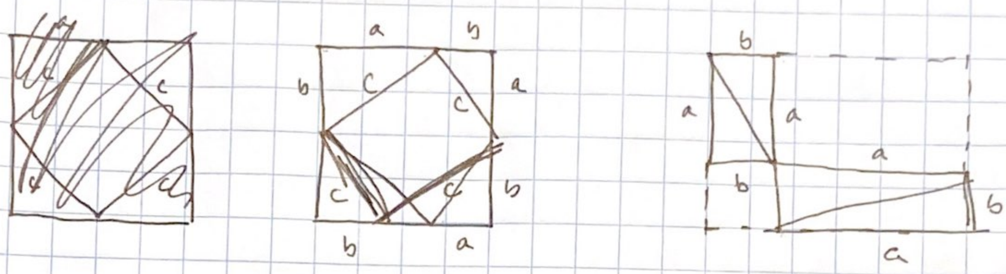
① Area decomposition of $(a+b) \times (a+b)$ square in:

1'st METHOD

$$a^2 + b^2 + 4\left(\frac{ab}{2}\right)$$

$$\Rightarrow c^2 + 4\left(\frac{ab}{2}\right)$$



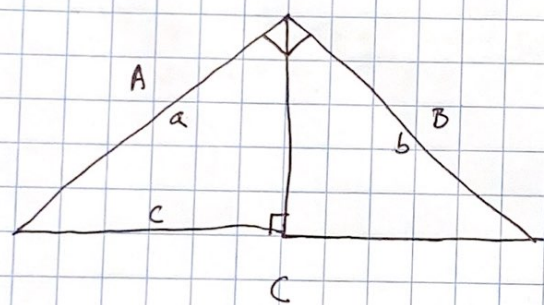

2ND METHOD

· using similar triangles by taking triangle ABC

s.t. $|AC| = b$

$|BC| = a$

$|AB| = c$



(2)

assume $\sqrt[7]{3}$ is rational.

Then there exists 2 positive integers s.t.

$$\left(\sqrt[7]{3}\right)^7 = \left(\frac{a}{b}\right)^7$$

$$3 = \frac{a^7}{b^7}$$

$$3b^7 = a^7$$

$$3b^7 = x$$

$a^7 \Rightarrow$ some integer

$\Rightarrow$ call this $x$

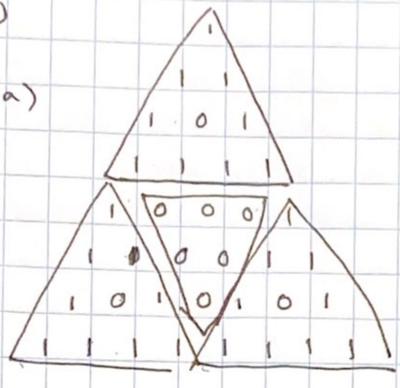
$$(3x)^7 = 3a^7$$

$$n^7 = 3a^7$$

$\Rightarrow$ ~~$n^7$ is a multiple~~ so $n$ is divisible by 7

(3)

(a)



(b) When $a$ is greater than 3 but less than 3.4494897, the period is 2. Period length is the number of unique values of $x_n$ when $n$ is large that it cycles. Let $a_n$ be the bifurcation parameter of which the period changes from $2^{n-1}$ to $2^n$. Then:

$a_1 = 3$

$a_2 = 3.4494897$

$a_3 = 3.5440903$

$a_4 = 3.564473$

the constant is:

$$\lim_{n \to \infty} \frac{a_{n-1} - a_{n-2}}{a_n - a_{n-1}} = 4.669201609$$

④

(a) A platonic solid is a solid where every face is a regular polygon that's the same size and shape. They are also convex and the same number of faces meet at each vertex. There are 5 platonic solids.

(b) $V = \frac{2E}{a}$  $F = \frac{2E}{b}$ $\Rightarrow$ $E = \frac{2ab}{2b - ab + 2a}$

(c) $F = \frac{2E}{b}$ $\Rightarrow$ $F = \frac{2}{b}\left[\frac{2ab}{2b - ab + 2a}\right]$

$$F = \frac{4ab}{2b^2 - ab^2 + 2ab}$$

(d)

| (a, b) | E | V | F | SOLID |
|---|---|---|---|---|
| (3, 3) ⇒ | 6 | 4 | 4 | Tetrahydron |
| (3, 4) | 12 | 8 | 6 | cube |
| (3, 5) | 30 | 20 | 12 | dodecahedron |
| (4, 3) | 12 | 6 | 8 | octahedron |
| (5, 3) | 30 | 12 | 20 | isocahedron |

⑤ · Let $aH = \{ah \mid h \in H\}$ be the left coset of $H$ containing $a$

* lemma: every coset of $H$ has the same number of elements as $H$

PROOF.

· Let $f: H \to aH$ such that $f(h) = ah$. If $f(h_i) = f(h_j)$ then $ah_i = ah_j$ so $h_i = h_j$ (injection)

· for all $h_k \in H$:

$$f(h_k) = ah_k$$

· every element is unique so every element in $aH$ is also unique (surjection)

$\Rightarrow f: H \to aH$ is a bijection

$\Rightarrow |aH| = |H|$

· Let $H$ contain $x$ elements and $G$ contain $y$ elements.

$\Rightarrow |H| = x$ and $|G| = y$

· Let $H, a_1H, a_2H, \ldots, a_{k-1}H$ be the distinct cosets of $H$

· from the previous proof, we know that each coset has the same cardinality as $H$

· Each element of $G$ appears in exactly one distinct coset of $H$. So:

$$y = xk$$

· $\Rightarrow x$ divides $y$ and $x, y, k$ are integers

· Therefore $|G|/|H|$ will also be an integer.

⑥

## Cauchy - Riemann Equations

- They have a real and imaginary parts of the equation which helps define harmonic ~~oscillator~~ functions
  - → Laplaces Equations (?)

⑦ William ~~Hamilton~~ Rowan Hamilton ; Dublin

⑧ Heron's Formula:

area of a triangle A $= \sqrt{s(s-a)(s-b)(s-c)}$ ; 1st century

⑨
- Studied in Cambridge
- teacher is Isaac Barrow
- the teacher "yeilded the Lucasian professorship to his pupil even" though he barely acknowledged Newton to be his superior.
- Became a warden and then master of the mint

⑩
- Leipzig (born)
- near the court of Hanover
- George I

⑪
(a) $\frac{2}{\pi} = \cos\frac{\pi}{4} \cos\frac{\pi}{8} \cos\frac{\pi}{16} \cos\frac{\pi}{32} \cdots$

(b) Neper ; Briggs

(12)

(a) A path ~~with~~ begins on a starting vertex and ends on an ending vertex while visiting each edge of the graph exactly one time.

(b) Every vertex, except for the starting and ending vertex, must have an even degree. The start and end vertices have an odd degree. So a path must have all vertices having even degrees except for two.

(c) This condition is necessary because the starting vertex only needs an exit edge without an associated entry edge. The ending vertex only needs an entry edge without the associated exit edge. This means that these two vertices have an odd degree. The other vertices that aren't the starting / ending point must have an exit edge ~~other~~ for each associated entry edge. This means that these vertices will have an even degree.