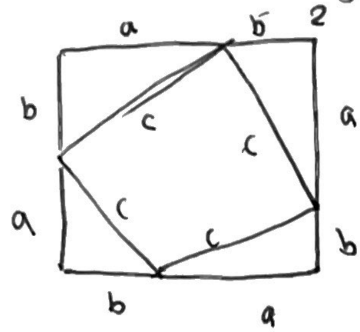
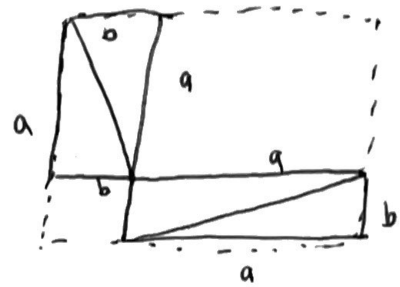
Exam 2

1 a) using the decomposition of an $(a+b) \times (a+b)$ square in

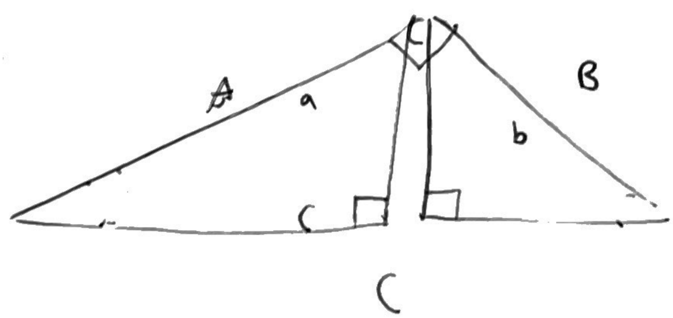
$\rightarrow a^2 + b^2 + 4 \cdot \left(\frac{ab}{2}\right)$

$\rightarrow c^2 + 4 \cdot \left(\frac{ab}{2}\right)$





b) using similar triangles by taking triangle ABC s.t. $|AC| = b$, $|BC| = a$, $|AB| = c$.



(2) Lemma: if n^7 is divisible by 3, n must also be divisible by 3.

If $\sqrt[7]{3}$ can be written as $\frac{m}{n}$,

where m and n are both divisible by 3, we can cancel out 3 till atleast one is not divisible by 3. There exists a pair of integers s.t. $\sqrt[3]{7} = \frac{m}{n}$, both are not divisible by 3.

$$3 = \frac{m^7}{n^7}$$

$$m^7 = 3n^7$$

$$m = 3a$$

$$(3a)^7 = 3n^7$$

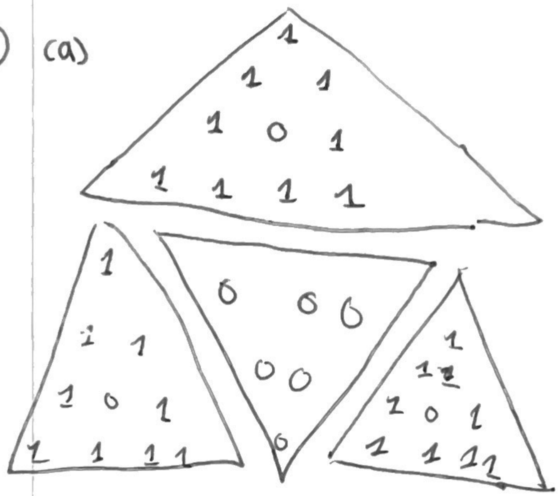
$$3^7 a^7 = 3n^7$$

$$n^7 = 3a^7$$

thus, n^7 is multiple of 7, so n is divisible by 7.

both n and m are so this is contradiction

③ (a)



* sorry my triangles look uneven *

(b) If a is greater than 3 but less than 3.449489 7, the period length is 2. Period lenghth is the amount of unique values of x_n when n is larger than its cycles. If a_n is the bifurcation parameter of period changes from 2^{n-1} to 2^n

$a_1 = 3$

$a_2 = 3.4494897$

$a_3 = 3.5440902$

$a_4 = 3.5644373$

$$\lim_{n \to \infty} \frac{a_{n-1} - a_{n-2}}{a_n - a_{n-1}} = 4.66920161$$

4 (a) platonic solid: every face is a regular polygon of the same size and shape. They are convex and the same number of faces meet at each vertex.

(b) $V = \frac{2E}{a}$, $F = \frac{2E}{b}$, so $E = \frac{2ab}{2b - ab + 2a}$

(c) $F = \frac{2E}{b}$ so $F = \frac{2}{b}\left(\frac{2ab}{2b - ab + 2a}\right)$

$$= \frac{4ab}{2b^2 - ab^2 + 2ab}$$

(d)

a	b	V	F	E	type
3	3	4	4	6	tetrahedron
3	4	8	6	12	cube
3	5	20	12	30	dodecahedron
4	3	6	8	12	octahedron
5	3	12	20	30	isocadedron

(5) Let $aH = \{ah \mid h \in H\}$ be the left coset of H containing a.

Lemma: Every coset of H has the same number of elements as H

proof: Let $F: H \to aH$ s.t. $f(h) = ah$. If $f(h_i) = f(h_j)$ $ah_i = ah_j$ so $h_i = h_j$ (an injection). For all h_k in H, $f(h_k) = ah_k$. Since every element is unique, so is every element in aH. This is a surjection, so, $f: H \to aH$ is a bijection and $|aH| = |H|$.

Let $|G| = n$ and $|H| = m$. Say $H, a_1H, \dots a_{k-1}H$ be the distinct cosets of H. Each coset has the same cardinality as H and each element of G only appears in exactly one coset so $n = mK$

m divides n, thus $|H|$ divides $|G|$

We explicity construct coset decompostions of G into cosets of H, each with the same # of elements, no overlaps. So, $|G|/|H|$ must be an integer.

(6) Cauchy-Riemann equations: solutions are analytic if $f = u(x, y) + iv(x, y)$

(7) William Rowan Hamilton, lived in Dublin

(8) Area of triangle $= \sqrt{s(s-a)(s-b)(s-c)}$
1st century (10 AD – 70 AD)

(9) Studied at Cambridge, teacher was Isaac Barron
The teacher yielded the Lucasian professorship to his student even though he hardly acknowledged him as his superior. Became a wonder master of the mind.

(10) Leipzig was birth city, spent most of his life near the court of Hanover, George I once employed him

(11) (a) $\frac{2}{\pi} = \cos\frac{\pi}{4} \cos\frac{\pi}{8} \cos\frac{\pi}{16} \ldots$

(b) Briggs and Napier

(12) (a) Eularian path: path that visits each edge in a graph exactly once, and it starts and ends on different vertices

(b) Every vertex has a degree that is even, exept two. Those two vertices start and end the path, and it has a degree that is odd.

(c) In a Eularian path P, you arrive at every vertex the same number of times you leave, except the starting and ending. If we call this value K, 2k edges have this vertex as its endpoint. The starting vertex leaves the path one more time than it enters (2k+1) and the ending vertex leaves the path one less time than it enters (2k-1). 2k is always even, 2k+1 and 2k -1 is always odd. Thus, (b) is true.