b) (2 points) Let a be the number of edges meeting each vertex, and let b be the number of edges surrounding each face. Express V (the number of vertices) and F (the number of faces) in terms of E (the number of edges), and a and b.
(c) (2 points) Find an expressions for F, in terms of a and b.
(d) (4 points) Obviously both a and b must be at least 3, and F (and hence V and E) must be positive. It is easy to see (you don't have to do it) that a, b must be both between 3 and 5, leaving 9 potential scenarios. Find those values of a and b that make sense, and thereby prove that there are exactly 5 Platonic solids. For each of them, find F (the number of faces) and give the name of the corresponding Platonic solid.

a) A Platonic solid is a solid in which all of its faces are identical regular Polygons, with the same number of edges/faces meet at each vertex.

b) Every edge must belong to 2 vertices:

$2E = aV \Rightarrow V = \frac{2E}{a}$

Every Edge has 2 faces of which it belongs to:

$bF = 2E \Rightarrow F = \frac{2E}{b}$

c) $V - E + F = 2$

$\frac{2E}{a} - E + \frac{2E}{b} = 2$

$E(a,b) = \frac{2}{2/a - 1 + 2/b}$

$F = \frac{2E}{b} = \frac{4}{(\frac{2}{a} - 1 + \frac{2}{b})b}$

d) $F(3,3) = \frac{4}{2-3+2} = 4$ Tetrahedron

$F(5,3) = \frac{4}{3(\frac{2}{5} - 1 + \frac{2}{3})} = 20$ Icosahedron

$F(3,4) = \frac{4}{4(\frac{2}{3} - 1 + \frac{2}{4})} = 6$ Hexahedron

$F(3,5) = \frac{4}{5(\frac{2}{3} - 1 + \frac{2}{5})} = 12$ Dodecahedron

$F(4,3) = \frac{4}{3(\frac{2}{4} - 1 + \frac{2}{3})} = 8$ Octahedron

$\cancel{F(4,4) = \frac{4}{4(\frac{2}{4} - 1 + \frac{2}{4})}} =$ Undefined

$\cancel{F(4,5) = \frac{4}{5(\frac{2}{4} - 1 + \frac{2}{5})}} < 0$

5. (10 points)
Prove Lagrange's theorem that if H is any subgroup of a group G, and $|H|$ and $|G|$ are their number of elements, respectively, then $|G|/|H|$ is always an integer.

~~It~~ Let g have m elements and H have n elements.
then $H = \{h_1 \ldots h_n\}$ so we let $H = G$, then $m/n = 1$ base case.
If $G \neq H$, then there exists an element $g_1 \in G$ not H. Let the left coset of $g_1 H = \{g_1 h_1 \ldots g_1 h_n)$ the members are all distinct
then for some $|G|/|H|$ must be an integer.

6. (10 points) What is the name of the following famous equation-pair?

$$u_x = v_y \quad , \quad u_y = -v_x \quad ,$$

or, in fuller notation

$$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y} \quad , \quad \frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x} \quad .$$

What is special about the function $u(x, y) + iv(x, y)$ where $u(x, y), v(x, y)$ satisfy the above system of two equations?

Cauchy-Riemann equations

This is significant because it allowed for the mapping X Y plane onto the U V plane with the introduction of complex numbers

7. (10 points) Who discovered the quaternions? What city did that person live in?

Hamilton

Dublin

8. (10 points) What is Heron's formula, what century did Heron live in?

Area of a triangle $A = \sqrt{s(s-a)(s-b)(s-c)}$

17th century

9. (10 points) Where did Isaac Newton study? Who was his teacher? What unusual action did that teacher do? What was Newton's position after he left Cambridge?

Cambridge

Isaac Barrow

Wardan of the Royal Mint

10. (10 points) In what city was Leibnitz born? Where did he spend most of his life? What King of England was once the employer of Leibnitz?.

Leipzig, Germany

Hanover

King George I

11. (10 points total)
(a) (5 points) State Viète's infinite product for $\frac{2}{\pi}$.

$$\frac{2}{\pi} = \frac{\sqrt{2}}{2} \times \frac{\sqrt{2+\sqrt{2}}}{2} \cdot \frac{\sqrt{2+\sqrt{2+\sqrt{2}}}}{2} \cdots$$

(b) (5 points) State the names of two people who initiated the use of logarithms

Napier + Briggs

12. (10 points altogether) (a) (3 points) Define a *Eulerian path* in a graph.

Eulerian path is a path where you can travel to each vertex using every edge exactly once and end at the different place from where you started.

(b) (3 points) State the necessary condition for a graph to have a Eulerian path

Every vertex must have degree that is an even number except for 2 vertexes, the starting vertex, and ending vertex.

(c) (4 points) Prove (or explain in your own words) why the condition in (b) is necessary.

This is because aside from the starting and ending vertex, you must travel into and out of the vertex this means that there must be 2n edges that meet at the vertex.

The starting vertex can not need to have even degree because you need to leave the vertex but you dont ever have to return to it.

The end vertex cannot have even degree because you only need to enter the vertex and not leave it.