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MATH 437 Exam II for Dr. Z.'s, Fall 2021, Dec. 6, 2021, 3:00-4:20pm, (on-line)

No Calculators! No Cheatsheets! YOU MAY USE YOUR HISTORY NOTEBOOK (But not your Math Notebook).
Show your work! An answer without showing your work will get you zero points.

Do not write below this line (office use only)

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1. (out of 10)

2. (out of 10)

3. (out of 10)

4. (out of 10)

5. (out of 10)

6. (out of 10)

7. (out of 10)

8. (out of 10)

9. (out of 10)

10. (out of 10)

11. (out of 10)

12. (out of 10)

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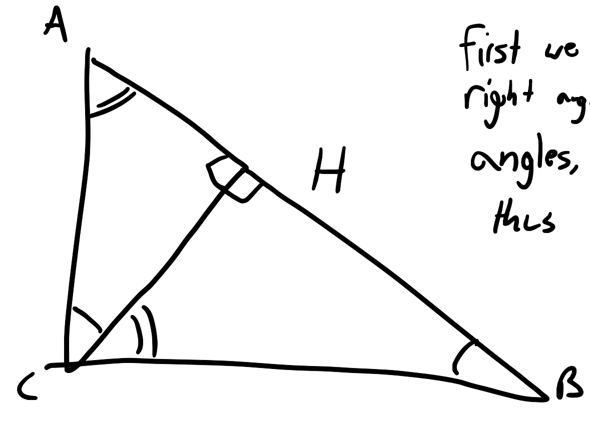
total: (out of 120)

1. (10 points) Give **two** proofs of the Pythagorean theorem.

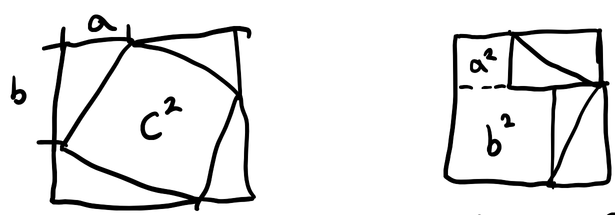


First we have a triangle ABC, with an right angle from C to the hypotnens H. Through similar angles, we say $\angle CAH = \angle BCH$ and $\angle ACH = \angle CBH$

thus $\frac{BC}{AB} = \frac{BH}{BC}$; $\frac{AC}{AB} = \frac{AH}{AC}$

$BC^2 = BH \cdot AB$ $\quad AC^2 = AB \cdot AH$

$$BC^2 + AC^2 = BH \cdot AB + AB \cdot AH$$
$$= AB(BH + AH)$$
$$= AB(AB)$$
$$BC^2 + AC^2 = AB^2$$



Thus $c^2 = a^2 + b^2$

2. (10 points) Prove that $\sqrt[7]{3}$ is irrational.

$$\sqrt[7]{3} = \frac{p}{q} \qquad q \neq 0$$

$$3 = \frac{p^7}{q^7}$$

implies

$$3q^7 = p^7$$

which is contradiction

3. (10 points total)
(a) (5 points) Construct the Pascal triangle mod 2 Fractal using the first 8 rows (i.e, the row for $n = 0$ through row for $n = 7$). Highlight the middle 0 section, and show that the remaining part consists of three identical triangles with 4 rows,

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
1 6 15 20 15 6 1
1 7 21 35 35 21 7 1

mod 2 Pascal

1
1 1
1 0 1
1 1 1 1
1 0 0 0 1
1 1 0 0 1 1
1 0 1 0 1 0 1
1 1 1 1 1 1 1 1

(b) (5 points) Define the Feigenbaum constant. Explain everything!

On the logistical map of $X_{n+1} = KX_n(1-X_n)$, where K is the reproduction constant. if $K<1$, population goes extinct, if $1<K<3$ the population will stabalize, and for $K>3$ it will have an ultimate period of 3, as K increases, it goes to ultimate period of 4, and further increase it will b 8. thus r_K will transition from period 2^{K-1} to 2^K

The $\lim_{n\to\infty} \frac{r_{K+1}-r_K}{r_K - r_{K+1}} = 4.669...$

4. (10 points altogether)
(a) (2 points) Define a **Platonic soild**
(b) (2 points) Let a be the number of edges meeting each vertex, and let b be the number of edges surrounding each face. Express V (the number of vertices) and F (the number of faces) in terms of E (the number of edges), and a and b.
(c) (2 points) Find an expressions for F, in terms of a and b.
(d) (4 points) Obviously both a and b must be at least 3, and F (and hence V and E) must be positive. It is easy to see (you don't have to do it) that a, b must be both between 3 and 5, leaving 9 potential scenarios. Find those values of a and b that make sense, and thereby prove that there are exactly 5 Platonic solids. For each of them, find F (the number of faces) and give the name of the corresponding Platonic solid.

a) Platonic Solid is a Polyhedron with perfect identical polygon faces

b) 1 edge per 2 vertexs, so $\frac{V}{2}$ and $\frac{E}{a}$ as $\frac{V}{2} = \frac{E}{a}$ gives vertexs, thus $V = \frac{2E}{a}$
1 edge per 2 faces in solid so $\frac{F}{2}$ and $\frac{E}{b}$ as this relation becomes $F = \frac{2E}{b}$

c) Using $V - E + F = 2$ $\quad E = \frac{F \cdot b}{2} \quad V = \frac{2\left(\frac{F \cdot b}{2}\right)}{a} = \frac{Fb}{a}$

$$\frac{Fb}{a} - \frac{Fb}{2} + F = 2 \qquad F = \frac{2}{\left[\frac{2b - ba + 2a}{2a}\right]} = \boxed{\frac{4a}{2b - ba + 2a}}$$

$$F\left[\frac{b}{a} - \frac{b}{2} + 1\right] = 2$$

d) $3 \leq a, b \leq 5$ $\quad$ F V E

$\checkmark (3,3) = \frac{12}{6 - 9 + 6} = \frac{12}{3} = 4 \; 4 \; 6$

$(3,4) = \frac{12}{8 - 12 + 8} =$ octa

$(3,5) = \frac{12}{10 - 15 + 10} = \frac{12}{5}$ icos

$(4,3) =$

$(4,4) =$

$(4,5) =$

$(5,3)$

$(5,4)$

$(5,5)$

5. (10 points)
Prove Lagrange's theorem that if H is any subgroup of a group G, and $|H|$ and $|G|$ are their number of elements, respectively, then $|G|/|H|$ is always an integer.

We take H to be subgroup of G, construct left cosets of H in G

that
$G = a_1 H + a_{2}h + \ldots + a_n H$ whor $a_i H \neq a_j H$ when $i \neq j$
All left cosets form a partition of G, using Lemma 2 they have cardinality and no cossets share terms, $|G| = |G : H| |H|$, fully devides. If G and H have same number of elements internally and none overlap, thus linear independent and $\frac{|G|}{|H|}$ is an integor

6. (10 points) What is the name of the following famous equation-pair?

$$u_x = v_y \quad , \quad u_y = -v_x \quad ,$$

or, in fuller notation

$$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y} \quad , \quad \frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x} \quad .$$

What is special about the function $u(x,y) + iv(x,y)$ where $u(x,y), v(x,y)$ satisfy the above system of two equations?

Partial differential equations, taking the derivative, relates flow of fluids

7. (10 points) Who discovered the quaternions? What city did that person live in?

William Hamilton in Dublin

8. (10 points) What is Heron's formula, what century did Heron live in?

$A = \sqrt{s(s-a)(s-b)(s-c)}$ 1st century

9. (10 points) Where did Isaac Newton study? Who was his teacher? What unusual action did that teacher do? What was Newton's position after he left Cambridge?

1) Cambridge

2) Isaac Barrow

3) let him take proffessorship

4) Warden

10. (10 points) In what city was Leibnitz born? Where did he spend most of his life? What King of England was once the employer of Leibnitz?.

Leipzig, spent life in Hanover, King George I

11. (10 points total)
(a) (5 points) State Viète's infinite product for $\frac{2}{\pi}$.

$$\frac{2}{\pi} = \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{2+\sqrt{2}}}{2} \cdot \frac{\sqrt{2+\sqrt{2+\sqrt{2}}}}{2} \cdot \frac{\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2}}}}}{2} \cdots$$

(b) (5 points) State the names of two people who initiated the use of logarithms

Henry Briggs + John Nappier

12. (10 points altogether) (a) (3 points) Define a *Eulerian path* in a graph.

A eulerian path is a path where each pathed is walked exactly once visiting each vertex

(b) (3 points) State the necessary condition for a graph to have a Eulerian path

The starting and ending vertex must be the only odd pair where all vertices must be in pairs even

(c) (4 points) Prove (or explain in your own words) why the condition in (b) is necessary.

The start and ending vertex is odd as it needs To leave or enter once, and can have added even paths. if not it will have path not travelled.

The even middle vertices need to be even to have equal incoming and outgoing paths. Ex