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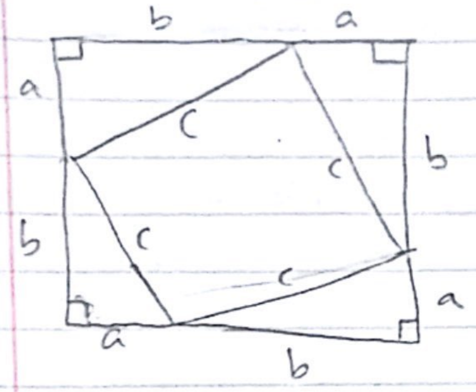
Exam 2

First proof



Area of big square $(a+b)^2$

area of small pieces

$\frac{1}{2}ab + \frac{1}{2}ab + \frac{1}{2}ab + \frac{1}{2}ab + c^2$

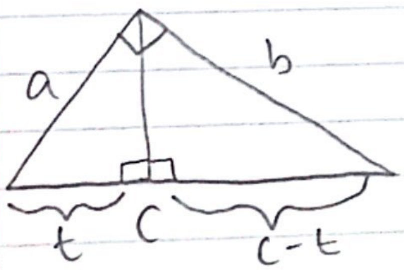
$= 2ab + c^2$

So $(a+b)^2 = 2ab + c^2$

$a^2 + 2ab + b^2 = 2ab + c^2$

$a^2 + b^2 = c^2$

Second proof



$\frac{t}{a} = \frac{a}{c} \rightarrow a^2 = ct$

$\frac{c-t}{b} = \frac{b}{c} \rightarrow b^2 = c^2 - ct$

Now $a^2 + b^2 = ct + c^2 - ct$

thus $a^2 + b^2 = c^2$

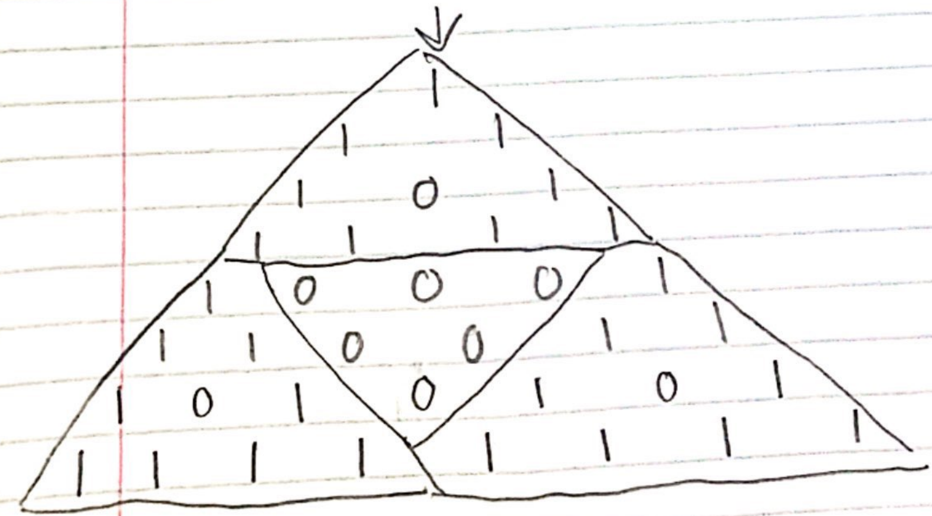
using similar triangles

2. Assume $\sqrt[7]{3}$ is rational then
$\sqrt[7]{3} = \frac{a}{b}$ where $b \neq 0$ and $\frac{a}{b}$ is in simpliest form.
Then we have $3 = \frac{a^7}{b^7}$ or $3b^7 = a^7$

Since a^7 is divisble by 3 then a must be divisible by 3. So let $a = 3 \cdot k$ for some $k \in \mathbb{Z}$
Now we have $3b^7 = (3k)^7 \rightarrow 3b^7 = 3^7k^7 \rightarrow b^7 = 3^6k^7$
Now we can see that b^7 is divisible by 3 so b is divisble by 3 but that contradicts that $\frac{a}{b}$ is in simpliest form since there is a common factor of 3 so $\sqrt[7]{3}$ is irrational.

3a.

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
1 6 15 20 15 6 1
1 7 21 35 35 21 7 1



3b. The feigenbaum constant is a Bifurcation parameter when $f(x) = ax(1-x)$ changes periods in powers of 2. It is useful for population change.

4a. A platonic solid is a polyhedron that has congruent faces and the same number of faces meet at each vertex

4b. Every edge belongs to two vertecies so
$2 \cdot E = a \cdot V$ and $2 \cdot E = b \cdot F$
so $V = \frac{2E}{a}$ and $F = \frac{2E}{b}$

4c. using $V - E + F = 2$
$$\frac{2E}{a} - E + \frac{2E}{b} = 2 \rightarrow E\left(\frac{2}{a} - 1 + \frac{2}{b}\right) = 2$$
$$\rightarrow E = \frac{2}{\frac{2}{a} - 1 + \frac{2}{b}}$$
$$\text{so } F = \frac{2\left(\frac{2}{\frac{2}{a} - 1 + \frac{2}{b}}\right)}{b}$$

4d. $a = 3,\ b = 3,\ \frac{2\left(\frac{2}{\frac{2}{3} - 1 + \frac{2}{3}}\right)}{3} = 4$ Tetrahedron

$a = 3,\ b = 4,\ \frac{2\left(\frac{2}{\frac{2}{3} - 1 + \frac{2}{4}}\right)}{4} = 6$ cube / hexahedron

$a = 4,\ b = 3,\ \frac{2\left(\frac{2}{\frac{2}{4} - 1 + \frac{2}{3}}\right)}{3} = 8$ octahedron

$a = 3,\ b = 5,\ \frac{2\left(\frac{2}{\frac{2}{3} - 1 + \frac{2}{5}}\right)}{5} = 12$ dodecahedron

$a = 5,\ b = 3,\ \frac{2\left(\frac{2}{\frac{2}{5} - 1 + \frac{2}{3}}\right)}{3} = 20$ Icosahedron

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5. Let G be a group with abstract multiplication denoted by $*$, and has the identity element e, and has a subgroup H. The left coset is $aH = \{a*h_1, a*h_2, \dots\}$ where $a \in G \setminus H$

Then $G = e*H \cup a_1*H \cup a_2*H \cup \dots \cup a_k*H$.

where $a_2 \in G \setminus H \wedge a_1*H$ and $a_3 \in G \setminus H \wedge a_1*H \wedge a_2*H$ and so on.

Then we get two lemmas, for any member a and b, either $a*H = b*H$ or $a*H$ has nothing to do with $b*H$. Secondly, $|a*H| = |b*H|$.

Using left-coset decomposition and the fact that there are the same number of elements in each coset and each coset has no intersection then $\frac{|G|}{|H|}$ must be an integer. thus

Langrange's theorem.

6. Cauchy-Riemann equations. Its real and imaginary part have to satisfy $u_x = v_y$, $u_y = -v_x$ in a given region, and have to satisfy certain conditions as to boundary and singularities

7. Hamilton, Dublin

8. $A = \sqrt{s(s-a)(s-b)(s-c)}$, 62 AD

9. Studied at Cambridge under Isaac Barrow. Barrow acknowledged Newton as his superior. Warden and then later master of the mint.

10. Born in Leipzig and spent most of his life near the court of Hanover. Under King George I.

11. $\frac{2}{\pi} = \cos\frac{\pi}{4}\cos\frac{\pi}{8}\cos\frac{\pi}{16}\cos\frac{\pi}{32}\ldots$

11b. John Neper and Henry Briggs

12a. A Eulerian path in a graph is made up of verticies and edges where you start at a vertex and visit each edge exactly once and end up at a different vertex then you started off with. and you can visit each vertex multiple times.

12b. Eulerian path - Every vertex must have an even degree meaning an even amount of edges that are connected to the vertex but the start and end vertex must have an odd degree

12c. For a Eulerian cycle, for each vertex you must go in and out but for a Eulerian path you must go out, in, out for the starting vertex and in, out, in at the ending vertex and the other vertex you're just stopping by so you're just going in and out. Of course this is for the simple case, we can have in, out, in, out, in etc as long as the start/end verticies have odd degree and the rest have even degree.