History of Math | Exam 2 | Karon Amin

1)

$\triangle ADC \sim \triangle ACB$

$$\frac{AC}{AB} = \frac{AD}{AC}$$

$a^2 = cd$

$b^2 = ce$

$\triangle BDC \sim \triangle BCA$

$$\frac{BC}{BA} = \frac{BD}{BC}$$

$b^2 = ce$

$a^2 + b^2 = cd + ce = c(d+e) = c^2$

$a^2 + b^2 = c^2$

2) Prove $\sqrt[2]{3}$ is irrational

Assum it is rational, hence we can write

$$3^{1/2} = \frac{m}{n} \quad \text{for} \quad m, n \in \mathbb{Z}$$

Then $3 = \frac{m^2}{n^2}$ or $3n^2 = m^2$ which means

3 divides m^2. If 3 divides m^2, then 3 divides

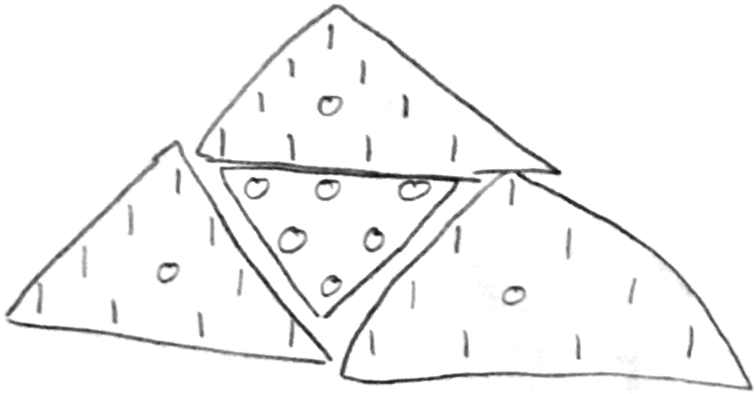
m since 3 is a prime number. Hence we

can write

$$m = 3k \quad \text{for som} \quad k \in \mathbb{Z}$$

Then $3n^2 = (3k)^2 \Rightarrow n^2 = 3k^2$ hence 3

3) a)



If the triangle above is $P(8)$ meaning a pascal triangle with 8 rows, then we can see that

$$P(8) = \begin{matrix} & P(4) & \\ & 0 & \\ P(4) & & P(4) \end{matrix} \qquad \text{where} \quad P(4) = \begin{matrix} & & & 1 & & & \\ & & 1 & & 1 & & \\ & 1 & & 0 & & 1 & \\ 1 & & 1 & & 1 & & 1 \end{matrix}$$

b) Assume we have some recurrence

$$x_{n+1} = a x_n (1 - x_n)$$

For different values of a, once x_n is large, it cycles between a set of numbers in order. This is the period.

When a is > 3 but less than $3.449...$, the period is 2. The bifurcation parameter for period 2 is 3. This is the first bifurcation parameter $a_1 = 3$. $a_2 = 3.449...$, which is when the period length jumps to 4. a_n is the bifurcation parameter of which the period changes from 2^{n-1} to 2^n. We define the feigenbaum constant as

$$\lim_{n \to \infty} \frac{a_{n-1} - a_{n-2}}{a_n - a_{n-1}} = 4.669...$$

4) Platonic solid is a convex polyhedron where
a) every face is the same [same number of edges surrounding it]
and every vertex has the same number of edges adjacent to it.

b) $V = \frac{2E}{a}$ and $F = \frac{2E}{b}$

c) We know $V - E + F = 2$

$$\frac{2E}{a} - E + F = 2 \Rightarrow \frac{2E}{a} - \frac{aE}{a} + F = 2$$

$$\frac{2E - aE}{a} + F = 2 \Rightarrow F = \frac{2a}{a} - \frac{2E - aE}{a}$$

$$\boxed{F = \frac{2a - 2E + aE}{a}}$$

d) Cube $[a, E] \to [3, 12]$

$$F = \frac{6 - 24 + 36}{3} = \frac{18}{3} = 6$$

[a, b]	V	E	F	Type
[3, 4]	8	12	6	cube
[3, 3]	4	6	4	Tetrahedron
[4, 3]	6	12	8	Octahedron
[3, 5]	20	30	12	Dodecahedron
[5, 3]	12	30	20	Icosohedron

5) Let $aH = \{ah \mid h \in H\}$

Then aH is the left coset of H which contains a.

We know ~~by~~ the lemma that every coset of H has the same number of elements as H

Let ther order of G $|G| = n$ and $|H| = m$ and let $H, a_1H, a_2H, \ldots, a_{k-1}H$ be the distinct cosets of H. We know that each coset must have the same number of elements as H, and that each element of G appears just once in any one coset. Therefore $n = mk$, $k \in \mathbb{Z}$, which means m divides n, therefore $|H|$ divides ~~___~~ $|G|$, hence why $|H|/|G|$ must be an integer.

6)

7) ~~Haml~~ William Rowon Homilton, lived in Dublin

8) Heron's formula for the area of a triangle

$$A = \sqrt{s(s-a)(s-b)(s-c)}$$

a, b, c are the lengths of a triangle

s is the semi-perimeter

$$s = \frac{a+b+c}{2}$$

Lived around 10 AD - 70 AD or the 1st century

9) University of Cambridge's Trinity College

Issac Barrow was his teacher

The teacher yielded the lucasion proffessorship to his pupil

He became Warden of the Royal Mint

10) - Leipzig, Germany
- Near the court of Hanover
- George I

11) a) $\frac{2}{\pi} = \cos\frac{\pi}{4} \cos\frac{\pi}{8} \cos\frac{\pi}{16} \cdots$

b) Briggs and Neper

12) a) A eulerian path is a path that visits every ~~vertex~~ edge exactly once and ends at a vertex

b) The necessary condition is for every vertex to have an even degree for an eulerian circuit, and for every vertex to have an even degree expect two start and end points in a eulerian path.

c) Because every vertex you visit in the middle of the path must have an ~~...~~ outgoing edge for every incoming edge, hence an even degree condition for all vertexes not including the start and end vertex